20th Australasian Fluid Mechanics Conference
Perth, Australia
5-8 December 2016

Randomness Representation with Kolmogorov Complexity in Laminar-Turbulent Transition Process of a Mixing Layer

M. Ichimiya[1] and I. Nakamura[2]

[1]Department of Mechanical Engineering, Tokushima University, Tokushima 770-8506, Japan

[2]Department of Mechanical Engineering, Nagoya University, Nagoya 464-8601, Japan

Abstract

The Kolmogorov complexity measure of randomness was introduced for the quantitative representation of laminar-turbulent transition process of a mixing layer. The mixing layer was formed downstream of a two-dimensional nozzle exit. A randomness factor had been proposed. It was defined as the ratio of the energy of the continuous spectrum to the total energy in the power spectrum distribution of the velocity fluctuations. The present author tested an applicability of the randomness factor to the transition progress of the mixing layer. As a result, the randomness factor did not show monotonic variation. Therefore, a measure which shows the monotonic variation has been expected to obtain. In this study, the exact and general concept of complexity proposed by Kolmogorov in 1983, which is based on program bit length – i.e., an algorithmic informational concept – is applied to a real experimental flow field. Velocity data time sequence obtained from a hot-wire anemometer was compressed using a compression program run on a Windows PC. From the compressed data sequence, we calculated the approximated Kolmogorov complexity, AK and by use of compressed data, the normalized compression distance between two data was obtained. The AK indicated the regularity of the laminar flow and the randomness of the turbulent flow quantitatively. In the natural transition process in the mixing layer, the AK did not change monotonically in the downstream direction. They therefore involve some uncertainty for measurement of the transition process. The information distance, normalized compression distance (NCD), however, increased monotonically downstream. Thus, the NCD appears to be a good measure of the transition process in the mixing layer.

Introduction

The laminar-turbulent transition phenomenon has long been investigated because it occurs very often in the natural world and in the industrial machinery field. The transition in free shear flow is also an important issue. Many kinds of free shear flow have been shown [8]. One example is a mixing layer formed between the jet issued from a nozzle and the surrounding quiescent fluid. This type of mixing layer is classified as a 'jet boundary' in Ref. [10].

The present study concerns the laminar-turbulent transition in the mixing layer as one of the fundamental problems in the transition in free shear flow. The laminar-turbulent transition process in the mixing layer is classified into three regions. First, the linear region appears where disturbances grow exponentially and a periodic fluctuation, i.e., a fundamental wave, is observed. Next, the nonlinear region appears where the harmonic and subharmonic waves of the fundamental wave are observed. Finally, the irregular region appears where an irregular fluctuation dominates.

In the case of a boundary layer, during the last stage of the laminar-turbulent transition process, isolated turbulent patches, "turbulent spots," are formed and develop downstream. Therefore, the intermittency factor, which is the time fraction of the turbulence, has been used as the indicator of the transition process. It is known to increase monotonically downstream. On the other hand, regarding the transition process of the free shear flow as in the present investigation, Sato and Saito [9] proposed the randomness factor as the indicator based on the variation of the power spectrum density of the velocity fluctuation, although it did not change monotonically [2].

Therefore, to obtain a new measure which changes monotonically through the transition process in the free shear flow, the present authors payed attention to a concept of Kolmogorov complexity in the information science field.

According to this concept, the complexity in the velocity signal was obtained, then its streamwise variation was expressed and its effectiveness was examined.

Kolmogorov Complexity and Compressibility

In this section, we briefly describe Kolmogorov complexity and normalized compression distance [6,7] to facilitate understanding of the contents of this paper. Kolmogorov related computational and information theories to deal with the randomness of finite sequences with computers defined by Kolmogorov complexity, $K(X)$, of discrete finite strings, X, belonging to a set. $K(X)$ is the length of the shortest program for outputting the object X. If there is some regularity in the object, the program to calculate the object can be shortened if one reflects the regularity in the programming. However, if there is no regularity, that is, if the object is genuinely random, there is no way to show the object by the computer by creating a program that shows the object as it is. In this way, the measurement of the randomness of the object becomes possible with the Kolmogorov complexity.

