

Boundary layer flow with vegetation

L. E. Holmedal¹, Marius Holmemo¹, H. Wang¹, P. -Y. Henry¹ and D. Myrhaug¹

¹Department of Marine Technology
Norwegian University of Science and Technology, 7491 Trondheim, Norway

Abstract

A one dimensional numerical model with a $k - \epsilon$ closure has been modified to account for the presence of submerged aquatic vegetation when modelling seabed boundary layers. This hydrodynamic model (i.e. without vegetation) has previously been validated against laboratory and field data. The presence of the vegetation was incorporated through the inclusion of an extra form drag term. In the $k - \epsilon$ equations, the presence of the vegetation result in an additional production term due to the turbulence generated in the wakes behind individual plants. In addition, an extra dissipation term is added to assist the dissipation of this extra wake-scale turbulence. The form drag and wake-production term are generated when spatially averaging the momentum equations and the conservation equation for turbulent kinetic energy over a large enough area. This is done in addition to the regular temporal averaging in order to account for the heterogeneity in the flow due to the complex geometry of the canopy. The model has been validated against flume data for unidirectional flow.

Introduction

The effect of vegetation on seabed boundary layers was first studied in the field of meteorology in investigations of wind blowing over terrestrial canopies. Presently there is a large research activity on this topic within coastal and river engineering, as well as ecology and biology. For exposed coastal locations the existence of seaweed is important for protecting the coast against erosion. However the harvesting of sea weed for food and alginate is receiving increased attention worldwide. In order to provide a sustainable management of the seaweed resources it is necessary to gain more knowledge of the physical processes which are taking place within the vegetated seabed boundary layer when it is exposed to waves and currents.

The presence of plants has often been simplified by regarding it as an addition to the general bed roughness. However, for sufficiently dense aquatic vegetation, it is well established that this is not the case (see e.g. Nepf, 2012). For numerical modelling of flow through vegetation, the effect of the vegetation therefore has to be included in a different way. Wilson and Shaw (1977) assumed that the heterogeneity in the flow due to individual plants could be accounted for by spatially averaging the governing equations in addition to the already introduced temporal averaging. Raupach and Shaw (1982) later modified the derivation, although they came to the same conclusion: The spatial averaging of the momentum equations results in an additional form drag due to the energy extracted from the mean flow due to the work against the plants. Spatial averaging of the conservation equation for turbulent kinetic energy results in an additional production term transforming the energy from the mean flow into additional turbulence in the wakes behind individual plant stems.

Shimizu and Tsujimoto (1994) performed numerical simulations (using a $k - \epsilon$ turbulence closure) of uniform flow through a sub-

merged canopy. They applied the additional terms introduced by the spatial averaging in the momentum and k -equation. An additional term was added to the ϵ -equation to account for the dissipation of the wake-produced turbulence. The inclusion of the additional terms to the k - and ϵ -equation introduces additional coefficients to help improve the generality of the model under different vegetation conditions. Shimizu and Tsujimoto (1994) obtained these additional constants by fitting their model results to experimental data from flume experiments. After determining the coefficients, their model was run against additional experimental trails to verify its generality. Furthermore, the transition zone that appears when flow enters a submerged canopy from a non-vegetated region was investigated.

Lopez and Garca (2001) used both a $k - \epsilon$ and a $k - \omega$ turbulence closure to model open channel flow through submerged vegetation. The inclusion of the additional form drag, wake-production and dissipation are similar to Shimizu and Tsujimoto (1994). However, they used a more theoretical approach to determine the additional constants. They found that the $k - \epsilon$ and the $k - \omega$ models predicted the flume data with similar accuracy. In this study, the numerical k - model used by Holmedal and Myrhaug (2013) to predict sea bed boundary layers has been modified to account for submerged vegetation.

Model

Governing equations

The model used in this study is a one dimensional finite difference model utilizing a staggered grid. The velocity components in the x and y direction are calculated at each node while the k and ϵ values are calculated between each node. The nodes are spaced in a logarithmic fashion throughout the water column with higher density closer to the bed in order to capture the boundary layer profile.

