

## Computation of Power from an Array of Wave Energy Converters Using a Boundary Element Method

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### Abstract

The power from array of wave energy converters can be calculated theoretically from semi-analytic hydrodynamic added mass and damping coefficients of various devices (fixed or floating). However, the semi-analytic approach requires the body to be an axi-symmetric in shape. The newly released open source solver NEMOH is based on the Boundary Element Method (BEM); it solves problems of the interactions of bodies with water waves [1]. NEMOH can be used to calculate hydrodynamic coefficients as an alternative to the semi-analytic method. In contrast to the semi-analytic technique, one can model a body with arbitrary shape in BEM. In this paper, we compare the performance of NEMOH with the solution obtained by solving the boundary value problem exactly.

Similarly to the formulation leading to a BEM, in the semi-analytic method we formulate the problem of water-wave interaction with bodies using linear inviscid wave theory. The unknown velocity potential is then expanded in terms of eigenfunctions in various zones incorporating different boundary conditions in the problem domain. The unknown coefficients from the eigenfunction expansion are then expressed in a matrix by matching the hydrodynamic pressure and velocity across various zones. At this stage, the systems of equations are truncated to a finite number of terms. The number of terms required is obtained by examining the convergence of the solution.

We apply NEMOH to an array of two heaving devices. In this case we compare the solution from NEMOH with the formulation of based on multiple scattering. This formulation of multiple scattering is comprised of the wave diffraction and radiation problems separately where formulations have been adopted from Mavrakos and Koumoutsakos [2] and Mavrakos [6] respectively. We also compare the elapsed CPU time with increasing resolution in NEMOH with increasing number of terms in eigenfunction expansion.

### Introduction

Wave energy is the emerging technology field that seeks to harness the 2 TW of wave energy available on the planet's ocean shores [3] by use of Wave Energy Converters (WEC). Research effort in WEC arrays typically makes the simplification of potential flow, followed further by the subset of assumptions of linear wave theory. One "semi-analytical" technique formulated in [2] treats the array problem as a series of scattering events and can be used to solve the boundary value problem exactly. Meanwhile, BEMs have existed for decades and can be applied to solve

problems of the same nature. NEMOH is one such software, released in 2014 with the notable benefit of being open source. The work outlined examines the performance of NEMOH compared to solutions based on the eigenfunction matching technique for the cases of a single and two heaving (vertically moving) cylinders. This technique has been widely used for solving the complete hydrodynamic problem for a group of axi-symmetric floating bodies (for example see, Siddorn and Eatock Taylor [11]). From the resulting hydrodynamic coefficients, calculations for power and  $q$  factor are then compared in this paper. The paper forms part of a wider initiative to better understand WEC array performance.

### Hydrodynamic Theory of WEC Arrays

#### The Boundary Value Problem

Under the standard assumptions of linear wave theory, one seeks the solution of the Laplace's equation to obtain the hydrodynamic force on the floating body:

$$\Delta\Phi = 0. \quad (1)$$

A regular monochromatic wave of height  $\eta(x, t)$  travels along positive  $x$  and  $z$  is positive upwards. It is prevalent in the study of array hydrodynamics to then assume an impermeable seabed, a non-zero horizontal velocity,  $w$  (i.e., in 2D the velocity field being defined as  $\mathbf{V} = u\hat{i} + w\hat{j}$ ) at the uniform seabed, a smooth and unbreaking free surface and atmospheric free surface pressure, conveyed respectively as

$$\left. \frac{\partial\Phi}{\partial z} \right|_{z=0} = 0, \quad (2)$$

$$\left. \frac{\partial\eta}{\partial t} \right|_{z=\eta(x,t)} + u \left. \frac{\partial\eta}{\partial x} \right|_{z=\eta(x,t)} = w, \quad (3)$$

$$\rho \left. \frac{\partial\Phi}{\partial t} \right|_{z=\eta(x,t)} + \rho g \eta = 0. \quad (4)$$

We factor out the time dependent part of the velocity potential ( $\Phi$ ) and associated quantities (like the free surface elevation,  $\eta$ ) as  $\Phi(x, y, z, t) = \text{Re}\{\phi(x, y, z) \cdot e^{-i\omega t}\}$  and then seek the solution of the time-independent velocity potential from the above representation of the boundary value problem (i.e., equations (1) to (4)).

