On the spurious dissipation of internal waves in ocean circulation models

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Abstract

Ocean circulation models employ turbulence closure schemes to represent unresolved sub-gridscale processes, and to maintain model stability. Here we investigate solutions that resolve part of the internal wave spectrum, and show that internal waves generated at boundaries of an ocean model spuriously decay as a result of the artificially high horizontal viscosity and diffusivity typically associated with the turbulence closures. We configure a 200m resolution regional-scale model with a near-inviscid and weakly diffusive interior such that spurious decay of the resolved wave field is minimised, and compare with a significantly more viscous and diffusive configuration as used in previous studies. The reduced viscosity and diffusivity results in an order of magnitude increase in the interior wave energy, with waves sourced from both the ocean surface and bottom propagating over the entire depth of the model ocean with negligible dissipation. The results thus point to the need to re-examine turbulence closures and other parameterisations that may conflict with resolved internal wave dynamics.

Introduction

Internal waves are an important mechanism for vertical and downscale transfer of energy in the ocean. Internal waves can transport energy from the upper and lower boundary of the ocean (where much of the energy is injected) to the ocean interior, where wave breaking and other nonlinear processes can lead to turbulent mixing [16]. Furthermore, they are amongst the larger scales of “unbalanced” flow, and can therefore provide a conduit from large-scale “balanced” flow to the small-scale turbulence where dissipation occurs [15]. Internal waves are generated by surface wind stresses [18], tidal interactions with bathymetry [e.g. 14], geostrophic flows over rough topography on the sea floor [8], and small-scale unbalanced flow at the ocean surface including submesoscale eddies, fronts and filaments [e.g. 1, 7, 13].

Only recently have computational capabilities expanded sufficiently to permit regional ocean circulation models to be run at sufficiently high (100m-1km) horizontal resolution to resolve a significant portion of the internal wave spectrum [9, 7, 12] — herein we will call these models wave-resolving. In their 200m resolution model, [9] find that the resolved waves generated via geostrophic flow over topography dissipate in the interior of the model ocean. They extrapolate this result to the global ocean to suggest that the resolved waves with scales exceeding 1km can contribute to sustaining turbulent mixing in the ocean interior.

As with all large-scale ocean models, the subgrid-scale turbulence in wave-resolving numerical models must be parameterised to maintain numerical stability. Typical values of Laplacian horizontal diffusivities and/or viscosities employed in these high resolution models [e.g. 9, 7, 12] are of $O(1) m^2 s^{-1}$ throughout the depth of the ocean. In comparison, values for vertical viscosity/diffusivity are of $O(10^{-5}) m^2 s^{-1}$ and are based on observed ocean dissipation rates of $O(10^{-10}) W kg^{-1}$ [17] in the interior. For example, for a typical interior stratification $N^2 = 10^{-6} s^{-2}$ and assumed mixing efficiency of $\Gamma = 0.2$, the predicted vertical diffusivity associated with $\epsilon = 10^{-10} W kg^{-1}$ is $\kappa_v = \Gamma \epsilon / N^2 = 2 \times 10^{-3} m^2 s^{-1}$ via the so-called ‘Ostrov relation’ [10]. If turbulence was entirely isotropic then vertical and horizontal diffusivities would be equal: $\kappa_h = \kappa_v$. In reality, the presence of stratification tends to flatten turbulent structures, leading to an aspect ratio $L/H > 1$ and thus $\kappa_h \sim (L/H)^2 \kappa_v > \kappa_v$. An extreme limit on physically realistic values of $\kappa_h$ may be determined by supposing the turbulence is quasi-geostrophic. Quasi-geostrophic flow is characterised by an aspect ratio $L/H \sim N^2/\epsilon \sim 10^{-3}/10^{-4} = 10$ and thus $\kappa_h$ could be at most $\kappa_h = 10^2 \kappa_v = 2 \times 10^{-3} m^2 s^{-1}$. Therefore, realistic values of horizontal turbulent diffusivity/viscosity are at most $O(10^{-3}) m^2 s^{-1}$ which is at least three orders of magnitude smaller than the $O(1) m^2 s^{-1}$ values typically used in wave-resolving models — these large values are chosen purely to ensure numerical stability at the top and bottom boundaries.