It also has been demonstrated that for a given string the Kolmogorov complexity was the bit length of the binary strings obtained when compressed ultimately within a range where it could be decoded [10].

As a way to approximate $K(X)$, a method to utilize a practical compression program has been proposed from the viewpoint of the previous section [5]. That is, as an approximation of $K(X)$, to use a bit length, $C(X)$, of the compressed data, X, obtained by the practical compression programs.

In this paper, to examine experimental data, i.e., digitized output of hot-wire anemometers, and approximate their complexity, we used the ratio of the length of the compressed data length by the program, C, and the original data length,

$$\mathrm{AK}(X) = \frac{C(X)}{|X|} \tag{1}$$

and designated the approximated Kolmogorov complexity, AK.

In practice, to distinguish the complexity between elements belonging to different sets is more important than to consider a single complexity itself. The fundamental aspect of the distinction is the distance defined in the set. Based on the information distance, the normalized compression distance (NCD) represented by the following equation is considered:

$$\mathrm{NCD}(X,Y)=\frac{C(XY)-\min\{C(X),C(Y)\}}{\max\{C(X),C(Y)\}} \tag{2}$$

Where data *XY* can be made by simply connecting data *Y* after data *X*. By utilizing NCD, clustering of practical data becomes possible. The expressions are applied to a classification in various research fields.

Experimental and Analytical Methods

Mixing Layer for Analysis

In this section, we describe the flow field. The mixing layer is formed at the beginning of the jet issued into a quiescent fluid from a rectangular nozzle. We measured the distributions of the mean and fluctuating velocities, spreading of the jet, production, convection and dissipation rates of the fluctuating energy [3]. We also revealed the progression of the laminar-turbulent transition process in the flow-normal direction, and obtained the randomness factor [3] which was proposed by Sato and Saito [9] as an indicator of the transition progress, then examined its validity in the transition process.

The jet is ejected to the quiescent fluid from the exit (x = 0) of a two-dimensional nozzle with the width of 310 mm and height, *h*, of 10 mm. The coordinate system and the upper half of the flow field are shown in figure 1.

The Reynolds number based on the height, *h*, and the nozzle exit velocity, U_0, was kept at 5000 ($U_0 \simeq 7.5$ m/s). X-shaped hot-wire probes with two tungsten sensing elements, each 5 µm in diameter and 1 mm in length, were used for the measurements. Output voltage was sampled at a frequency of 5 kHz for about 52 seconds. The measurements were conducted in a range of flow-normal distance from the centerline, $y \geqq 0$. Results are shown here in the range $x/h \leqq 20$, where self-preservation is well established.

Compression Work

The analyzed data were numerical, obtained from the first 32768 data from 262144 data taken from the streamwise fluctuating velocity sampled every 0.2 ms. At first their minimum values were subtracted, and then normalized by the nozzle exit velocity so as to remove the decimal point and negative sign, and they were finally converted to 3-digit integers each (a total of 98304 integers). The final data were compressed by Windows PC using 7z compression format. The optimal compression format was examined previously [4] where results from 7z mostly satisfied the distance axioms with NCD.

Results and Discussion

Conventional Measure for Laminar-Turbulent Transition of Mixing Layer

Sato and Saito proposed a randomness factor defined as the ratio of the energy of the continuous spectrum to the total energy in the power spectrum distribution of the velocity fluctuations [9].

In figure 2, the contour map of the randomness factor in the *x*-*y* plane is shown. As a guide of the region where the mean velocity gradient $-\partial U/\partial y$ exits, positions where U/U_m equals 0.99 near the centerline ($y/(h/2)$ = 0) and 0.05 in the outer (large *y*) region are drawn as white solid lines, where U_m is the velocity on the local centerline. The black dashed line is the $y/(h/2)$ position where the rms value of the fluctuating velocity, *u*', becomes maximum respective of *x*. As can be seen from figure 2, after it once decreased in the region of $3 < x/h < 5$, where the line spectral region increased gradually and the periodical fluctuation became dominant. The randomness factor increased farther downstream, where the line spectrum changed to the continuous spectrum, that is, the periodic fluctuations of the velocity changed to irregular fluctuations. That is, the randomness factor does not vary monotonically.

