20th Australasian Fluid Mechanics Conference
Perth, Australia
5-8 December 2016

Optimal parameters in the inverse distance weighting method for mesh deformation

Xuan Zhou[1]

[1] Institute of Applied Physics and Computational Mathematics
Beijing, China

Abstract

Mesh deformation is of great importance to the numerical simulation of unsteady computational fluids involving moving boundaries. The Inverse Distance Weighting (IDW) method is a simple and direct interpolation approach for mesh deformation. The method has two exponential factors respectively on the translation and rotation terms, which significantly affect the deformation performance, but are usually determined by trials. We investigate the optimal parameters of the IDW method from two-dimensional numerical simulations with different mesh types, boundary shapes and deformation patterns. The results demonstrate that the optimal parameters are independent of mesh type and boundary shape, but are related to deformation pattern. The optimal factor of the translation term is in the range of [1, 2] for both rotational and translational movements, while the optimal factor of the rotation term is 2 for rotational movements, and tends to infinity for translational movements which means the rotation term can be neglected. These results should be guidance to the parameter selection in the IDW method, and is therefore very important to the performance of the IDW method.

Keywords

Inverse distance weighting, mesh deformation, dynamic meshes, optimal parameter, interpolation method

Introduction

Numerical simulations involving moving boundaries require a computational mesh moving together with boundary shapes, such as the fluid-structure interaction, aeroelastic computation and aerodynamic shape optimization. Mesh regeneration and interpolation operation of unstructured meshes are relatively expensive on computational cost. Therefore, mesh deformation methods have been widely used to adapt a computational mesh to a new displaced boundary without changing mesh connectivity. Various mesh deformation approaches found in literature can be generally sorted into the partial differential equation (PDE) method [1,2], physical analogy technique [3,4], algebraic method [5,6] and their combinations [7].

The PDE method is popular in mesh smoothing and simple to implement, but the mesh deformation ability of the method is quite limited. The physical analogy is the most prevalent deformation method for its physical foundation and general robustness, except it is computationally expensive. The algebraic techniques simply define the movement of gird nodes as a function of boundary motions, which compute the displacement of a volume node (the mesh node located in the computational domain except the boundaries) by its relative position from the boundaries of the computational domain. Recently, algebraic methods have been developed rapidly for their high efficiency and easy implementation, and can be classified into the Delaunay interpolation [8], the radial basis function interpolation [9,10] and the methods based on inverse distance weighting interpolation (IDW) [11]. The Delaunay interpolation method is based on the Delaunay mesh connecting boundary nodes, which is efficient but only effective for convex moving boundaries. The interpolation method of the radial basis function (RBF) is based on nodes rather than on meshes, and can be easily implemented in parallel [12]. However, an equation system with the scale of boundary nodes needs to be solved and different radial basis functions should be adopted in different applications. Moreover, some novel mesh deformation method have been developed in recent years, the sphere relaxation algorithm have been introduced to improve the deformed mesh quality of the boundary mesh [13].

The IDW method is a weighted average interpolation technique of scattered data points, which has been widely applied in computational graphics and geoscience for its high efficiency, simple implementation and parallelization. Lu [14] presented an adaptive IDW approach to further improve the performance. The explicit mesh deformation method based on the IDW was developed by Melville [15] and Allen [16], which is an interpolation using inverse distance weighting factors. The IDW method treats the deformation of a volume mesh as a projection of the displacement from boundaries into the volume mesh, which is also a node based method without considering the connectivity of elements. Moreover, the IDW method keeps the elements near deforming boundary (boundary mesh) move together with the boundary, which preserves high mesh quality near boundaries. The IDW method has attracted lots of attentions in recent years. McDaniel [17] applied a hybrid version of the methods developed by Melville and Allen to guarantee the quality of viscous meshes. Witteveen [18,19] used the IDW interpolation as a mesh optimization method to improve the orthogonality of the element adjacent to boundary structures. Luke [20] presented a tree-code optimization of the IDW to maintain the orthogonality of the boundary layer in viscous meshes and improve the algorithm efficiency.

Inverse Distance Weighting Method

The mesh deformation method based on the IDW has two basic forms. The first one [21] defined the displacement of a volume node as the weighted average of all boundary node displacements, which is simple and direct.

$$u_i = \frac{\sum_{j=1}^{N_d} w_j(r_{ij}) u_j}{\sum_{j=1}^{N_d} w_j(r_{ij})} \tag{1}$$

where u_i and u_j are the displacements of volume node i and j, respectively. N_d is the number of boundary nodes, $w_j(r_{ij})$ is the weighting factor which refers to the inverse distance from node i to node j and r_{ij} is the distance from node i to node j. However, the form of the IDW method requires a summation over all boundary nodes for each volume node, which makes the method inefficient for large scale meshes [22]. Furthermore, improvements are needed to guarantee the mesh quality for irregular and complicated mesh deformations.

