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Vortex-induced Vibration of Four Cylinders in an In-line Square Configuration in Steady Flow

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Abstract

Vortex-induced vibration (VIV) of four rigidly connected circular cylinders in an in-line square configuration at a Reynolds number of 150 and a low mass ratio of 2.5 is investigated numerically for spacing ratios (L) of 1.5, 2, 2.5, 3 and 4. Among all the studied spacing ratios, the lock-in regime for L=1.5 is found to be the widest and the corresponding maximum amplitude is the largest. The response amplitudes in the lock-in regime for L=2 are the smallest, because of the strong interaction among the vortices. For L=2.5, 3 and 4, the lock-in regime is similar to that of a single cylinder and the response amplitudes in the lock-in regime are slightly higher than that of a single cylinder. The critical spacing ratio for vortex shedding to occur from the two upstream cylinders is much smaller than that for flow past a four stationary cylinders. The vortex shedding flows outside the lock-in regime for large spacing ratio of L=4 are in an anti-phase pattern, which is similar to that of flow past four stationary cylinders and leads to zero cross-flow displacement.

Introduction

Vortex-induced vibration (VIV) of a group of cylinders in fluid flow is of importance in engineering applications. It affects the stability of structures such as overhead cables, heat exchanger tube arrays, offshore riser pipes and offshore mooring cables. VIV of a circular cylinder has been studied extensively in the past decades and most of the studies are focused on the VIV of an elastically mounted rigid cylinder in flow. These works are summarized in [2, 14, 17, 19, 20]. The response of an elastically mounted cylinder in fluid flow is dependent on the Reynolds number, the mass ratio, the damping ratio and the reduced velocity. The Reynolds number is defined as $\mathrm{Re} = UD/\nu$, with U, D and ν the free stream velocity, the cylinder diameter and the kinematic viscosity of the fluid, respectively. The mass ratio m^* is the ratio of the cylinder mass to the displaced fluid mass. For a circular cylinder, $m^* = m/(\rho\pi D^2/4)$, where, m is the cylinder mass per unit length, ρ is the density of the fluid. The reduced velocity is defined as $V_\mathrm{r} = U/f_\mathrm{n}D$ with f_n being the structural natural frequency of the cylinder.

Many numerical studies on VIV of circular cylinders were conducted at very low Reynolds numbers using the two-dimensional numerical models [3, 8, 9, 16, 23]. Different low Reynolds number studies focus on different aspects of the VIV. For instance, Singh and Mittal [16] focused their study on the hysteresis behaviour of the cylinder, Borazjani and Sotiropoulos [3] focused their study on the interference between two tandem vibrating cylinders. Zhao [24] found both VIV and galloping for two cylinders in side-by-side arrangement and Zhao et al. [23] studied VIV of a square cylinder in fluid flow. The maximum response amplitudes in these low Reynolds number studies are generally much lower than those found in the high Reynolds number experimental tests. Two-dimensional numerical models based on the Reynolds-averaged Navier-Stokes (RANS) equations were also used to simulate VIV of circular cylinders at high Reynolds numbers in the subcritical Reynolds number regime and it was found RANS equations provide reasonable good results of the response amplitude and frequency. By solving the RANS equations, Pan et al. [13] and Guilmineau and Queutey [7] obtained good results of 1-dof VIV except in the upper branch and Zhao and Cheng [21] predicted super upper branch of the response in the 2-dof VIV very well. Numerical studies on VIV of two cylinders of different configurations are also conducted mainly at low Reynolds numbers [10, 11, 24, 25]. A number of experimental studies are conducted to understand the wake flow patterns for flow past four cylinders. Sayers [15] measured vortex shedding frequency for flow past four cylinders and found that the small change in the flow incident angle may lead to sudden change in the vortex shedding frequency. Flow past four cylinders in square arrangements is also studied by numerical simulations. Farrant et al. [4] and Han et al. [6] simulated flow past four cylinders in an inline square arrangement at a low Reynolds numbers of 200. The in-phase and anti-phase vortex shedding were well predicted in these numerical simulations. Oscillatory flow past four cylinders in a square arrangement was also studied numerically due to its engineering importance [1]. While flow past four cylinders has been studied extensively, the studies of the VIV of four cylinders in fluid flow are very rare. Zhao and Cheng [22] studied response of four cylinders in a square arrangement with a constant spacing ratio and various flow approaching angles numerically. The lock-in regime of the four cylinders was found to be affected significantly by the flow approaching angle.