Approximated Kolmogorov Complexity

With the complexity analysis, it is possible to determine the randomness of the flow regardless of the change in spectral distribution. First, the approximated Kolmogorov complexity (AK) that directly presents the data compressibility is examined.

In figure 3, the contour map is shown. The fact that AK is small shows that during the compression process any regularity is detected in the data, then the randomness becomes small. In the area around the dashed black line, AK is large, i.e., of greater complexity. The condition is quantitatively shown where regularity remains in the flow before and immediately after the beginning of the transition, the regularity then disappears and the flow approaches random in accordance with the transition progress. Such upstream regularity is due to the smallness of the velocity fluctuation. Details of the transition progress including the fact that the flow just behind the nozzle exit is still laminar were given in the previous report [2,3].

Since AK does not necessarily monotonically change, it is not possible to measure the turbulent transition by AK alone.

Normalized Compression Distance

In this section, NCD was obtained. In figures 4 and 5 the contour map, distributions in the x-direction are shown, respectively. Values in the respective figures are NCD (*X*, *Y*) whose *Y* data are taken at that position. The position of the reference data, *X*, is x/h = 0.5, $y/(h/2)$ = 0. In order to investigate NCD just behind the nozzle in detail, figure 5 is drawn in a semi-logarithmic way. Since NCD at the reference position is 0.013, the contour map of figure 4 is drawn with the exception of around the reference position. In the distribution on the centerline in figure 5, the value rapidly increases from 0.013 in the reference position, x/h = 0.5, to more than 0.9 in $x/h \geqq 1$.

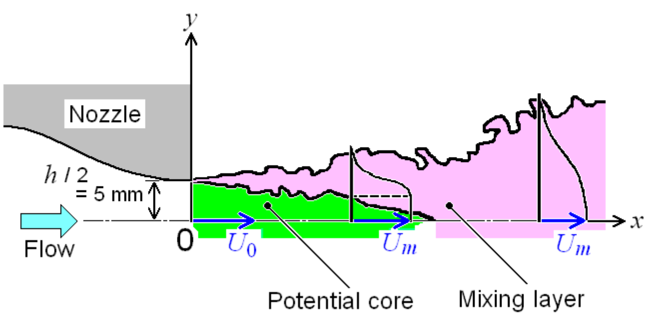

Figure 1 Schematic diagram of two-dimensional mixing layer and coordinate system.

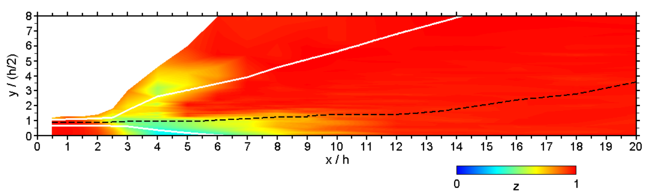

Figure 2 Contour map of randomness factor.

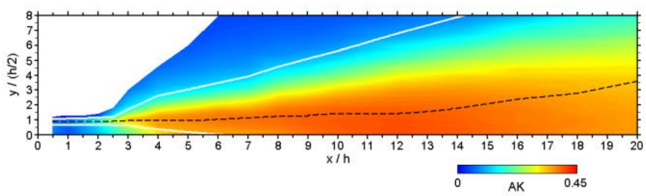

Figure 3 Contour map of approximated Kolmogorov complexity.

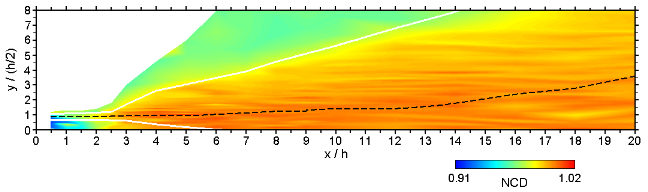

Figure 4 Contour map of NCD between streamwise fluctuating velocities.