The second form of the IDW method developed by Allen [23] treated the displacement of a volume node as a combination of the translation and rotation derived from the displacements of some reference nodes on boundaries, usually the closest nodes are chosen. The displacement of a volume node in the deformed mesh is given as following [15], which refers to one deformed or moving boundary (there may be many boundaries),

$$u_i = u_i^T \left(1.0 - \psi_i\right)^{st} + u_i^R \left(1.0 - \psi_i\right)^{sr} \tag{2}$$

where u_i^T, u_i^R are the translational and rotational displacement of node i, respectively, which are the weighting average displacements of the reference boundary nodes associated with node i, as respectively defined in eq.(3) and eq.(4). The reference boundary nodes associated with node i are illustrated in Fig.1, which usually refer to the closest boundary nodes (e.g. p_2, p_4) of all boundaries and their neighbor nodes (e.g. p_1, p_3, p_5, p_6). ψ_i describes the relative position of node i from boundaries and controls the extent of boundary deformations transferred to volume nodes. The specific expression of ψ_i can be found in Ref. [15].

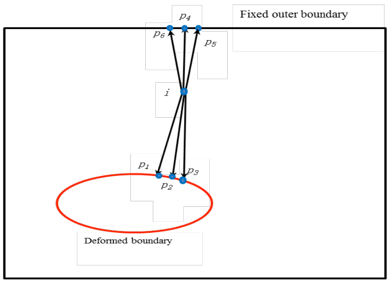


Fig.1An illustration of the reference boundary nodes of node i: the closest boundary nodes (p2, p4) and their neighbor nodes (p1, p3, p5, p6)

$$u_i^T = \sum_{j=1}^{Nc} \alpha_{ij} u_{ij} \tag{3}$$

$$u_i^R = \left[R_i - I\right]\left(x_i - \sum_{j=1}^{N_c} \alpha_{ij} x_{ij}\right) \tag{4}$$

The weighting factor α_{ij} is a reciprocal function of the distance from node i to boundary node j. [$\boldsymbol{R}_i$] is the rotation matrix of the reference boundary nodes associated with node i [15], [$\boldsymbol{I}$] is the identity matrix. N_c is the number of the reference boundary nodes associated with node i. u_{ij} is the displacement of the reference boundary node j associated with node i. $\boldsymbol{x}_i$ is the position vector of node i. $\boldsymbol{x}_{ij}$ is the position vector of the reference boundary node j associated with node i. The st and sr ($st > 0$, $sr > 0$) are the scaling exponential factors of translation and rotation components in eq.(2), respectively. These factors adjust the contributions of translation and rotation components to the displacement of each node. They also determine how and how far the boundary displacement spreads into the computational domain. Therefore, the values of st and sr directly affect the ability and quality of mesh deformation, and are very important to the performance of the IDW method. The two factors were previously given by several numerical attempts, and they were only specified for some typical cases in literature. Allen [16] proposed $2 \le sr, st \le 5$ as the range of optimal parameter values, and st = 2.0 and sr = 4.0 were chosen for all numerical examples. While Ji [21] suggested $sr, st \ge 2$ and $sr = 2^{6-\psi_j}$ which are different from Allen. Moreover, the ranges and values mentioned above cannot generally ensure the best mesh deformation in various numerical applications. Systematic and comprehensive investigations on the parameters have not been found yet.

In this work, the optimal values of st and sr in the IDW method are investigated systematically and comprehensively using numerical simulations, which involve different mesh types, boundary structures and deformation patterns. As the far field is assumed to be large enough in the numerical simulation, the impact of the shape of the far field boundary is not discussed in this work. In order to illustrate mesh deformation clearly, numerical simulations in two-dimension (2D) are carried out in this work. The optimal parameters are finally determined by considering both the ability and quality of mesh deformation. The results should be guidance to the parameter selection in the IDW method, and is therefore very important to the performance of the IDW method.

Optimal parameters of rotation and translation

In order to investigate the optimal parameters of rotation and translation systematically, three different boundary shapes and mesh types have been considered. Effects of the parameter values on both the quality and ability of mesh deformation have been discussed. The boundary structures in the examples in this section are rotated clockwise at their centers or aerodynamic centers, five degrees at each step. The optimal parameters for various boundary shapes have been investigated at first. Afterwards, the optimal parameters for the three mesh types have been studied as well.