By utilizing the boundary layer approximation and using the quadratic drag law for the form drag on the vegetation, the momentum equations are given as

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial}{\partial z} \left(\nu_t \frac{\partial u}{\partial z} \right) - f_{Dx} \quad (1)$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\partial}{\partial z} \left(\nu_t \frac{\partial v}{\partial z} \right) - f_{Dy} \quad (2)$$

where f_{Dx} and f_{Dy} are the direction specific form drag in x - and y -direction, respectively. Hence,

$$f_{Dx} = \frac{1}{2} C_{Da} u |u| \quad (3)$$

$$f_{Dy} = \frac{1}{2} C_{Dv} v |v| \quad (4)$$

By including the additional wake production and dissipation in the k and ε equations, these are given as

$$\frac{\partial k}{\partial t} = -\frac{\partial}{\partial z} \left(\frac{v_t}{\sigma_k} \frac{\partial k}{\partial z} \right) + v_t \left(\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right) - \varepsilon + P_w \quad (5)$$

$$\frac{\partial \varepsilon}{\partial t} = -\frac{\partial}{\partial z} \left(\frac{v_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial z} \right) + C_{\varepsilon 1} \frac{\varepsilon}{k} v_t \left(\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right) - C_{\varepsilon 2} \frac{\varepsilon^2}{k} + D_w \quad (6)$$

where $(C_{\varepsilon 1}, C_{\varepsilon 2}, \sigma_k, \sigma_\varepsilon) = (1.44, 1.92, 1.00, 1.30)$, and P_w and D_w are expressed as

$$P_w = C_{fk} \frac{1}{2} C_D a |\mathbf{u}|^3 \quad (7)$$

$$D_w = C_{\varepsilon 1} \frac{\varepsilon}{k} C_{f\varepsilon} \frac{1}{2} C_D a |\mathbf{u}|^3 \quad (8)$$

Here, \mathbf{u} is the resulting velocity vector (u, v) .

The coefficients C_{fk} and $C_{f\varepsilon}$ are taken from Lopez and Garcia (2001) and has the value of 1 and 1.33, respectively.

Above the canopy ($z > h$) the drag coefficient is defined zero, $C_D = 0$, hence reducing the momentum equations and the k - ε equations to what they would have been for an unobstructed tidal flow.

By assuming that the upper part of the water column can be modelled as potential flow, the horizontal pressure gradients are given by

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad (9)$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (10)$$

Boundary conditions

At the sea bed:

At the sea bed the log law for a rough surface is used, hence

$$u(z) = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right), \quad (11)$$

The horizontal velocity components are zero due to the no-slip condition $u = v = 0$ at $z = z_0$

The boundary conditions for k and ε follow from the log law

$$k = \frac{u_*^2}{\sqrt{C_\mu}} \quad (12)$$

$$\varepsilon = C_\mu^{3/4} \frac{k^{3/2}}{\kappa z} \quad (13)$$

At the free-surface:

| | $H[m]$ | $h[m]$ | $a[1/m]$ | $S, \times 10^{-5}$ | C_D | λ_f |
|----|--------|--------|----------|---------------------|-------|-------------|
| ST | 0.0747 | 0.041 | 10.0 | 213 | 1.0 | 0.41 |
| LG | 0.34 | 0.12 | 1.09 | 360 | 1.13 | 0.13 |

Table 1: Experimental parameters from the flume experiments by Shimizu and Tsujimoto (1994). Here ST denotes Shimizu and Tsujimoto (1994) and LG denotes Lopez and Garcia (2001). The parameters H , h , a , S , C_D and λ_f are the water depth, canopy height, frontal area per volume, energy slope, drag coefficient and roughness density, respectively.