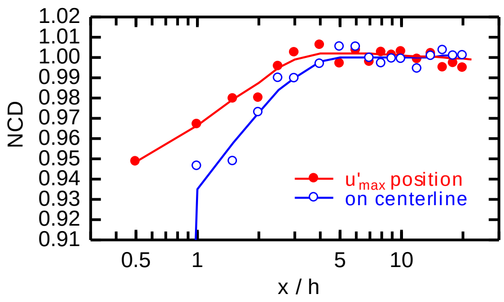

Figure 5 Variations of NCD between streamwise fluctuating velocities in streamwise direction.

In figure 4, NCD is greater in a region of great fluctuating velocity. Unlike AK and the randomness factor, in the streamwise variation, after monotonically increasing, the value is also substantially constant. Since it does not take the same value in two positions, NCD is appropriate as a measure for the laminar-turbulent transition.

Relationship Between Different Velocity Components at Same Position

If two data, *X* and *Y*, whose information distance is shown by NCD, have the same quantity between different positions as in the previous section, the NCD shows spatial variation with the complexity of the quantity. On the other hand, the NCD between different quantities at the same position shows the information distance between the two quantities. In this way, the information distance between the two quantities is shown spatially. In this section, the relationship between different velocity components; streamwise and flow-normal components, at the same position will be considered.

For the spatial variation of the two velocity components, a correlation function and a probability density function, pdf, were also considered. To estimate the difference of the latter distribution between two components, the divergence (Kullback–Leibler divergence) [1] was adopted. This is a measure to show quantitatively how the pdf distribution of the fluctuating velocity in the flow-normal direction, *v*, differs from that in the streamwise direction, *u*. The divergence is shown by the following equation:

$$D = \int P(v) \log \frac{P(v)}{P(u)} dv \tag{3}$$

The divergence in which $P(u)$ and $P(v)$ are changed is also obtained. Though both were not same, in the present paper only the divergence according to Eq. (3) will be discussed since both quantitative trends were the same.

Both fluctuating velocities are normalized by the nozzle exit velocity, U_0. Then, contour maps of a correlation function, the divergence, and the NCD are shown in figures 6–8, respectively. In each figure, the value where a correlation or relationship between two components is strong (the correlation function is

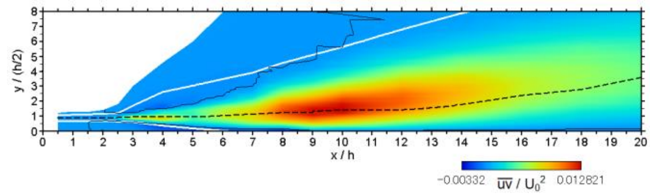

Figure 6 Contour map of correlation function between streamwise and flow-normal velocity components. The black line indicates the value of zero.

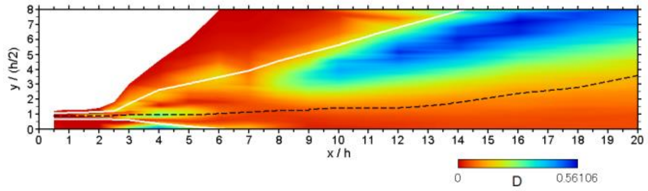

Figure 7 Contour map of divergence of pdf between streamwise and flow-normal velocity components.

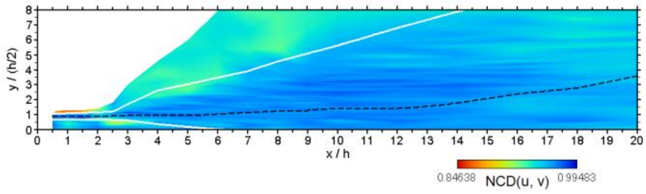

Figure 8 Contour map of NCD between streamwise and flow-normal velocity components in same position.

large, the divergence is small, and the NCD is small) shows a red-color side.