A WEC extracts power via its Power Take Off (PTO) mechanism which dictates its response to excitation. As is typical, we will assume WECs with PTOs and mooring systems inducing a net linear damping and restoring force.

#### WEC interactions in arrays

The vertically oriented, surface piercing truncated cylinders of uniform density that are analysed are representative of WEC devices which can be envisioned as large (several meters wide) heaving buoys.

As incident waves  $\phi_0$  pass through an array, the positioning of the devices relative to each other will cause a diffraction of the original wave  $\phi_d$ . This can be conveyed in terms of a force coefficient  $f_j$  in the  $j^{\text{th}}$  degree of freedom for each device. Secondly, in the radiation problem, a radiated wave  $\phi_{ij}$  emanates from the body depending on the device's allowed number of degrees of freedom  $J$ . Purely heave motions (in  $z$ ;  $J = 3$ ) will emit an axisymmetrical wave and purely surge motions (in  $x$ ;  $J = 1$ ) a bisymmetrical wave. Each radiated wave of a device will disturb the other devices which will in turn produce more radiated waves which disturb the originating devices and so on, in a multiple-scattering interaction. The net relation between each device and every other is conveyed in terms of added mass  $\mu_j$  and damping coefficients  $\lambda_j$  in the devices'  $j^{\text{th}}$  degree of freedom. The resulting potential will by superposition, be given as:

$$\phi = \phi_0 + \phi_d + \sum_{ij} \phi_{ij} \quad (5)$$

### Solution methods

We briefly describe the solution methodologies in each approach (viz., analytical and BEM) below.

#### Multiple Scattering and Eigenfunction expansion

The phenomena of diffraction and radiations from an array of finite number of WECs can be dealt in exact theory through the principle of multiple scattering also widely used in different areas of physics. For example, Mavrakos and Koumoutsakos [2] implemented the idea of multiple scattering, in order to find the wave excitation force imparted on different members in an array based on the diffraction problem of a single device. In this approach, the quantities associated with each order of scattering is obtained from the previous order of scattering and then the force and moment on any device in the array is found by combining all of these quantities from all order of scatterings. Prior to that, the fluid region beneath and surrounding a device are represented by different series of eigenfunction expansions in terms of unknown Fourier coefficients. The matching of velocity and pressure across the interface of these various regions gives the solution of these unknown coefficients in each order of scattering.

Using the above formulations, we can express the total velocity potential around a device in the array and in a cylindrical coordinate system as

$$\phi(r, \theta, z) = -i\omega \frac{H}{2} \sum_{m=-\infty}^{+\infty} i^m \psi_m(r, z) e^{im\theta}, \quad (6)$$

where,

$$\frac{1}{d} \psi_m(r, z) = \sum_{j=0}^{\infty} \left[ Q_{mj} \frac{I_m(\alpha_j r)}{I_m(\alpha_j b)} + F_{mj} \frac{K_m(\alpha_j r)}{K_m(\alpha_j b)} \right] Z_j(z), \quad (7)$$

$d$  is the water depth. With,

$$\begin{cases} Q_{mj} = \sum_{s=1}^{\infty} Q_{mj}^s \text{ and} \\ F_{mj} = \sum_{s=1}^{\infty} F_{mj}^s. \end{cases} \quad (8)$$

Here, we have three different indices namely,  $m$ ,  $j$  and  $s$  to sum over the coefficients. These are as follows.

- $m$  is the index of the Fourier series: in particular,  $m=0$  represents a monopolar (axisymmetric) wave, whereas,  $m = \pm 1$  represents a bipolar wave field.
- $j$  is the index of the vertical eigenfunctions  $Z_j(z)$ . These vertical eigenfunctions are in turn functions of roots ( $\alpha_j$ ) of the wave dispersion relation  $\omega^2 = gk \tanh(kd)$  for  $j=0$  and the transcendental equation  $\omega^2 + \alpha_j g \tan(\alpha_j d) = 0$  for  $j \geq 1$ .
- $s$  is the index of the scattering order.

Once the unknown Fourier coefficients in all fluid regions for each order of scattering are determined, the hydrodynamic force in a given direction (i.e., along  $r$  or  $z$ ) of the device can be obtained by integrating the pressure on the desired surface of the device.