In this study, VIV of four rigidly coupled cylinders in an inline square arrangement at a low Reynolds number of 150 and a low mass ratio of 2.5 is studied numerically. The flow around the cylinders and the response of the cylinders are predicted by solving the two-dimensional Navier-Stokes (NS) equations and the equation of motion, respectively. The one-degree-of-freedom VIV in the cross-flow direction are simulated for spacing ratios of 1.5, 2, 2.5, 3 and 4 and reduced velocities ranging from 1 to 30. The effects of the spacing ratio on the response of the cylinders and the vortex shedding flow pattern are discussed.

Numerical Method

VIV of four rigidly coupled cylinders in an inline square arrangement as shown in Figure 1 is considered. In this study the 1-dof vibration of the cylinders in the cross-flow direction are simulated numerically.

The governing equations for simulating the flow are the two-dimensional incompressible Navier-Stokes (NS) equations. The Arbitrary Lagrangian Eulerian (ALE) scheme is applied to account for the moving boundaries of the cylinder surfaces. In this study the velocity (u, v), the time t, the coordinate (x, y) and the pressure p are nondimensionalized as $(u,v)=(\tilde{u},\tilde{v})/(f_n D)$, $t=\tilde{t}f_n$, $(x,y)=(\tilde{x},\tilde{y})/D$, $p=\tilde{p}/(f_n^2 D^2)$, respectively, where the tildes denote the dimensional parameters, D is the diameter of the circular cylinders, f_n is the structural natural frequency of the system. By using the above nondimensionalization method, the NS equations in the ALE are expressed as

$$\frac{\partial u_i}{\partial t}+\left(u_j-\hat{u}_j\right)\frac{\partial u_i}{\partial x_j}+\frac{\partial p}{\partial x_i}=\frac{V_r}{\mathrm{Re}}\frac{\partial^2 u_i}{\partial xj \partial xj}, \qquad (1)$$

$$\frac{\partial u_i}{\partial x_i}=0 \qquad (2)$$

where x_1=x and x_2=y are the Cartesian coordinates in the in-line and the transverse directions of the flow as defined in Figure 1 (a), respectively; u_i is the fluid velocity component in the x_i-direction and $\hat{u}_i$ is the moving velocity of the mesh nodes. The motion of the cylinder is predicted by solving the two-degree-of-freedom equation of the motion for the displacements of the cylinder system:

$$\frac{\partial^2 Y}{\partial t^2}+4\pi\zeta\frac{\partial Y}{\partial t}+4\pi^2 Y=\frac{1}{2\pi}\frac{V_r^2\sum C_L}{m^*}, \qquad (3)$$

where X_1=X and X_2=Y are the displacements of the four-cylinder system in the x- and y-directions, respectively; m^* = m/m_d is the mass ratio with m being the total mass of the four cylinders and $m_d=\rho\pi D^2$ is the displaced mass of the fluid by the four cylinders; $\zeta=2c/\sqrt{Km}$ is the damping ratio with c and K being the damping constant and spring constant of the system, respectively; $\sum C_L$ is the sum of the lift coefficients in the y-direction of the four cylinders. The drag and lift coefficients of a cylinder are defined as $C_D=F_D/(\rho DU^2/2)$ and $C_L=F_L/(\rho DU^2/2)$ with F_D and F_L being the drag and lift forces in the inline and the cross-flow directions, respectively and ρ being the fluid density. For 1-dof VIV in the cross-flow direction, only cross-flow component of Eq. (3) is solved. The governing equation for calculating the displacements of the nodes of the FEM mesh is [21]

$$\nabla\cdot\left(\gamma\nabla S_y\right)=0, \qquad (4)$$

where S_y represents the displacement of the nodal points in the y-direction, and γ is a parameter that controls the mesh deformation, which is set to be γ=1/A, with A being the area of the element. The displacement of the mesh nodes is the same as the displacement of the cylinder on the cylinder surface and zero on other boundaries. By giving the displacements at all the boundaries, Eq. (4) is solved by a Galerkin FEM. Initially, the velocity and the pressure are zero in the whole computational domain and the cylinder's displacement and velocity are zero in all the simulations.

