Abstract
Mesh deformation is of great importance to the numerical simulation of unsteady computational fluids involving moving boundaries. The Inverse Distance Weighting (IDW) method is a simple and direct interpolation approach for mesh deformation. The method has two exponential factors respectively on the translation and rotation terms, which significantly affect the deformation performance, but are usually determined by trials. We investigate the optimal parameters of the IDW method from two-dimensional numerical simulations with different mesh types, boundary shapes and deformation patterns. The results demonstrate that the optimal parameters are independent of mesh type and boundary shape, but are related to deformation pattern. The optimal factor of the translation term is in the range of [1, 2] for both rotational and translational movements, while the optimal factor of the rotation term is 2 for rotational movements, and tends to infinity for translational movements which means the rotation term can be neglected. These results should be guidance to the parameter selection in the IDW method, and is therefore very important to the performance of the IDW method.

Keywords
Inverse distance weighting, mesh deformation, dynamic meshes, optimal parameter, interpolation method

Introduction
Numerical simulations involving moving boundaries require a computational mesh moving together with boundary shapes, such as the fluid-structure interaction, aeroelastic computation and aerodynamic shape optimization. Mesh regeneration and interpolation operation of unstructured meshes are relatively inefficient for large scale meshes [11]. The Inverse Distance Weighting Method (IDW) [12] is based on nodes rather than on meshes, and can be easily implemented in parallel [12]. However, an equation system with the scale of boundary nodes needs to be solved and different radial basis functions should be adopted in different applications. Moreover, some novel mesh deformation method have been developed in recent years, the sphere relaxation algorithm have been introduced to improve the deformed mesh quality of the boundary mesh [13].

The IDW method is a weighted average interpolation technique of scattered data points, which has been widely applied in computational graphics and geoscience for its high efficiency, simple implementation and parallelization. Lu [14] presented an adaptive IDW approach to further improve the performance. The explicit mesh deformation method based on the IDW was developed by Melville [15] and Allen [16], which is an interpolation using inverse distance weighting factors. The IDW method treats the deformation of a volume mesh as a projection of the displacement from boundaries into the volume mesh, which is also a node based method without considering the connectivity of elements. Moreover, the IDW method keeps the elements near deforming boundary (boundary mesh) move together with the boundary, which preserves high mesh quality near boundaries. The IDW method has attracted lots of attentions in recent years. McDaniel [17] applied a hybrid version of the methods developed by Melville and Allen to guarantee the quality of viscous meshes. Witteveen [18,19] used the IDW interpolation as a mesh optimization method to improve the orthogonality of the element adjacent to boundary structures. Luke [20] presented a tree-code optimization of the IDW to maintain the orthogonality of the boundary layer in viscous meshes and improve the algorithm efficiency.

Inverse Distance Weighting Method
The mesh deformation method based on the IDW has two basic forms. The first one [21] defined the displacement of a volume node as the weighted average of all boundary node displacements, which is simple and direct.

\[ u_i = \frac{\sum_{j=1}^{N_b} w_j(r_{ij})u_j}{\sum_{j=1}^{N_b} w_j(r_{ij})} \]  

(1)

where \( u_i \) and \( u_j \) are the displacements of volume node \( i \) and \( j \), respectively. \( N_b \) is the number of boundary nodes, \( w_j(r_{ij}) \) is the weighting factor which refers to the inverse distance from node \( i \) to node \( j \) and \( r_{ij} \) is the distance from node \( i \) to node \( j \). However, the form of the IDW method requires a summation over all boundary nodes for each volume node, which makes the method inefficient for large scale meshes [22]. Furthermore, improvements are needed to guarantee the mesh quality for irregular and complicated mesh deformations.

The second form of the IDW method developed by Allen [23] treated the displacement of a volume node as a combination of the translation and rotation derived from the displacements of
In this work, the optimal values of \( st \) and \( sr \) in the IDW method are investigated systematically and comprehensively using numerical simulations, which involve different mesh types, boundary structures and deformation patterns. As the far field is assumed to be large enough in the numerical simulation, the impact of the shape of the far field boundary is not discussed in this work. In order to illustrate mesh deformation clearly, numerical simulations in two-dimension (2D) are carried out in this work. The optimal parameters are finally determined by considering both the ability and quality of mesh deformation. The results should be guidance to the parameter selection in the IDW method, and is therefore very important to the performance of the IDW method.

Optimal parameters of rotation and translation

In order to investigate the optimal parameters of rotation and translation systematically, three different boundary shapes and mesh types have been considered. Effects of the parameter values on both the quality and ability of mesh deformation have been discussed. The boundary structures in the examples in this section are rotated clockwise at their centers or aerodynamic centers, five degrees at each step. The optimal parameters for various boundary shapes have been investigated at first. Afterwards, the optimal parameters for the three mesh types have been studied as well.

