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Free Convection Flow of Non-Newtonian Power-law Liquid Film with Nanoparticles along an Inclined Plate

B. Vasu[1], Rama S R Gorla[2], P V S N Murthy[3] and V. R. Prasad[4]

[1]Department of Mathematics
Motilal Nehru National Institute of Technology, Allahabad - 211004, India

[2]Department of Mechanical Engineering,
Cleveland State University, Ohio, 44115, USA

[3]Department of Mathematics,
Indian Institute of Technology, Kharagpur-721302, India.

[4]Department of Mathematics,
Madanapalle Institute of Technology and Science, Madanapalle-517325, India

Email: bvasu@mnnit.ac.in

Abstract

In the present paper, the first and second laws of thermodynamics are employed in order to study hydrodynamics along with heat and mass transfer of gravity-driven non-Newtonian Ostwald-de-Waele power law liquid film suspending with nanoparticles along an inclined plate. Revised Buongiorno's model is adopted for nanofluid transport on free convection of film flow. The model, which includes the effects of Brownian motion and thermophoresis, is revised so that the nanofluid particle fraction on the boundary is passively rather than actively controlled. Boussinesq approximation is considered to account for buoyancy. A convective boundary condition is employed which makes this study unique and the results are realistic and practically useful. The modeled boundary layer conservation equations are transformed to dimensionless, coupled and highly non-linear system of differential equations, and then solved numerically. The numerical results are presented graphically and discussed quantitatively for various values of thermo-physical parameters. Our results shows that, when the buoyant force on the heated fluid adjacent to the surface more than outweighs the gravitational force, the lighter fluid tends to raise and a flow reversal is observed in the vicinity of the surface. A comparison of the present results is made with the earlier published results and is found to be in good agreement.

Keywords: Film flow, Nanofluid, Free Convection, Power-law model, Brownian motion, Entropy analysis.

Introduction

A falling film is the gravity flow of a continuous liquid film down a solid tube having one free surface. Non-Newtonian nanofluid behavior is encountered in a great variety of everyday life as well as in industrial operations. By far the largest effort has been devoted to Newtonian fluid mechanics. Recently, modest attention has been devoted to gravity-driven thin film flow of the non-Newtonian nanofluids, as compared with its Newtonian counterpart. Non-Newtonian transport phenomena arise in many branches of chemical and materials processing engineering. Such fluids exhibit shear-stress-strain relationships which diverge significantly from the Newtonian (Navier-Stokes) model. Most non-Newtonian models involve some form of modification to the momentum conservation equations. These include power law, thixotropic and viscoelastic fluids. Such rheological models however cannot simulate the microstructural characteristics of many important liquids including polymer suspensions, liquid crystal melts, physiological fluids, contaminated lubricants etc.

In the development of energy-efficient heat transfer fluids, the thermal conductivity of the heat transfer fluids plays a vital role. Despite considerable research and development efforts on heat transfer enhancement, major improvements in cooling capabilities have been constrained because traditional heat transfer fluids used in today's thermal management systems, such as water, oils, and ethylene glycol, have inherently poor thermal conductivities, orders-of magnitude smaller than those of most solids. A method of improving heat transfer rates is to use solid particles in the base fluids. Nanofluids are engineered by suspending nanoparticles with average sizes below 100 nm in traditional heat transfer fluids such as water, oil, and ethylene glycol. *Nanofluids* (*nano*particle *fluid* suspensions) is the term coined by Choi [5] to describe this new class of nanotechnology-based heat transfer fluids that exhibit thermal properties superior to those of their host fluids or conventional particle fluid suspensions. Nanofluids are uniformly stable and suspended in a liquid for heat transfer intensification, in industrial sectors including power generation, thermal therapy for cancer treatment, chemical sectors, ventilation etc.