A rectangular computational domain with a height of 40D in the cross-flow direction and a width of 60D in the flow direction is used, corresponding to a blockage ratio of 0.05. Previous studies showed that the domain width has little effects on the result of the cylinder response if the blockage ratio is less than 0.05 [12]. The computational domain is divided into four-node quadrilateral bi-linear elements as shown in Figure 1. The numbers of the finite element nodes vary from 49725 to 50083 for L=1.5 to 4. Refined elements are used close to the cylinder surfaces in order to capture the strong variations of the flow field. 96 elements are distributed along the surface of each cylinder. The minimum nondimensional mesh size at the cylinder surface is 0.002. Initially the velocity and the pressure in the whole computational domain are zero. The velocity is V_r at the inlet boundary and is equal to the vibration speed of the cylinders on the cylinder surfaces. At the outflow boundary the pressure is set to be zero and the gradient of the velocity in the flow direction is zero. At the two lateral boundaries, the gradient of the pressure and the velocity in the cross-flow direction are zero.

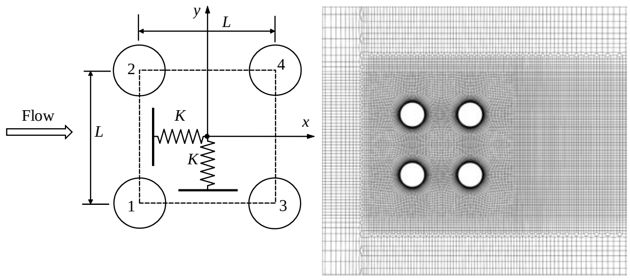

Figure 1. Sketch for VIV of four cylinders in an inline arrangement; Computational mesh near the four cylinders in an in-line square configuration for L=2.5

Numerical Results

VIV of four cylinders in an inline square arrangement are simulated for a constant Reynolds number of 150, a constant low mass ratio of 2.5, and spacing ratios of 1.5, 2, 2.5, 3 and 4. Simulations are carried out for reduced velocities ranging from 1 to 25, which cover the lock-in regimes for all the spacing ratios. The four cylinders are rigidly connected with each other and they vibrate as a single body. The 1-dof VIV in the cross-flow direction is simulated. The validation of the numerical model in the simulations of flow past a single stationary cylinder, VIV of a single cylinder and VIV of two cylinders at low Reynolds numbers have been demonstrated in [25] and will not be repeated here. The density of the computational mesh used in this study shown in Figure 1 (b) is the same as that used in the study of VIV of two cylinders in fluid flow at Re=150 in [25].

Figure 2 shows the variations of the response amplitude and frequency with the reduced velocity for the 1-dof vibrations in the cross-flow direction. The nondimensional amplitude in the y-directions is defined as $A_y=(Y_{max}-Y_{min})/2$, where the subscripts "max" and "min" stand for the maximum and minimum displacement, respectively. The response frequencies are determined by the fast Fourier transform (FFT) of the vibration displacement. For a broad branded FFT spectrum of an irregular vibration, the frequency is determined to be the peak frequency of the spectrum with the highest amplitude. The nondimensional frequency f_y is the ratio of the response frequencies in the y-directions to the structural natural frequency. Based on the high amplitude and the closeness between the natural frequency and the vibration frequency, the lock-in regime for a single cylinder is $3.5 \le V_r \le 8$. For L=1.5, the four cylinders behave as a single body with greater dimension than a single cylinder. It can be seen that the maximum response amplitude for L=1.5 is about twice that of a single cylinder. The lock-in regime for L=1.5 is also much wider than that of a single cylinder. For L=1.5, the response frequency increases with the increasing reduced velocity until V_r=7, after which the response frequency changes little until V_r=25.

The response amplitudes for L=2 are the smallest among other spacing ratios. The response frequency for L=2 is close to the natural frequency in the wide range of reduced velocity of $3 \le V_r \le 25$. The low amplitude for L=2 can be explained by the FFT spectra of the vibration displacement and the lift coefficient shown in Figure 3. The spectra of displacement and lift coefficient are plotted together in order to see the correlation between the force and the response. For L=2 and $V_r > 10$, the spectra of the displacement and lift coefficient are broad banded as shown in Figure 3 (e),leading to the reduction in the response amplitude. The responses are generally very irregular for a broad banded spectrum, and the frequency component corresponding to the response frequency is not the one with the highest peak value. This explains that the response amplitude for L=2 is the lowest among other spacing ratios. The response amplitude for L=1.5 is reduced suddenly as the reduced velocity is increased from 20 to 22. This is due to that the peak frequency of the response displacement and that of the lift coefficient are not the same as shown in Figure 3 (c). Zhao and Cheng [22] also reported that the peak response frequency is different from the peak frequency of the lift force for irregular response of four cylinders in the turbulent flow regime.