Here we show that large values of viscosity/diffusivity leads to rapid decay of the vertically propagating internal waves that are resolved in these models (order 1km wavelengths). As a result these waves dissipate energy in the interior of the model ocean rather than propagating freely. The dissipation is spurious because it is a direct linear consequence of artificially elevated viscosity/diffusivity, rather than a result of wave breaking on large-scale velocity and density gradients or wave-wave interactions which are the physical mechanisms believed to dissipate wave energy in the ocean interior [11].

Results: Wave-resolving numerical model

We use the Massachusetts Institute of Technology Global Circulation Model [MITgcm; 4] to simulate the hydrostatic primitive equations in a 500km square, 4km deep zonally re-entrant domain at 200m horizontal resolution. A snapshot of the domain and model solution is shown in figure 1. The vertical grid consists of 200 points with grid spacing of 1.5m at the surface, increasing to 40m at mid-depth, and reducing to 20m in the deepest 1km of the domain. This numerical configuration is able to resolve both the surface and bottom generation, and subsequent propagation, of internal waves with horizontal wavelengths of approximately 2km or larger. The model is forced by temperature-restoring ‘sponges’ of 30km width just inside the northern and southern boundaries. These sponges maintain a stratification $(N^2 = 10^{-6} s^{-2})$ in the deep; $N^2 = 2 \times 10^{-3} s^{-2}$ just below the thermocline and surface temperature difference across the domain $(\Delta T = 10K)$, the vertical profiles of which are set to roughly approximate the parameter regime of the Southern Ocean. We perform two sets of simulations, one with a flat bottom at 3700m depth, and the other with rough topography. The rough topography consists of a 40km by 160km, 800m high Gaussian hill in the centre of the domain, plus white noise roughness of root-mean-square (rms) height 300m on wavelengths from 10km to 100km. The bottom topography is flat within the sponges near the northern and southern boundaries as shown in figure 1.

Subgrid turbulence in the model is parameterised via Laplacian diffusion and viscosity. Here we consider two different configurations. In the first we have vertically uniform viscosity and dif-
fusivity with horizontal values of $A_v = k_h = 3 \text{m}^2\text{s}^{-1}$ and vertical values $A_z = k_z = 2 \times 10^{-2} \text{m}^2\text{s}^{-1}$ — we call this the viscous configuration. In the second set of simulations, we maintain the value of the horizontal viscosity at $A_h = 3 \text{m}^2\text{s}^{-1}$ near the boundaries to ensure model stability, but allow it to decay in a Gaussian fashion (with an $e$-folding depth of 200m) with vertical distance from the surface or ocean bottom. We set negligible uniform background values of $A_h = k_h = A_v = 10^{-6} \text{m}^2\text{s}^{-1}$ in the interior. We find a uniform background value of horizontal diffusivity $k_h = 0.1 \text{m}^2\text{s}^{-1}$ is sufficient to maintain model stability in the interior. We call this set of simulations the inviscid configuration. These parameter choices are not intended to be a realistic representation of sub-grid processes — the parameters are chosen purely to minimise spurious decay of the resolved wave field while maintaining numerical stability.

In figure 2 we compare snapshots of the divergence field ($\partial_y w$) for flat (a,b) and rough (c,d) topography simulations, in the viscous (a,c) and inviscid (b,d) configurations. Internal waves can be identified as banded structures (phase lines) in the divergence fields. It is immediately apparent that the internal wave field is much stronger in the inviscid configuration than in the corresponding viscous configuration, particularly at small scales. There is strong surface wave generation in the flat-bottom model (a,b). Many of these waves propagate all the way to the ocean bottom, and in some cases reflect off the bottom, when not spuriously dissipated as in (a). Bottom wave generation in the rough viscous simulation (c) is weak, with little propagation away from the bottom. In comparison, the rough inviscid simulation (d) shows strong small scale bottom generation and propagation.