Three typical boundary shapes with different aspect ratios are considered. The initial computational meshes are unequal and unstructured, as shown in Fig.2, which illustrates the boundary structures of a NACA0012 airfoil, a rectangle and a square.

In this work, the optimal values of \( st \) and \( sr \) are

\[
u_i = u_i^t \left( 1.0 - \psi_i^t \right) + u_i^r \left( 1.0 - \psi_i^r \right)
\]

where \( u_i^t \) and \( u_i^r \) are the translational and rotational displacement of node \( i \), respectively, which are the weighting averages of the displacement of the reference boundary nodes associated with node \( i \), as respectively defined in eq.(3) and eq.(4). The reference boundary nodes associated with node \( i \) are illustrated in Fig.1, which usually refer to the closest boundary nodes (e.g. \( p_1, p_2, p_3, p_4 \)) of all boundaries and their neighbor nodes (e.g. \( p_5, p_6, p_7, p_8 \)). \( \psi_i^t \) describes the relative position of node \( i \) from boundaries and controls the extent of boundary deformations transferred to volume nodes. The specific expression of \( \psi_i^t \) can be found in Ref. [15].

The weighting factor \( \alpha_{ij} \) is a reciprocal function of the distance from node \( i \) to boundary node \( j \). \([R] \) is the rotation matrix of the reference boundary nodes associated with node \( i \) [15], \([I] \) is the identity matrix. \( N_i \) is the number of the reference boundary nodes associated with node \( i \). \( u_i^t \) is the displacement of the reference boundary node \( j \) associated with node \( i \). \( x_i \) is the position vector of node \( i \). \( x_j \) is the position vector of the reference boundary node \( j \) associated with node \( i \). The \( st \) and \( sr \) ( \( st > 0 \), \( sr > 0 \) ) are the scaling exponential factors of translation and rotation components in eq.(2), respectively. These factors adjust the contributions of translation and rotation components to the displacement of each node. They also determine how and how far the boundary displacement spreads into the computational domain. Therefore, the values of \( st \) and \( sr \) directly affect the ability and quality of mesh deformation, and are very important to the performance of the IDW method. The two factors were previously given by several numerical attempts, and they were only specified for some typical cases in literature. Allen [16] proposed \( 2 \leq sr, st \leq 5 \) as the range of optimal parameter values, and \( st = 2.0 \) and \( sr = 4.0 \) were chosen for all numerical examples. While Ji [21] suggested \( sr, st \geq 2 \) and \( sr = 2^{st} \) which are different from Allen. Moreover, the ranges and values mentioned above cannot generally ensure the best mesh deformation in various numerical applications. Systematic and comprehensive investigations on the parameters have not been found yet.

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Subsequently, similar to the investigations of translation and rotation, the optimal parameters of combined movements are related to those of rotation and translation individually, and are independent of boundary structures and mesh types.
In conclusion, the optimal parameters of combined movements fall in the range between that of translation and rotation individually. Specifically, the optimal parameters approximate to that of translation when the weight of translation is larger than rotation, and approximate to that of rotation when the weight of rotation is larger than translation.

Conclusions and discussion

The optimal parameters of the mesh deformation method based on the IDW interpolation are investigated, and the two basic deformation forms of rotation, translation and their combination are studied. The numerical simulations demonstrate that the optimal parameters are independent of boundary shape and mesh type, but depends on deformation form. The optimal deformation ability and mesh quality of rotation are achieved at \( s_r = 2 \) and \( s_f \in [1, 2] \), which is different from the Ref. [21] in which \( s_f \geq 2 \) and \( s_r = 2^{6-v} \). The optimal deformation ability and mesh quality of translation are reached when \( s_t = 1 \) and \( s_r \) tends to infinity. The structured and equal unstructured meshes are suggested to weaken the translation component in rotation to achieve optimal mesh deformation ability.

Although only two deformation forms, i.e. rotation and translation, have been investigated in this work, other deformation forms can be decomposed into these two basic forms, and their optimal parameters can be determined by the optimal parameters in rotation and translation individually. The optimal value of \( s_t \) is in the range of \([1, 2]\) for both translation and rotation. The rotation component is essential for rotation and could be ignored for translation, in other words, the value of \( s_r \) is small for rotation while tends to infinity for translation. For the clarity and simplicity of 2D meshes, the optimal parameters in 2D mesh deformation have been studied. However, the optimal parameters in 2D can be used as reference values for the optimal parameters in 3D due to the fact that the IDW method is independent of mesh connectivity. These results should be guidance to the parameter selection in the IDW method, and is therefore very important to the performance of the IDW method.

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References