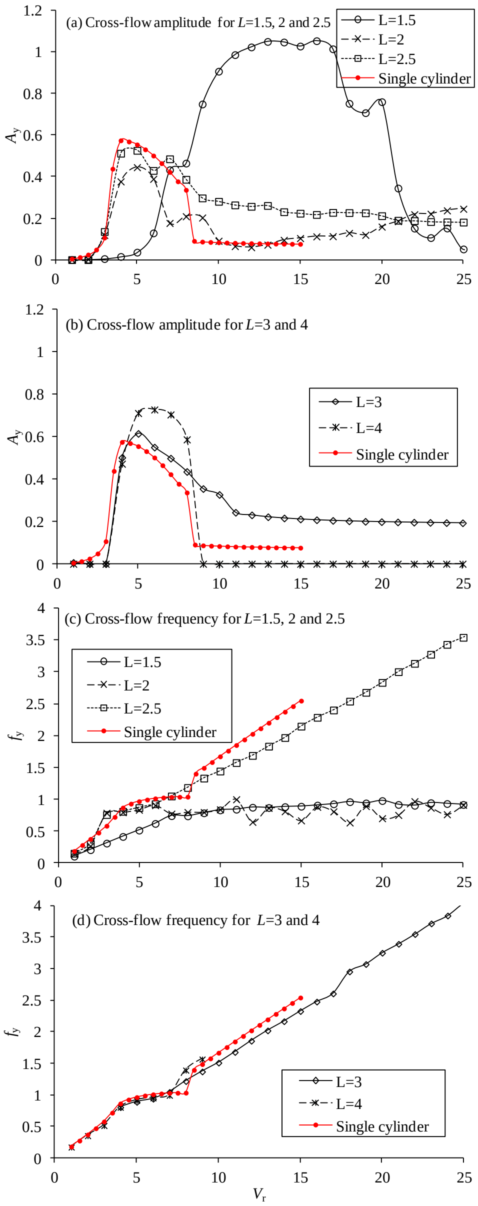

Figure 2. Variations of response amplitude and frequency with reduced velocity in the cross-flow direction for 1-dof VIV

The vortex shedding for the flows past four stationary cylinders is in an anti-phase pattern for L=2.5 to 4 which leads to zero lift force. However, the response amplitudes for a vibrating cylinder at these spacing ratios are the evidence of asymmetric vortex shedding flow. Fig.4 shows the vorticity contours for L=1.5 and V_r=15 for the 1-dof VIV. Two negative vortices are shed from the cylinder when the cylinder is moving downwards and two positive vortices are shed when the cylinder is moving upwards. The response amplitude corresponding to figure 4 is much higher than that of a single cylinder because of the larger dimension of the four cylinder structure.

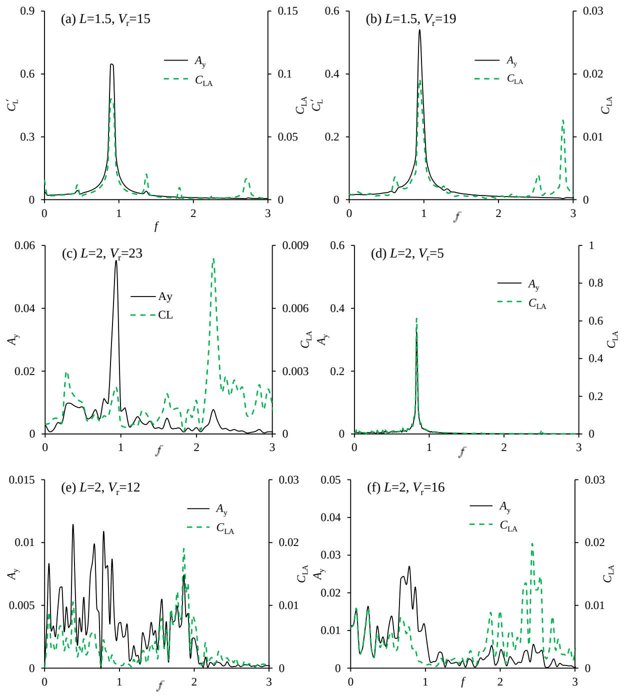

Figure 3. FFT spectra of the displacement and the lift coefficient for the 1-dof VIV of four cylinders