A comprehensive survey of convective transport in nanofluids was made by Buongiorno [1], who considered seven slip mechanisms that can produce a relative velocity between the nanoparticles and the base fluid: inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity. Of all of these mechanisms, only Brownian diffusion and thermophoresis were found to be important. An analytic study on the onset of convection in a horizontal layer of a porous medium with the Brinkman model and the Darcy model filled with a nanofluid was presented by Kuznetsov and Nield [11]. Godson et al. [7] presented the recent experimental and theoretical studies on convective heat transfer in nanofluids, their thermophysical properties and applications, and clarified the challenges and opportunities for future research. Chamkha et al [3] analyzed Natural Convective Boundary Layer Flow over a Sphere Embedded in a Porous Medium Saturated with a Nanofluid. Gorla et al [8] studied the mixed Convection Flow of Non-Newtonian fluid from a Slotted Vertical Surface with Uniform Surface Heat Flux. Recently, Chamkha et al [4] considered the unsteady free convective boundary layer flow of a nanofluid over a vertical cylinder. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. Ram Reddy [16] considered the effect of Soret parameter on mixed convective flow along a vertical plate in a nanofluid under convective boundary condition. A clear picture about the nanofluid boundary layer flows is still to emerge. Recently Nayak [13] studied Soret and Dufour effects on mixed convection unsteady MHD boundary layer flow over stretching sheet in porous medium with chemically reactive species. On the other hand, the impetuous research on convective flow in porous media is surveyed in the books by Neild and Bejan [14], Pop and Ingham [15] and *Incropera* and DavidP. *Dewitt [6]*. As with Newtonian nanofluid dynamics, non-Newtonian modelling of nanofluid transport phenomena has also attracted significant attention very recently. An integral approximate solution for the boundary layer equations in the case of a power-law type non-Newtonian laminar falling film was provided by Murthy and Sarma [12]. Heat transfer from an inclined plane to non-Newtonian fluid falling films was studied both theoretically and experimentally by Stucheli and Widmerfl [20]. Gorla and Nee [9] analyzed the entrance region heat transfer to a laminar, non-Newtonian falling liquid film with a fully developed velocity profile by perturbation solution. The recent book by Shang [17] systematically summarized the research results in free convection film flows and heat transfer. The objective of present work is to model fluid flow and heat transfer of a laminar non-Newtonian falling liquid film filled by a nanofluid.

Mathematical Model:

Consider the accelerating laminar flow of a non-Newtonian power-law liquid film suspending nanoparticles down along an inclined plane surface, as shown schematically in Fig.1. The incompressible and inelastic fluid is assumed to obey the Ostwald-de-Waele power law model and the action of viscous stresses is confined to the developing momentum boundary layer adjacent to the solid surface. The non-Newtonian nanofluid model incorporates the effects of Brownian motion and thermophoresis. It is also assumed that all the fluid properties are constant except that of the influence of the density variation with temperature and the nanoparticles volume faction in the body force term (Boussinesq's approximation).

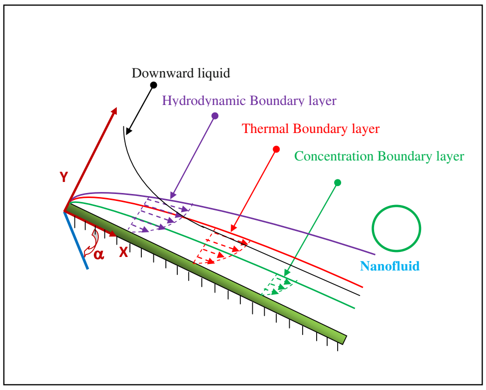

Figure 1: Physical model and coordinate system

The basic conservation equations for mass, momentum, energy and nanoparticle volume fraction in non-dimensional form within the viscous boundary layer are:

$$\frac{n}{n+1}\eta\frac{\partial U}{\partial \eta}-U-2\frac{\partial V}{\partial \eta}=0 \qquad (1)$$

$$2n\left(\frac{\partial U}{\partial \eta}\right)^{n-1}\frac{\partial^2 U}{\partial \eta^2}+\frac{n}{n+1}\eta U\frac{\partial U}{\partial \eta}-2V\frac{\partial U}{\partial \eta}-U^2+1=0 \qquad (2)$$

$$\frac{1}{\Pr_x}\frac{\partial^2\theta}{\partial\eta^2}+\frac{n}{2(n+1)}\eta U\frac{\partial\theta}{\partial\eta}+Nb_x\frac{\partial\phi}{\partial\eta}\frac{\partial\theta}{\partial\eta}-V\frac{\partial\theta}{\partial\eta}+Nt_x\left(\frac{\partial\theta}{\partial\eta}\right)^2=\xi U\frac{\partial\theta}{\partial\xi} \qquad (3)$$

$$\frac{1}{Le_x}\frac{\partial^2\phi}{\partial\eta^2}+\frac{n}{2(n+1)}\eta U\frac{\partial\phi}{\partial\eta}-V\frac{\partial\phi}{\partial\eta}+\frac{1}{Le_x}\frac{Nt_x}{Nb_x}\left(\frac{\partial^2\theta}{\partial\eta^2}\right)=\xi U\frac{\partial\phi}{\partial\xi} \qquad (4)$$