Similar arguments can be extended to the radiation problems involving many devices as in Mavrakos [6]. The solution in this case depends on the solution of the radiation problem of a single device. But instead of splitting the region beneath the single cylindrical buoy in various number of ring elements (as been done in Mavrakos [6]) for solving the radiation problem for a single device, we use the integral formulation of Yeung [7].

#### The formulation based on Boundary Element Method (BEM) in NEMOH

In most of the cases the numerical investigations on hydrodynamic interactions among multiple floating objects are based on BEM. Prominent commercial BEM packages include WAMIT, ANSYS Aqwa and WAVE DYN. In late 2014, Ecole Centrale de Nantes released to the public their BEM called NEMOH which had previously been in private development for 30 years. Released as open source under Apache License 2.0, it is the first and currently only open source potential flow BEM applicable to the problem of WEC arrays.

As with all potential flow BEMs, it is necessary only to discretise the boundary of the domain of interest as opposed to the whole domain itself. Through Green's Second Identity, a so-called boundary integral equation can be formulated for a given problem, allowing solution for any point within the fluid surrounding the device(s). This procedure of reduction of the problem by one dimension greatly increases the efficiency of the calculation and is touted as the BEM's main advantage over the Finite Element Methods (FEM).

Details on the theoretical aspects of the NEMOH solver, including a discussion on the discretisation of the boundary and use of a numerical solution are given in [1].

#### Power absorption and array performance

Assuming that each device in the array is optimally damped, the overall power from the array is given by Evans [8] as

$$P_{max} = \frac{1}{8} \mathbf{X}^T \mathbf{B}^{-1} \mathbf{X}. \quad (9)$$

Where,  $\mathbf{X}$  is the row vector of complex excitation force coefficients of the form,  $\mathbf{X} = \{X_1, X_2, X_3, \dots, X_N\}$  and  $\mathbf{B}$  is the matrix of size  $(N \times N)$  of the damping coefficients,

$$\mathbf{B} = \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1N} \\ \vdots & \ddots & & \\ B_{N1} & & & B_{NN} \end{bmatrix}, \quad (10)$$

assuming there are  $N$  devices in the array. In general  $N$  represents the total number of modes of oscillations (maximum 6) in the literature. While the degrees of freedoms are restricted to 1 (i.e., in

the index 3 for the heave as in this paper),  $N$  becomes same as the total number of devices in the array.

The reciprocity relation tells us that for any two devices  $i$  and  $j$  in the array,  $B_{ij} = B_{ji}$ . The  $q$  or ‘interaction’ factor, as first defined in [5] for an array setup of a given device positioning, incident wave direction and incident wave frequency, conveys the extent to which the interactions are ‘constructive’ ( $q>1$ ) or whether they are ‘destructive’ ( $q<1$ ); and it is given by

$$q = \frac{P_{array,max}}{\sum_n P_{n,max}}, \quad (11)$$

where  $P$  is power. It is of interest to investigate this variation of the  $q$  factor as one changes the separation, the configuration of the array while the number of the devices is more than two and the direction of the incident wave. Many researchers have investigated these variations (for example see, Wolgamot et al. [10]). However, in this paper we only focus on the separation keeping our array layout to be very basic with only two devices subjected to incident wave in line with the array.

## Comparison of models

### Twin Buoy

We consider the basic array of only two devices to investigate the interaction phenomena in an array of WECs using the above-mentioned methodologies. It is apparent that the scattering phenomena can be overly complex with an increasing number of devices in the array, yet we use this most basic array set up to investigate the applicability of the adopted methodologies and then gain confidence for more complex set ups.

The schematic of the problem is shown in figure 1. In this paper we only consider the case for which the angle of wave incidence is defined by  $\beta = 0$ . Then we investigate the effect of changing the spacing ( $l$ ) on the interactions between twin buoys.

Figure 2 shows the comparison between NEMOH and multiple scattering (MS) based on eigenfunction expansion (EE) (abbreviated as MS-EE in the figures) of the excitation force coefficients in heave for  $l=5b$ . Then we present the non-dimensional added mass and damping coefficients for the same spacing in figure 3. In all cases, a good agreement is obtained. There are small discrepancies which are yet to be resolved. Both of the solutions from NEMOH and MS-EE are converged with the set of parameters as given in the following.