First of all, if we compare the correlation function (figure 6) and the divergence (figure 7), they show an opposite trend in the region of $x/h \leqq 5$, though they evidence the same trend downstream, $5 \leqq x/h$ and $1 < y/(h/2) < 2$. Next, the correlation function (figure 6) and the NCD (figure 8) show an opposite trend in the whole region. Finally, the divergence (figure 7) and the NCD (figure 8) show an opposite trend in the region of $x/h \leqq 5$ and $y/(h/2) \leqq 2$, though they exhibit the same trend $4 \leqq y/(h/2)$. In this way, since the correlation function, the divergence and the NCD display no consistency, they show the difference of the two components from their respective viewpoint.

The reason for these distributions will be considered from velocity signals and pdf profiles at characteristic position. As an example, the three distributions in the flow-normal direction at $x/h = 10$ are shown in figure 9. With the increase of the distance from the centerline, all quantities increase first then decrease, though the position where the trend changes differs. To examine the three quantities, the velocity signals and pdf distributions at three representative positions, $y/(h/2) = 0.08$, 1.48 and 3.98, are shown in figures 10 and 11, respectively.

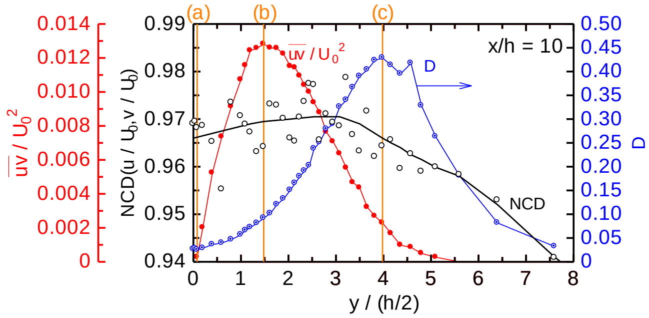

Figure 9 Flow-normal variations of correlation function, divergence and NCD, $x/h = 10$.

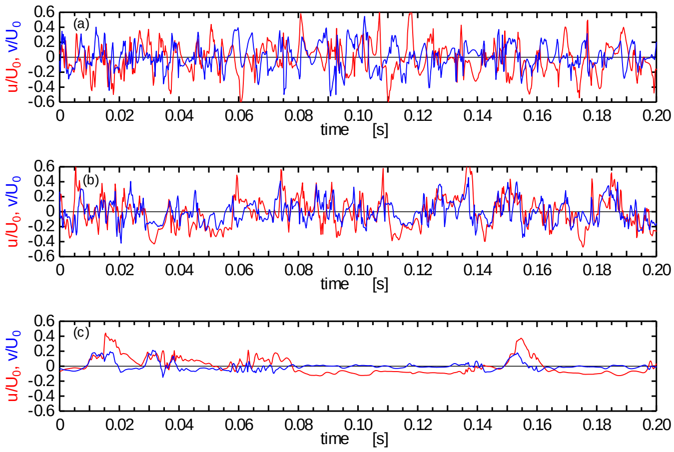

Figure 10 Instantaneous velocity signals, x/h = 10. (a)$y/(h/2)$ = 0.08, (b)$y/(h/2)$ = 1.48, (c)$y/(h/2)$ = 3.98.

In the velocity signals in the streamwise and flow-normal directions at $y/(h/2)$ = 0.08 and 3.98, the duration with the same sign is almost same as that with the opposite sign. This made the correlation function almost zero. On the other hand, at $y/(h/2)$ = 1.48, the signs for both velocity are almost the same. This made the correlation function larger.

Next, for the pdf distributions, figure 11, the difference of the two (streamwise and flow-normal) profiles becomes remarkable with the increase of the flow-normal distance. This made the divergence larger.

Finally, for the velocity signals, both signals become intermittently turbulent with the increase of the flow-normal distance. Therefore, the complexity aspect becomes similar, and the NCD decreases.