At the free surface zero gradient is applied to all the parameters, hence

$$\frac{\partial u}{\partial z} = 0, \quad \frac{\partial v}{\partial z} = 0, \quad \frac{\partial k}{\partial z} = 0, \quad \frac{\partial \varepsilon}{\partial z} = 0 \quad (14)$$

Results and discussion

The insides of a flume are normally smooth. However, the turbulence generated from the shear layer at the bottom of the flume will only dominate a thin region closest to the bed before the stem-wake turbulence becomes the dominant source of turbulence. Therefore, the choice of wall function to couple the bed shear stress to the initial values of k and ε will not be of significant importance for the overall results. Hence, the rough boundary condition is assumed valid if a small bottom roughness z_0 is applied. Nepf et al. (1997) measured the turbulence intensities for emergent vegetation, and found that the bed shear turbulence dominated less than 10 % of the water depth even for sparse stem spacings with a volume fraction of only 0.6 %.

For the case of a canopy represented by rigid cylinders, the model has been validated against flume data from Shimizu and Tsujimoto (1994) taken in steady unidirectional flow. The numerical results from the k - ε models of Shimizu and Tsujimoto (1994) and Lopez and Garcia (2001) are also included as part of the validation. Table 1 shows the different parameters needed for the validation. The pressure gradient is given by

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = gS, \quad (15)$$

where $g = 9.81[m/s^2]$ and S is the energy slope given in the experiments.

The experimental parameters presented in Lopez and Garcia (2001) are the ones used by Dunn et al. (1996) in their experiments. The canopy was represented by rigid vertical cylinders mounted onto a false bottom plate. Their arrangement was in a staggered pattern with variable density. A honeycomb grid was positioned upstream of the canopy in order to straighten the inflow onto the canopy. Velocity profiles were measured at four different locations in the longitudinal direction of the tank. Each profile consisted of 10 measurement points each. These profiles were then spatially averaged to obtain the resulting mean velocity profile for the canopy.

Although Dunn et al. (1996) measured changing values for the canopy drag coefficient, the mean value of $C_D = 1.13$ is used for the present model following Lopez and Garcia (2001). The modelled velocity profile, using the same input parameters as the numerical model of Lopez and Garcia (2001) given in in Table 1, has been plotted against the predicted velocity profile from their k - ε model and the experimental data from Dunn et al. (1996). The comparison is displayed in figure 1.

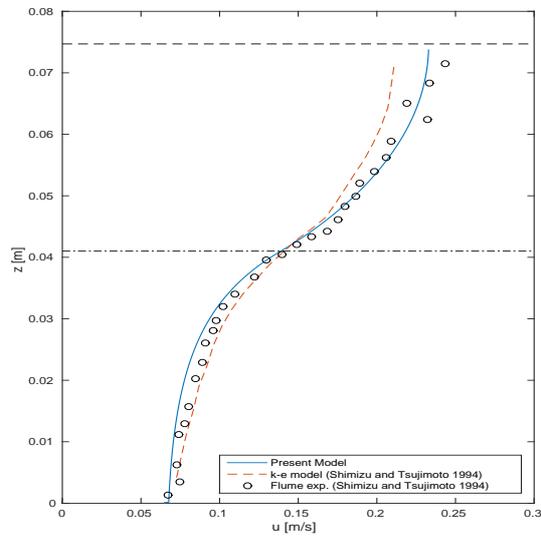


Figure 1: Comparison of the predicted velocity profile between the present $k-\varepsilon$ model, the $k-\varepsilon$ model and flume experiments from Shimizu and Tsujimoto (1994). Here u is the spatially averaged mean velocity and z is the water depth. The horizontal dashed and dash-dot line indicate the water surface H and canopy height h , respectively.

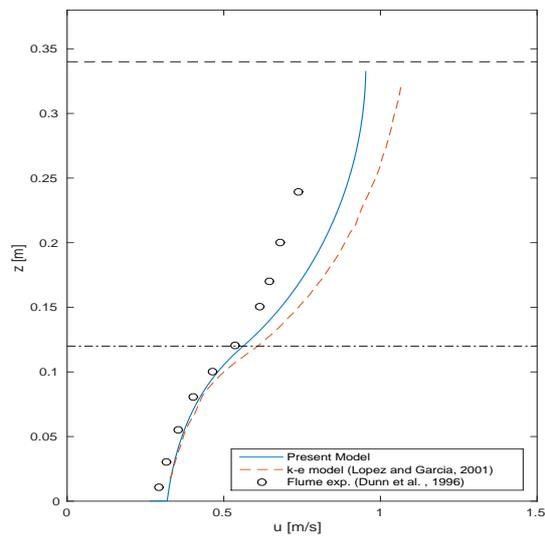


Figure 2: Comparison of the predicted velocity profile between the present $k-\varepsilon$ model, the $k-\varepsilon$ model from Lopez and Garcia (2001) and experimental data from Dunn et al. (1996). Here u is the spatially averaged mean velocity and z is the water depth. The horizontal dashed and dash-dot line indicate the water surface H and the canopy height h , respectively.