Subject to the boundary conditions:

$$U=0,\ V=0\quad \frac{\partial\theta}{\partial\eta}=-\gamma(1-\theta),\ Nb_x\frac{\partial\phi}{\partial\eta}+Nt_x\frac{\partial\theta}{\partial\eta}=0 \quad \text{at}\ \eta=0 \qquad (5)$$

$$U\to 1,\qquad \theta\to 0,\quad \phi\to 0\quad \text{as}\qquad \eta\to\infty$$

where $\Pr_x=\frac{x u_{x,\infty}}{\alpha}\mathrm{Re}_x^{\frac{-2}{n+1}}$ is the local Prandtl number, $Le_x=\frac{x u_{x,\infty}\mathrm{Re}_x^{\frac{-2}{n+1}}}{D_B}$ is the local Lewis number, $Nb_x=\frac{\tau D_B(C_w-C_\infty)}{x u_{x,\infty}\mathrm{Re}_x^{\frac{-2}{n+1}}}$ is the local Brownian motion parameter, $N_t=\frac{\tau D_T(T_w-T_\infty)}{T_\infty x u_{x,\infty}\mathrm{Re}_x^{\frac{-2}{n+1}}}$ is the local thermophoresis parameter, $\alpha=\frac{k}{(\rho C)_f}$, $\tau=\frac{(\rho C)_p}{(\rho C)_f}$, g is the acceleration due to gravity, ρ is the fluid density, κ is the thermal conductivity, μ is the viscosity of the fluid, T and C are the temperature and concentration, ρ_f is the density of the base fluid and ρ_P is the density of the nanoparticles. $D_B$and D_T are the Brownian diffusion coefficient and the thermophoresis is diffusion coefficient, respectively, $(\rho C)_f$ and $(\rho C)_P$ are the heat capacity of the base fluid and the effective heat capacity of the nanoparticles material, respectively. α is the thermal diffusivity, n power law index respectively. The deviation of n from unity indicates the degree of deviation from Newtonian rheology and the particular case n = 1 represents a Newtonian fluid with dynamic coefficient of viscosity K

Here, $u_{x,\infty}=\sqrt{2xg\cos\alpha}$

The above equations (1) to (5) can be further non-dimensionalized using new variables:

$$\xi=\frac{x}{x_0},\quad \eta=\frac{y}{x}\mathrm{Re}_x^{\frac{1}{n+1}},\quad U=\frac{u}{\sqrt{2gx\cos\alpha}},\qquad \mathrm{Re}_x=\frac{x^n u_{x,\infty}^{2-n}\rho}{\kappa}$$

$$V=\frac{v}{\sqrt{2gx\cos\alpha}}\mathrm{Re}_x^{\frac{1}{n+1}},\quad \theta(\xi,\eta)=\frac{T-T_\infty}{T_f-T_\infty},\ \phi=\frac{C-C_\infty}{C_w-C_\infty},$$

Where ξ - tangential coordinate, η - coordinate, U,V - velocities in x and y directions respectively, Re_x - local Reynolds number, θ,ϕ - non dimensional temperature and nanofluid volume fraction respectively

Entropy Analysis

In the nanofluids flows, the improvement of the heat transfer properties causes a reduction in entropy generation. However, a convection process involving a liquid film flow of nanofluids is inherently irreversible. According to Woods, the local volumetric rate of entropy generation (Ns) number in dimensionless form is given by:

$$N_s = (\theta')^2 + \frac{Br_x}{\Omega}(U')^{n+1} + \lambda\left(\frac{\varsigma}{\Omega}\right)^2(\phi')^2 + \lambda\left(\frac{\varsigma}{\Omega}\right)\theta'\phi' \qquad (6)$$

Where the dimensionless parameters $Br_x = \frac{\mu\, u_{x,\infty}^{n+1}(x\,\mathrm{Re}_x^{-1/n+1})^{1-n}}{k\left(T_f - T_\infty\right)}$, $\Omega = \frac{\left(T_f - T_\infty\right)}{T_\infty}$, $\varsigma = \frac{\left(C_w - C_\infty\right)}{C_\infty}$, $\lambda = \frac{D_B C_\infty}{k}$ are the Brinkman number, liquid film height, temperature, nanoparticle volume fraction different parameters respectively

Results and Discussion:

The nonlinear differential Equations (1) - (4) subject to the boundary conditions (5) have been solved numerically by using Matlab bvp4c solver for some values of thermophysical parameters, and discussed the effects of various physical parameters on the velocity, temperature, nanoparticle concentration fields and entropy generation. The resulting non-similarity solutions for the dimensionless velocity, temperature and nanofluid volume fraction are displayed in Fig. 2-8. In order to assess the accuracy of the numerical solution, we tabulated results of $dU/d\eta\, and\, d\theta/d\eta$ for different values of η when n = 0.5, Pr_x = 10, Nb_x = Nt_x = Lb_x = 0. A comparison of the present results with the local non-similarity solution as reported by Shang and Anderson [18] shown in Table 1 and urge the present results are excellent coincide. Therefore, we believe that the comparison supports very well validity of the present results.