Three typical boundary shapes with different aspect ratios are considered. The initial computational meshes are unequal and unstructured, as shown in Fig.2, which illustrates the boundary structures of a NACA0012 airfoil, a rectangle and a square.

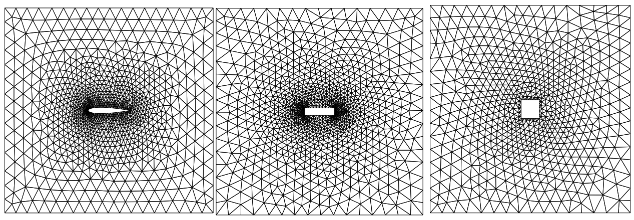


Fig.2 Initial computational meshes for different boundary shapes

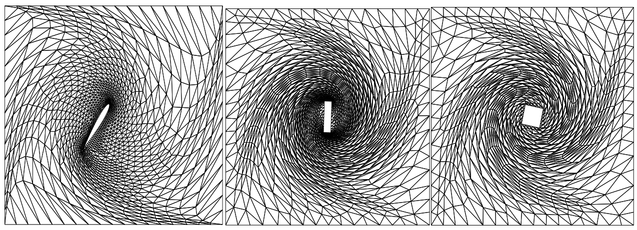


Fig. 3 The deformed meshes in the maximum rotation degrees

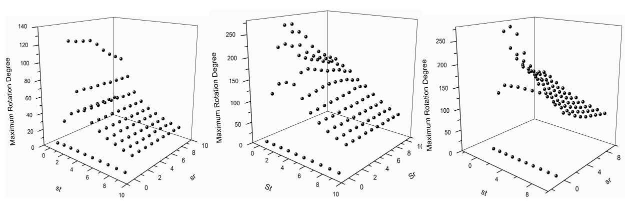
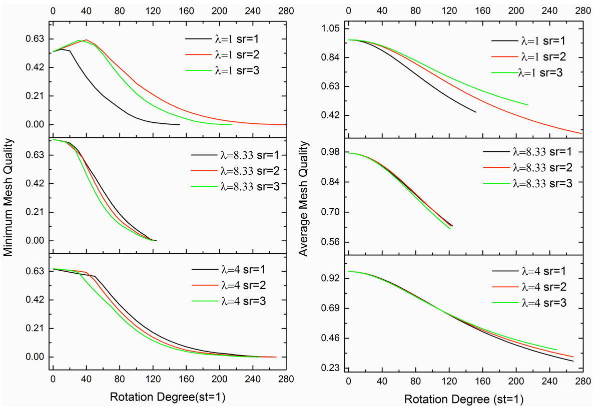
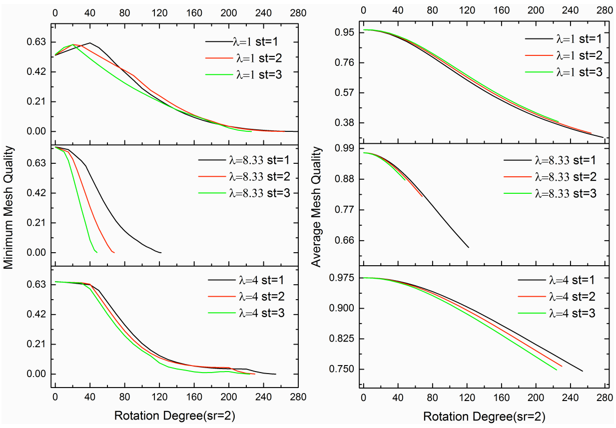


Fig. 4 The relationship between the parameters and the maximum rotation degree in different boundary shapes



(a) The average and minimum mesh quality (st=1)



(b) The average and minimum mesh quality (sr=2)

Fig. 5 The relationship between various st, sr and the mesh quality in different boundary shapes (rotational defromation)

Fig.5 (a) demonstrates the minimum and average quality of the computational meshes for different boundary shapes at *st* = 1. As the aspect ratio increasing, the variation between mesh qualities with different *sr* is decreased. The maximum rotation degrees are acquired at *sr* = 2 for all the three boundary shapes. Therefore, considering the mesh quality and maximum rotation degree simultaneously, the mesh deformation ability and quality are optimal at *st* = 1 and *sr* = 2. Fig.5 (b) depicts the minimum and average quality of the computational meshes for different boundary shapes when *sr* = 2. The variation of average mesh quality between different *st* is small. The optimal mesh deformation quality also has been achieved at *st* = 1 and *sr* = 2. Therefore, the mesh deformation ability and mesh quality are optimal at *st* = 1 and *sr* = 2 for all three boundary shapes**.**