It is seen from figure 1 that the boundary layer closest to the wall is extremely thin. Therefore, the characteristics of the no-slip boundary condition is not obvious. However, the turbulence generated from the shear layer at the bottom of the flume will only dominate a very thin region close to the bed before the stem-wake turbulence becomes the dominant source of turbulence (Nepf et al., 1997). Therefore, the choice of wall function to couple the bed shear stress to the initial values of k and ϵ will not be of significant importance for the overall results and the effect of the bottom roughness is insignificant.

Figure 1 shows that the experimental velocity profile are well predicted. Both k - ϵ models coincide well with the experimental results within the canopy height. Above the canopy both the present model and that of Lopez and Garcia (2001) overpredict the results. However, the present model predicts the above-canopy flow more accurately than Lopez and Garcia (2001).

The reason behind these different results produced by two apparently equal k - ϵ models may come from the procedure used by Lopez and Garcia (2001) to find the eddy viscosity ν_t . Instead of applying the generally accepted coefficient $C_\mu = 0.09$, they used an iterative calculation process, hence introducing C_μ as a depth dependent variable. They also used a Dirichlet boundary condition for the dissipation at the surface as opposed to the Neumann condition used in the present model.

Shimizu and Tsujimoto (1994) compared their numerical results with flume measurements over a canopy of equally spaced rigid cylinders. The velocity profiles were found using a hot-film anemometer. Figure 2 shows the modelled velocity profile from the present model, plotted against the numerical and experimental results presented by Shimizu and Tsujimoto (1994); the present model predicts the flume experiments accurately throughout the whole water column. The results also shows a good fit with the numerical results from Shimizu and Tsujimoto (1994). However, they used different values for the coefficients C_{fk} and $C_{f\epsilon}$ than the ones adopted from Lopez and Garcia (2001) for the present model. Instead of the more theoretical values of $C_{fk} = 1$ and $C_{f\epsilon} = 1.33$, they determined the coefficients from numerical fitting with flume data (not with the presented data in figure 2). By doing so they ended up with $C_{fk} = 0.07$ and $C_{f\epsilon} = 0.16$ which is significantly different from Lopez and Garcia (2001). Based on the results of figure 2, the coefficients from Lopez and Garcia (2001) are considered more suited for the present model.

When validating the present model against flume data for unidirectional flow, it was able to predict the velocity profiles within the canopy well. For the canopies of relative low density presented in this work, the velocity profile was accurately reproduced throughout the whole water column. For relatively dense canopies (not shown here) the above-canopy flow became overdamped compared to the flume data. A possible explanation to this observation is that the generation of canopy-scale vortices are limited by the free surface for small ratios. Because the canopy-scale vortices are not accounted for in the numerical model, this might explain the improvement in predicted velocities for the flume experiment with low ratio. Overall, the flume experiments indicated acceptable accuracy for the implementation of the canopy-related terms into the numerical model.

Conclusions

A one dimensional numerical model with a k - ϵ closure has been modified to account for the presence of submerged aquatic veg-

etation when modelling seabed boundary layers. The presence of the vegetation was incorporated through the inclusion of an extra form drag term. In the k - ϵ equations, the presence of the vegetation result in an additional production term due to the turbulence generated in the wakes behind individual plants. In addition, an extra dissipation term is added to assist the dissipation of this extra wake-scale turbulence. The form drag and wake-production term are generated when spatially averaging the momentum equations and the conservation equation for turbulent kinetic energy over a large enough area. Finally, the model has been validated against flume data for unidirectional flow and yields an overall good prediction of the flow within the canopy.

Acknowledgements

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