Table1: *Comparison of* $dU/d\eta\, and\, d\theta/d\eta$ for different values of η when n = 0.5, Pr_x = 10, Nb_x = Nt_x = Lb_x = **γ =** 0.

η	Shang and Anderson [18]		Present Results	
	$dU/d\eta$	$d\theta/d\eta$	$dU/d\eta$	$d\theta/d\eta$
0	1.104406	-1.139345	1.104409	-1.139346
0.1	1.001948	-1.137857	1.001952	-1.137859
0.2	0.905206	-1.127787	0.905208	-1.127789
0.3	0.814695	-1.101675	0.814696	-1.101677
0.4	0.730722	-1.054052	0.730727	-1.054055
0.5	0.653416	-0.982272	0.653418	-0.982275
0.6	0.582748	-0.887155	0.582750	-0.887158
0.7	0.518561	-0.773095	0.518564	-0.773099
1.0	0.361926	-0.397539	0.361929	-0.397541
1.6	0.17416	-0.026229	0.174163	-0.026232
2.0	0.108331	-0.001286	0.108332	-0.001286
2.5	0.061694	-0.000007	0.061695	-0.000007

Here, ($f(\eta_\delta)$ is the value of the normalized stream function at the outer edge of the boundary layer, and the dimensionless boundary layer thickness η_δ is defined as the value of η for which the normalized velocity in Fig. 2 becomes equal to 0.99.

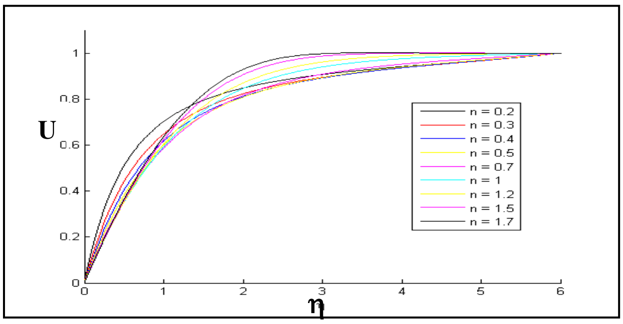

Figure 1 Effect of n on velocity distribution

From Fig 3, we observed that the temperature decreases as n increase i.e. the thermal boundary layer become thinner while power law index increased. Also the same trend has been observed for the nanofluid volume fraction for increase in n.

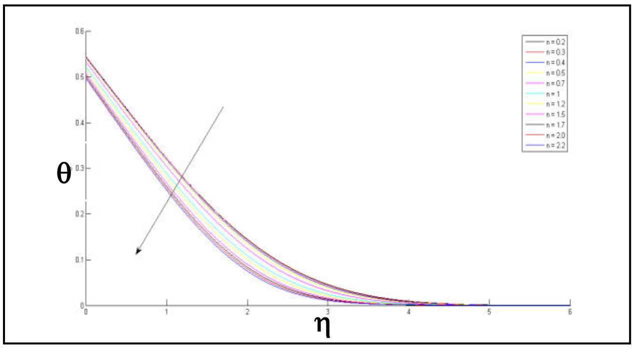

Figure 2 Effect of n on temperature distribution

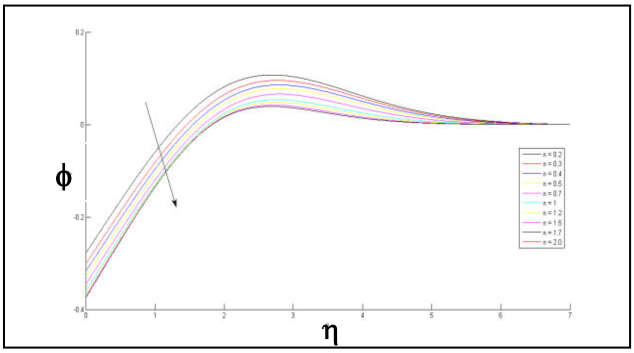

Figure 3 Effect of n on nanofluid volume fraction distribution

Figures 5 and 6 depicts the variation of the effect of the convective parameter (**γ)** on temperature (θ) and nanofluid volume fraction (ϕ). Fig. 5 it is observed that for larger amounts of the **γ**, θ is larger and the **γ** has a thickening effect on the thermal boundary-layer.