To summarize, near the edge of the mixing layer with the increase of the flow-normal distance, both velocity signals become intermittently turbulent, and the complexity in both directions becomes similar. On the other hand, as the pdf distribution is limited within a narrow frequency range, the slight difference in the pdf distribution profile contributes to an increase in the divergence. This makes for the opposite trend in the divergence and NCD.

Conclusions

The Kolmogorov complexity and its approximation were explained as a measure for the representation of the randomness of finite discrete strings. The velocity data in the laminar-turbulent transition process in the mixing layer formed downstream of the two-dimensional nozzle exit are compressed by the compression program, and the approximated Kolmogorov complexity and normalized compression distance were then calculated. The following conclusions were obtained.

(1) AK can express randomness in turbulent flows and regularity in laminar flows quantitatively.

(2) NCD of numerical data from hot-wire anemometer output in the natural transition process of the mixing layer varies

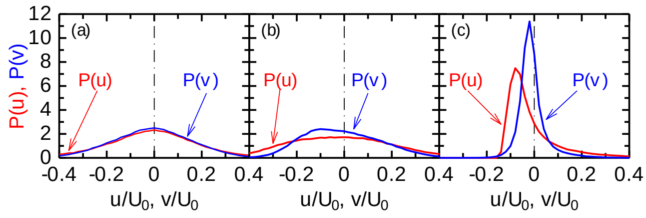

Figure 11 Probability density function, x/h = 10. (a)$y/(h/2)$ = 0.08, (b)$y/(h/2)$ = 1.48, (c)$y/(h/2)$ = 3.98.

monotonically in the downstream direction. It can be a measure of the transition progress.

(3) The complexity of the velocity signals for the streamwise and flow-normal directions at the same position measures a different property with the correlation function and probability density function.

References

[1] Cover, T. M. & Thomas, J.A., *Elements of Information Theory, 2nd ed.*, John Wiley & Sons, 2006.

[2] Ichimiya, M., Kamada, S., Okajima, A. & Osaki, T., Effect of local periodic disturbance on mixing layer downstream of two-dimensional jet (Spatial structure and quantitative representation of laminar-turbulent transition process), *J. Fluid Sci. Tech.*, **8**-1, 2013, 90-105.

[3] Ichimiya, M., Kato, T. & Morimoto, T., Effect of Local Periodic Disturbance on Mixing Layer at Exit of Two-Dimensional Jet, *J. Fluid Sci. Tech.*, **6**-6, 2011, 887-901.

[4] Ichimiya, M. & Nakamura, I., Randomness representation in turbulent flows with Kolmogorov complexity (In mixing layer), *J. Fluid Sci. Tech.*, **8**-3, 2013, 407-422.

[5] Li, M., Chen, X., Li, X., Ma, B. & Vitányi, P. M. B., The Similarity Metric, *IEEE Trans. Info. Theo.*, **50**-12, 2004, 3250-3264.

[6] Li, M. & Vitányi, P. M. B., Kolmogorov Complexity and its Applications, in *Handbook of Theoretical Computer Science Vol. A, Algorithms and Complexity*, editor van Leeuwen, J., Elsevier, 1990.

[7] Li, M., & Vitányi, P. M. B., *An Introduction to Kolmogorov Complexity and its Applications, 2nd ed.*, Springer, 1997.

[8] Rajaratnam, N., *Turbulent Jets*, Elsevier, 1976.

[9] Sato, H. & Saito, H., Fine-Structure of Energy Spectra of Velocity Fluctuations in the Transition Region of a Two-Dimensional Wake, *J. Fluid Mech.*, **67**-3, 1975, 539-559.

[10] Schlichting, H. & Gersten, K., *Boundary-Layer Theory, 8th Revised and Enlarged ed.*, Springer, 652, 2000.

[11] Vitányi, P., (translated in Japanese, Watanabe, O., Universal Similarity Measurement Based on Complexity, *Mathematical Sciences*, **519**, 2006, 54-59(in Japanese)).