The snapshots in figure 2 indicate that the interior divergence field contains a significant contribution from internal waves and is controlled by the choice of turbulent parameters. To generalise this result to the whole domain figure 3(a) displays vertical profiles of time and area averaged root-mean-square (rms) divergence for each of the model runs. The areal average is computed only over the area of the model domain outside the sponges, 60km $< y < 440km$, and the time average is computed over a period of 30 days post equilibration.\(^1\) For the viscous configuration, the rms divergence in the flat and rough cases only differs significantly in the lower half of the domain (below $\sim 2km$), suggesting that bottom-generated lee waves only propagate in this region. For the inviscid configuration, the rms divergence differs significantly throughout the interior, indicating that bottom-generated waves propagate throughout the water column.

The interior rms divergence is two to three times larger in the inviscid configurations vis-a-vis the viscous configurations with the same topography. In contrast, the time and area averaged kinetic energy ($E_K = (u^2 + v^2)/2$, figure 3b) for a given topography is essentially identical for the inviscid and viscous configurations, implying that the large (domain- and meso-) scales of the flow — which dominate the kinetic energy — are unaffected by the choice of turbulent parameters.

The viscous configuration exhibits an order-of-magnitude reduction in high wavenumber kinetic energy at 2000m depth (not shown), implying that the waves generated at the boundaries must be dissipating significant energy between the boundary and 2000m depth. The two terms leading to a loss of energy from the wave field (neglecting any wave-mean flow exchange) are the viscous dissipation of kinetic energy, $\varepsilon = A_h |\nabla \tilde{u}_h|^2 + A_z |\partial_y \tilde{u}_h|^2$, and the loss of available potential energy (APE) via

\(^1\)The 200m resolution model is initialised from a lower 500m resolution model with a fully equilibrated mesoscale eddy field. The 200m resolution model is run for 1 to 2 months to permit the submesoscale and wave fields to equilibrate, whereafter the time average is taken.

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**Figure 1:** Snapshot of the model solution for the rough topography simulation in the inviscid configuration. Vertical transects show the flow divergence $\partial_y w$ (right hand colour bar) at that location. The top and bottom flow speed is shown (left hand colour bar) on the corresponding surface.

**Figure 2:** Vertical transect of the divergence field (at $y = 150km$) in each of the four model runs: (a) flat viscous, (b) flat inviscid, (c) rough viscous, and (d) rough inviscid.
irreversible mixing, $\phi_d = \partial_t z' \left( k_b |\nabla_z b|^2 + k_v |\partial_z b|^2 \right)$ (e.g. equation 22 of [2]) where $z'(b, t)$ is the mean height of the isopycnal with buoyancy $b$ over the domain at time $t$. In our model $z'$ is strongly constrained by the sponges at the meridional boundaries and varies by only $\sim 0.1\%$ over the time averaging interval (30 days), greatly simplifying the calculation of APE. Vertical profiles of $\epsilon$ and $\phi_d$ for each of the model runs are plotted in figure 3(c,d). For the viscous configuration (black, red) — where $k_v = A_k$ throughout the domain — the viscous dissipation and irreversible mixing are comparable in magnitude, $\phi_d \sim \epsilon$. In the inviscid configuration (grey, blue) — where $k_v \gg A_k$ in the interior — it is the irreversible mixing that is the dominant mechanism of energy loss from the wave field, $\phi_d / \epsilon \sim 100$. The interior viscous dissipation is three to four orders of magnitude smaller in the inviscid configuration than in the viscous configuration. The irreversible mixing is 1 to 2 orders of magnitude smaller (consistent with the ratio of $k_v$ in the configurations, $3/0.1 = 30$). Thus as anticipated, the reduced wave-scale kinetic energy at 2000m in the viscous configuration is associated with increased viscous dissipation and irreversible mixing in the interior.

**Conclusions: Implications for the ocean wave field**

The values of interior dissipation observed in the viscous configuration ($\epsilon \sim O(10^{-11})$ W/kg) are comparable to those seen in previous modelling studies with similar parameter choices [e.g. 9]. We argue that this value of dissipation is spurious, since it is associated with the linear decay of propagating waves due to artificially elevated horizontal viscosity and diffusivity. The inviscid configuration, in which elevated viscosity is maintained within several hundred metres of the boundary but reduced in the interior, leads to an order of magnitude increase in the intensity of the interior wave field. Interior dissipation is correspondingly weaker implying that, without a physical justification for elevated values of viscosity, simulation-derived estimates of dissipation [9], mixing and consequent changes in the ocean circulation [5] should be treated with caution.