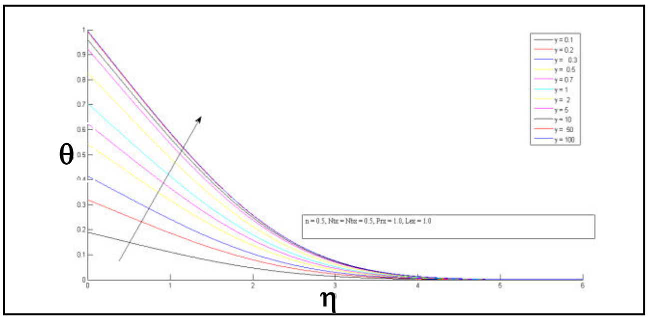

Figure 4 Effect of γ on temperature distribution

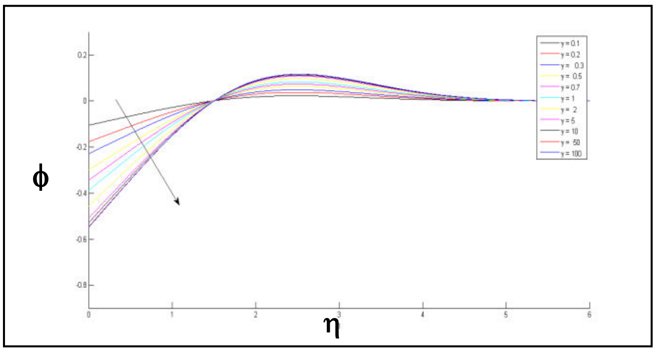

Figure 5 Effect of γ on nanofluid volume fraction distribution

Fig 6 show that an increase in the γ leads to a reversal nature for the volume fraction in the boundary layer i.e. ϕ decreased near the wall (up to $\eta \approx 2$) after that has an increased.

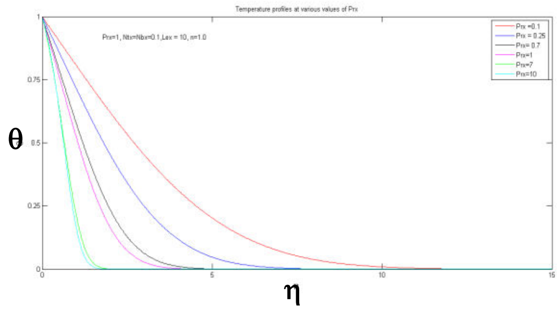

Figure 7 Effect of Pr_x on temperature distribution

The effect of the power law index *n* on the entropy generation number is illustrated in Fig. 7. An increase of the n yields higher entropy generation number *Ns*. A decrease in the entropy generation produced by fluid friction and joule dissipation occurs with increasing the value of the n.

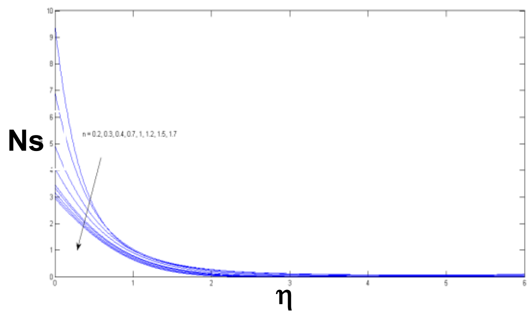

Figure 8 Effect of n on Entropy generation distribution

Concluding Remarks

This paper has focused on the heat transfer from an inclined plane surface to an accelerating liquid film of a power-law nanofluid. Although the thermal boundary layer equation generally fails to permit similarity solutions a novel similarity transformation devised by Shang [17] for the accompanying hydro-dynamical problem was adopted in combination with a local non- similarity solution method due to Keller [10], Sparrow et al [19]. The resulting transformed problem turned out to involve independent parameters. It is noteworthy that all other parameters like the streamwise location x the fluid properties and the component of the gravitational acceleration along the wall have been combined into Pr_x and the local Reynolds number Rex. Accurate numerical results were obtained for various combinations of Parameters. Special treatment of the low and high Prandtl number cases was essential in order to maintain the numerical accuracy and the results were practically indistinguishable from those of Shang [17] for n=1 over the entire Pr_x range. The thickness of the thermal boundary layer decreases monotonically with increasing Pr_x. The thermal boundary layer extends far out in the free stream for $Pr_x \leq 1$ and is on the other hand confined to the innermost part of the momentum boundary layer for $Pr_x \geq 1$. With an increase of **γ**, the temperature rises and the nanofluid concentration reduces near the wall due to the nanoparticle.

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