In all of these cases, we used 600 panels with all cylinders discretised into a 30-sided regular polygon. Whereas, in MS-EE, we used 7 terms for the Fourier coefficients ( $-3 \leq m \leq +3$ ), 7 order of scattering, 50 vertical eigenfunctions for diffraction and 20 vertical eigenfunctions for radiation problems.

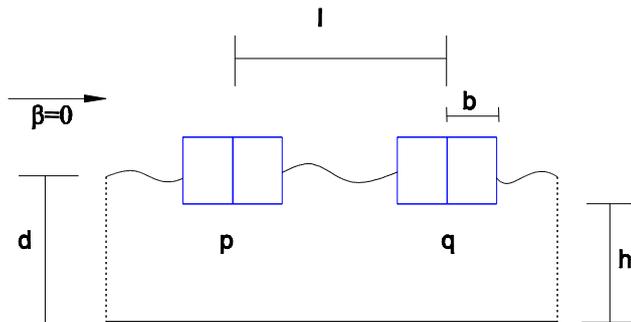


Figure 1. Schematic of array set up of twin buoys. In this paper,  $d=10.0b$ ;  $h = 9.5b$ .

Moreover, we found that by only considering the coefficients associated with  $m=0$ , we can gain a significant saving in CPU time while solving the radiation problem. Similar advantage can also be gained from NEMOH by restricting the degrees of freedom while calculating the wave excitation force coefficients.

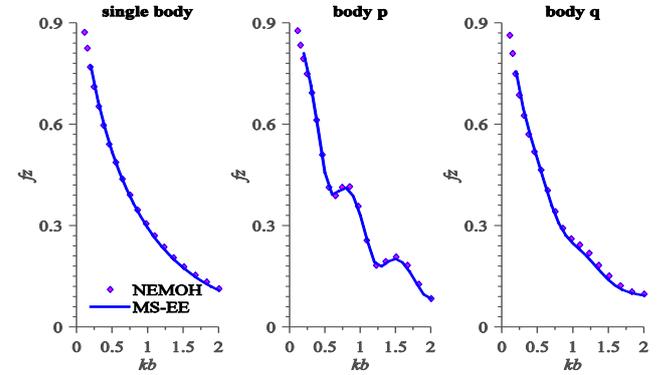


Figure 2. Comparison of the heave excitation force coefficients between NEMOH and eigenfunction expansion.

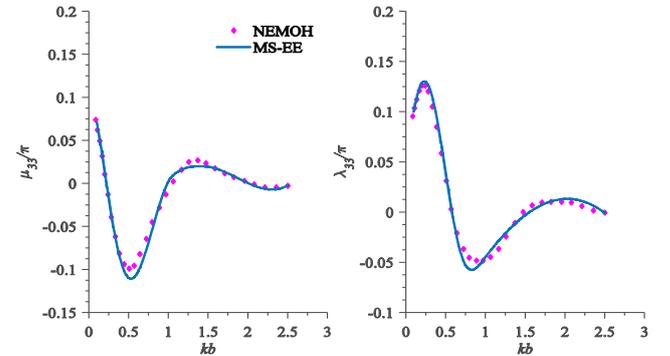


Figure 3. Comparison of added mass and damping coefficients for  $l=5b$  between NEMOH and eigenfunction expansion.

### Variation of $q$ factor

Finally we apply the formula of power absorption (equations 9-11) to investigate the  $q$  factor for two different spacing,  $l=5b$  and  $l=10b$ , in figure 4. We also include the prediction from the point absorber theory of Budal [5]. A satisfactory agreement is found in both cases.

We provide the elapsed CPU times for convergence studies in MS-EE and in NEMOH in table 1 for the diffraction problem. The convergence is studied in the MS-EE model in two stages: in one we keep the range of azimuth ( $m$ ) maximum changing the order of interactions (the 2<sup>nd</sup> major column) and in another we keep the number of interaction order ( $s$ ) maximum changing the azimuth range. In NEMOH the convergence is studied by changing only the number of panels on the surface of the body. Even though in all of the results in the paper we use the  $s=7$  and  $-4 \leq m \leq +4$  for a convergence upto 6 decimal points; our experience shows that convergence upto 2-3 decimal points can be achieved in MS-EE by even using  $s=3$  and  $-4 \leq m \leq +4$ . Under this set of input, with the converged result, NEMOH is still found to be slightly faster than MS-EE (5.9 m in NEMOH compared to 6.3 m in MS-EE).