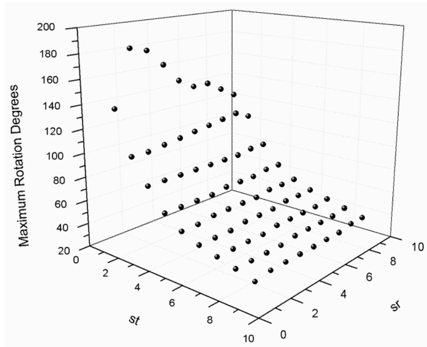
Subsequently, similar to the research of different boundary shapes, the effects of st and sr on the mesh deformation in different mesh types are investigated. Three typical mesh types which are structured mesh, equal unstructured mesh (the sizes of mesh elements are almost identical) and unequal unstructured mesh (the sizes of mesh elements are unequal) are considered.

In conclusion, the optimal parameters of rotation are sr = 2 and $st\in[1,2]$, which are independent of mesh type and structure boundary.

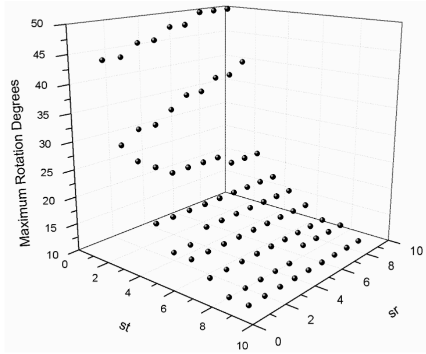
The optimal parameters of the combination of translation and rotation

Translation and rotation are two basic deformation forms in 2D, and any other deformation forms can be considered as a combination of translation and rotation. The effect of the parameters on the ability and quality of deformation in combined movements are also studied to determine the optimal parameters for general deformation forms.

The relationship between the parameter values and mesh deformation ability of two combined movements is depicted in Fig.6. In case (a), the boundary rotates 2° around the center and translates one unit upwards in each deformation step; In case (b), the boundary rotates 2° and translates 5 units length in each deformation step. The boundary structure is the same as in Fig.2 (b). The maximum rotation degrees are 180° and 50° for case (a) and (b), respectively.



(a) Rotates 2° and translates a unit length per step



(b) Rotates 2° and translates 5 units length per step

Fig.6The relationship between parameter values and the mesh deformation ability in combined movements

Fig.6 (a) shows that the largest rotation angle is reached at st = 1 and sr = 2. Note that the relationship between the parameters and maximum rotation degree is consistent with that of rotation. Therefore, the optimal parameters of combined movements approximate to that of rotation when the weight of rotation is larger than translation.

Fig.6(b) shows that the maximum rotation angle achieves the optimal value at st = 1, and keeps increasing with sr increasing. Moreover, the relationship between the parameters and maximum rotation degree is consistent with that of translation. Therefore, the optimal parameters of combined movements approximate to that of translation when the weight of translation is larger than rotation.

Similar to the investigations of translation and rotation, the optimal parameters of various boundary shapes and mesh types are studied as well. The results demonstrate that the optimal parameters of combined movements are related to those of rotation and translation individually, and are independent of boundary structures and mesh types.

In conclusion, the optimal parameters of combined movements fall in the range between that of translation and rotation individually. Specifically, the optimal parameters approximate to that of translation when the weight of translation is larger than rotation, and approximate to that of rotation when the weight of rotation is larger than translation.

Conclusions and discussion

The optimal parameters of the mesh deformation method based on the IDW interpolation are investigated, and the two basic deformation forms of rotation, translation and their combination are studied. The numerical simulations demonstrate that the optimal parameters are independent of boundary shape and mesh type, but depends on deformation form. The optimal deformation ability and mesh quality of rotation are achieved at sr = 2 and $st \in [1,2]$, which is different from the Ref. [21] in which $st \geq 2$ and $sr = 2^{6-\psi_j}$. The optimal deformation ability and mesh quality of translation are reached when st=1 and sr tends to infinite. The structured and equal unstructured meshes are suggested to weaken the translation component in rotation to achieve optimal mesh deformation ability.