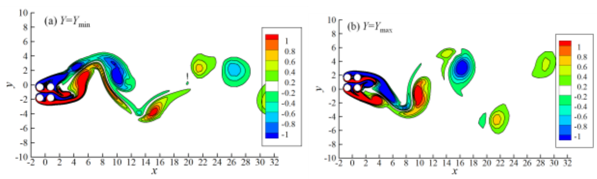

Figure 4. Vorticity contours for L=1.5 and Vr=15 in the 1-dof VIV

Figure 5 shows the vorticity contours for V_r=5 and different spacing ratios. The vortex shedding for L=1.5 and V_r=5 is similar to that of four stationary cylinders because the response is outside lock-in regime with a small amplitude. The biased flow is also observed for L=2 and 3 at V_r=5, and the biasness of the flow for L=3 appears much stronger than that for L=2. For each of the biased flows in Figure 5 (b) and (c), strong interaction among vortices occurs immediately behind the cylinder and the wake flow about 10 diameters downstream the cylinder is characterized by two rows of vortices. In Figure 5 , it can be seen that the critical spacing ratio for the vortex shedding to occur from the upstream cylinders is much smaller than that for flow past four stationary cylinders. The vortex shedding from the two upstream cylinders synchronizes in the lock-in regime. The biased flow disappears when the spacing ratio reaches 4 as shown in Figure 5 (d), indicating the weakness of the influences among the cylinders. The four rows of vortices are very regularly aligned in the wake of the cylinders in Figure 5 (d).

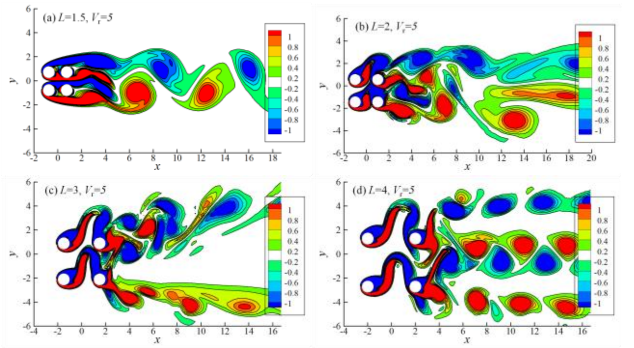

Figure 5.Vorticity contours of 1-dof VIV for V_r=5 and different spacing ratios

Conclusions

VIV of four circular cylinders in an in-line square configuration at a Reynolds number of 150 and a low mass ratio of 2.5 are investigated numerically. In this study, extensive simulation is carried out for the spacing ratios of *L*=1.5, 2, 2.5, 3 and 4 and reduced velocities ranging from 1 to 25, which covers the full lock-in regime for all the gap ratios. The lock-in regime for *L*=1.5 is found to be the widest among other spacing ratios. The lock-in regime for *L*=1.5 is wider and the response amplitude in the lock-in regime is higher, because the four cylinders behave as a single body. The response amplitudes in the lock-in regime for *L*=2 are the smallest among those in other spacing ratios, because of the strong interaction among the vortices. For *L*=2.5, 3 and 4, the lock-in regime is similar to that of a single cylinder and the response amplitudes in the lock-in regime is slightly higher than that of a single cylinder. For the intermediate spacing ratio of *L*=2, the response frequency is found to be close to the natural frequency in the wider range of reduced velocity $3 \le V_r \le 25$. However, the response amplitude for *L*=2 is smaller than those in other spacing ratios. The FFT analysis shows that the lift coefficient at *L*=2 is broad banded and the component of the lift coefficient that excites the vibration is not the peak frequency, leading to low response amplitude. The vortex shedding for VIV of four cylinders is different from that for flow past four stationary cylinders mainly in two aspects. Firstly, the critical spacing ratio for vortex shedding from the two upstream cylinders is between 1.5 and 2, which is much smaller than that for flow past the four stationary cylinders. Secondly, biased vortex shedding flow is observed in the wake of the four vibrating cylinders for *L*=2, 2.5 and 3. The strongest biasness of the flow is found for *L*=3, leading to the increase of the mean cross-flow position with the increasing reduced velocity.

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