Both of the MS-EE and NEMOH has been run in a desktop PC with Intel core i5-2500 3.30 GHz processor with 8 GB RAM.

As a final illustration, we provide the free surface elevation obtained from the MS-EE model by solving diffraction for a given wave frequency around a single isolated buoy in figure 5.

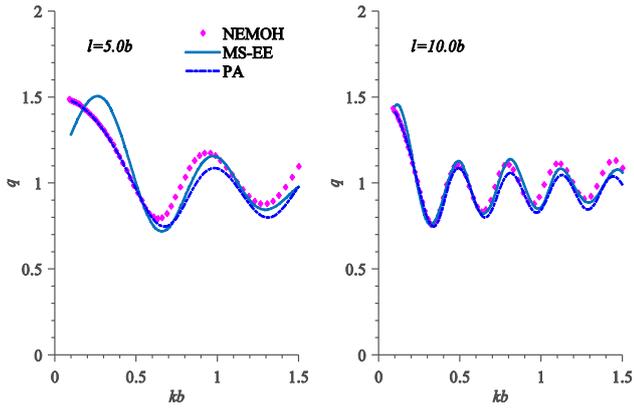


Figure 4. Comparison of  $q$  factor predicted by point absorber, multiple scattering and NEMOH.

MS-EE (upto 7 scattering order)		MS- EE ( $-4 \leq m \leq +4$ )		NEMOH	
$m$	Elapsed CPU (in minutes)	$s$	Elapsed CPU (in minutes)	Number of panels	Elapsed CPU (in minutes)
$-1 \leq m \leq +1$	4.7887	1	2.1078	500	0.15376
$-2 \leq m \leq +2$	8.2529	3	6.3146	1000	0.8877
$-3 \leq m \leq +3$	11.63065	5	10.0948	2000	5.9
$-4 \leq m \leq +4$	14.891	7	14.179		

Table 1. Details of elapsed CPU times in MS- EE and NEMOH with changing different parameters.

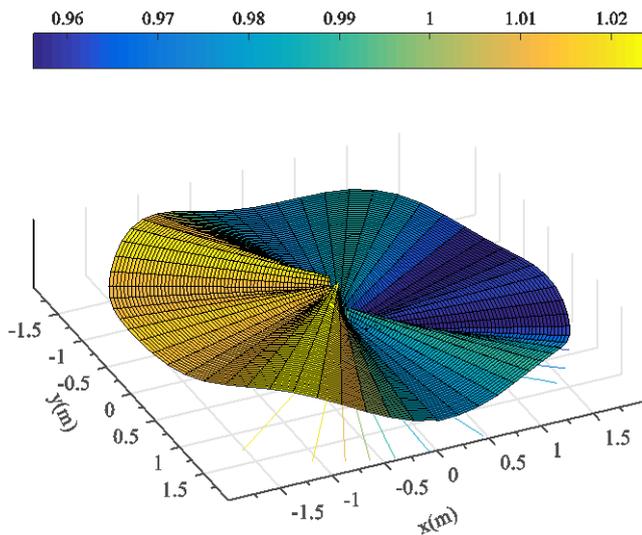


Figure 5. Scattered wave field around the isolated buoy obtained from MS- EE. The wave field is normalized by the incident wave amplitude ( $=0.01\text{m}$ ). The radius of the device ( $b$ ) is  $0.1\text{m}$  and all other parameters used as in Figure 1. The incident wave frequency is given by  $kb = 0.4$ .

## Conclusions

We compare the results from NEMOH (version 2.03) and the exact analytical theory based on linear wave theory applied to the case of a basic array of two WECs of heaving surface piercing truncated cylindrical devices. Past works which dealt with validation of the solution from NEMOH, mostly considered similar numerical models like WAMIT [9]. Here, we found that the predictions from NEMOH can as reliable as from the analytical model. On the other hand, we have found that the elapsed CPU time in NEMOH is slightly lower compared to the analytical model while obtaining the converged result. This study shows that NEMOH should also be applicable to arrays with more complex configurations.

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