Although only two deformation forms, i.e. rotation and translation, have been investigated in this work, other deformation forms can be decomposed into these two basic forms, and their optimal parameters can be determined by the optimal parameters in rotation and translation individually. The optimal value of st is in the range of [1, 2] for both translation and rotation. The rotation component is essential for rotation and could be ignored for translation, in other words, the value of sr is small for rotation while tends to infinite for translation. For the clarity and simplicity of 2D meshes, the optimal parameters in 2D mesh deformation have been studied. However, the optimal parameters in 2D can be used as reference values for the optimal parameters in 3D due to the fact that the IDW method is independent of mesh connectivity. These results should be guidance to the parameter selection in the IDW method, and is therefore very important to the performance of the IDW method.

Acknowledgements

This work was supported by the National Basic Research Program of China (Grant No. 2010CB832701), the National Natural Science Foundation of China (Grant No. 11272010) and the project 2012A0201011 of CAEP.

References

[1] C. Burg. Analytic study of 2D and 3D grid motion using modified Laplacian, International Journal for Numerical Methods in Fluids, 2006, 52(2): 163-197.

[2] S.L. Karman, M. Sahasrabudhe. Unstructured adaptive elliptic smoothing, in: 45th AIAA Aerospace Sciences Meeting and Exhibit, AIAA-2007-0559.

[3] F. J. Blom. Consideration on the spring analogy, International Journal for Numerical Methods in Fluids, 2000, 32(6): 647-668.

[4] K. Stein, et al. Automatic mesh update with a solid extension mesh moving technique, Computer Methods in Applied Mechanics and Engineering, 2004, 193: 327-333.

[5] S. Jakobsson, O. Amoignon. Mesh deformation using radial basis functions for gradient-based aerodynamic shape optimization, Computers & Fluids, 2007, 36(6): 1119-1136.

[6] X. Zhou, S.X. Li. A new mesh deformation method based on disk relaxation algorithm with pre-displacement and post-smoothing, Journal of Computational Physics, 2013, 235: 199-215.

[7] X. Zhou, S. Li, B. Chen. Spring-interpolation approach for generating unstructured dynamic meshes, Acta Aeronautica et Astronautica Sinica, 2010, 31(7): 1389-1395.

[8] N. Qin, X. Liu, H. Xia. An efficient moving grid algorithm for large deformation, Modern Physics Letters B, 2005, 19(28-29): 1499-1502.

[9] A. de Boer, M.S. van der Schoot, H. Bijl. Mesh deformation based on radial basis function interpolation, Computers and Structures, 2007, 85: 784-795.

[10] T.C.S Rendall, C.B. Allen. Efficient mesh motion using radial basis functions with data reduction algorithms, Journal of Computational Physics, 2009, 228(17): 6231-6249.

[11] A. S. W Jeroen. Explicit and robust inverse distance weighting mesh deformation for CFD, in: 48th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition, AIAA-2010-165.

[12] T.C.S. Rendall, C.B. Allen. Parallel efficient mesh motion using radial basis functions with application to multi-bladed rotors, International Journal for Numerical Methods in Engineering, 2010, 81(1): 89-105.

[13] X Zhou, S.X Li. A novel three-dimensional mesh deformation method based on sphere relaxation, Journal of Computational Physics, 2015, 298: 320-336.

[14] G.Y Lu, D.W. Wang. An adaptive inverse-distance weighting spatial interpolation technique, Computers & Geosciences, 2008, 34: 1044-1055.

[15] R. Melville. Nonlinear simulation of F-16 aeroelastic instability, in: 39th AIAA Aerospace Sciences Meeting, AIAA-2001-0570.

[16] C. B. Allen. Parallel universal approach to mesh motion and application to rotors in forward flight, International Journal for Numerical Methods in Engineering, 2007, 69(10): 2126-2149.

[17] D. R. McDaniel. Efficient mesh deformation for computational stability and control analyses on unstructured viscous meshes, in: 47th AIAA Aerospace Sciences Meeting including The New Horizons Forum and Aerospace Exposition, AIAA-2009-1363.

[18] J. A. Witteveen, H.Bijl. Explicit mesh deformation using Inverse Distance Weighting interpolation, in: 19th AIAA Computational Fluid Dynamics, AIAA-2009-3996.

[19] J. A. Witteveen. Explicit and robust Inverse Distance Weighting mesh deformation for CFD, in: 48th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposion, AIAA-2010-165.

[20] E. D. Luke, E. Collins, E. Blades. A fast mesh deformation method using explicit interpolation, Journal of Computational Physics, 2012, 231: 586-601.

[21] L. Ji, R. Wilson & K. Sreenivas. A parallel universal mesh deformation scheme, in: 28th AIAA Applied Aerodynamics Conference, AIAA 2010-4938.