The results in this paper raise the obvious question: is it possible to constrain the location and intensity of internal wave-driven mixing in the ocean using current generation models? In short, such model simulations would depend both on the mechanisms to generate and to dissipate internal waves. While models are now capable of generating a realistic wave field, we argue that a method to accurately represent wave dissipation (while maintaining numerical stability) remains obscure. Therefore, more advanced sub-gridscale parameterisations are required to understand the role of wave-induced mixing in these flows. The results of this work suggest that such parameterisations will need to be non-local since there is not necessarily a direct connection between the source location of waves and the location of eventual dissipation of the wave energy.

The inviscid configuration described herein is not proposed as a fully realistic simulation of the internal wave field, and has two significant shortcomings. Firstly, viscosity remains elevated in the near-boundary regions of the model to maintain numerical stability, which may result in some spurious decay of waves in this region. Second, the interior diffusivity is large compared with the viscosity, although as the choice of diffusivity ($k_v = 0.1 \text{m}^2 \text{s}^{-1}$) is expected to lead to negligible spurious decay for resolved wave scales, further reductions to $k_v$ are unlikely to substantially alter the resolved wave field. Preliminary model runs (not shown) with an interior diffusivity of $k_v = 0.001 \text{m}^2 \text{s}^{-1}$ appear to confirm this. The net loss of energy in the interior (via viscous dissipation and/or irreversible mixing) in the inviscid simulations of $O(10^{-12})$ W kg$^{-1}$ therefore provides an upper limit on the interior dissipation from these wave scales. The resolved waves in the present model generated at the surface and by geostrophic flows over the sea floor are thus not able to account for the much larger values of dissipation seen in the ocean interior in observational campaigns ($10^{-10}$ to $10^{-9}$ W kg$^{-1}$ at 2000m above the sea floor; [17]). This larger observed dissipation may instead be associated with smaller (sub-1km) scale waves that remain un-resolved in the present model, or with non-linear wave-wave interactions [11] in the presence of a more intense wave field at the resolved scales (which could be generated by strong tidal flows, not included in the present model).

While the interior dissipation magnitudes in the numerical model are spurious, it is notable that the largest energy loss in the rough topography cases is the viscous dissipation near
the topography. In the viscous configuration 90% of the viscous dissipation occurs below 3500m depth, while in the inviscid configuration this figure is 99.6%. The peak magnitude of the bottom dissipation increases by almost a factor of 10 (from $10^{-2}$ to $10^{-3}$ W kg$^{-1}$) in the inviscid configuration despite the viscosity in this region remaining the same as in the viscous configuration. Since the kinetic energy is unchanged, it is unlikely that the spectrum of upward-propagating locally generated waves changes or that the dissipation of these waves increases. Instead, the increase in bottom dissipation suggests that waves sourced from outside the bottom boundary layer — such as surface generated waves — are viscously dissipated at the bottom boundary. The lack of a similar increase in bottom dissipation for the inviscid configuration in the flat-bottom cases indicates that the rough topography itself is vital in driving elevated dissipation, presumably via processes such as critical reflection [3] and scattering [6] of incoming waves.

Our results (e.g. figure 2) also indicate that spontaneous internal wave generation at the ocean surface [e.g. 13] can give rise to a wave field of comparable magnitude to that generated by geostrophic flow over rough topography. These spontaneously generated waves propagate over the entire depth of the model ocean when spurious decay is avoided, transferring energy from the surface to the deep ocean as suggested above. Thus, particularly in regions of the ocean with a relatively smooth sea floor where tidal and geostrophic lee wave generation are minimal, spontaneous surface generation may play an important role in the energy budget.

The present work has shown that high-resolution modelling studies that seek to describe the generation and propagation of internal waves must carefully consider the subgrid turbulence closure used in the model interior. These results should prove useful for future studies of surface and geostrophic lee wave generation.

References


