19th Australasian Fluid Mechanics Conference
Melbourne, Australia
8-11 December 2014

Computational Optimisation and Analysis of A Truncated Hypersonic Nozzle for X3 Expansion Tunnel

H. Wei[1], W. Y.K. Chan[1], P. A. Jacobs[2], and R. G. Morgan[1]

[1]Centre for Hypersonics, School of Mechanical and Mining Engineering
The University of Queensland, Queensland 4072, Australia

[2]Queensland Geothermal Energy Centre of Excellence, School of Mechanical and Mining Engineering
The University of Queensland, Queensland 4072, Australia

Abstract

Large or full size model testing capability is essential for ground test facilities to replicate key flow features encountered in real flight. The X3 expansion tunnel at the University of Queensland is undergoing upgrades to accommodate both full-scale Hayabusa re-entry vehicle model testing and high Mach number scramjet combustion experiments. The former necessitates a core flow of 400mm in diameter, and the latter entails flow angles below 2° within a core flow diameter of no less than 200mm. To meet the requirements, a Mach 12 nozzle was designed. With conventional full-length nozzle design methodologies, the length of the nozzle would be too large to be feasible, due to not only spatial constraints but also the fact that a significant proportion of the nozzle exit flow would be the boundary layer. Therefore, computational optimisation of a truncated nozzle was carried out, and without enough flow-straightening, it became more difficult to achieve small flow angles. A Navier-Stokes flow solver with a 5-species air finite-rate reaction scheme was employed for the objective function evaluation, with the initial design input a truncated inviscid nozzle contour derived using the method of characteristics. To suppress contour contraction and the generation of shock waves and centreline disturbances in the nozzle flow, associated penalty functions were implemented. CFD simulation of nozzle exit flow development in the dump tank indicated that the nozzle could be further shortened since free expansion in the test section is equivalent to that within the nozzle over the truncated section of the same length, resulting even lower cost and smaller axial vacuum force.

Introduction

Design Objectives

The traceability of ground test data to flight relies on the reproducibility of various non-dimensional flow parameters [3]. Limited by facility size and cost, it is common practice that small-scale models are used, making it impossible to simultaneously recreate vital aspects of hyper-velocity flow in laboratories. For example, while binary collision processes scale with ρL, ρL^2 must be maintained between flight and ground facilities to match three-body chemical reactions. Thus a full-scale model test capability will enhance the similitude between ground testing and flight, and in turn the applicability of test data to vehicle design.

The X3 expansion tunnel at the University of Queensland is currently equipped with a Mach 10 nozzle capable of producing a core flow of roughly 250mm in diameter. Scheduled for the facility are full-scale Hayabusa spacecraft [9] model testing and high Mach number scramjet combustion experiment. Accordingly, a 400mm diameter core flow is necessary for Hayabusa model accommodation, and flow angle must be constrained below 2° within the 200mm inner core flow for scramjet testing. Therefore, the mission of the new nozzle is to stretch X3 facility's competence in both Mach number and core flow size while retaining low flow angle. The target nozzle exit Mach number is 12, with a core flow diameter of 400mm, and flow angle below 2° for the 200mm inner core flow. The inlet diameter of the nozzle is 182.6mm, with the inflow air at Mach 7.5 1000K and 5kPa, which is assumed to be in equilibrium.

While the nozzle is designed to achieve the performance mentioned above, its length and diameter must be minimised. Longer nozzle length enables smoother flow expansion and smaller flow angle, but compromises core flow size due to unregulated growth of the boundary layer. During expansion tunnel operations, the nozzle, together with the dump tank it is connected to, is evacuated to only a few Pascals, causing a large axial vacuum force. With nozzle inlet diameter and ambient pressure fixed, the magnitude of the axial vacuum force is solely dependent on the nozzle exit diameter. To quantify the gravity of the problem, a nozzle with a fair exit diameter of 400mm yields a relatively large axial vacuum force of 48kN. With the nozzle length held constant, a smaller exit diameter generally leads to better flow angularity, but weaker flow expansion and therefore smaller core flow and lower exit Mach number. Given the contradiction between growing boundary layer thickness and exit flow angle, together with that between axial vacuum force and core flow size as well as exit Mach number, a careful balance must be achieved between them.

Design Options

In the early years of hypersonic flow research, two-dimensional nozzles were favoured over axisymmetric nozzles for the use of flexible walls to facilitate variable Mach numbers, the ease of windows installation, and the fear of focusing effects of axisymmetric nozzles [14]. However, flow distortions due to high heating rates at the nozzle throat and non-uniformities at the test section rendered these nozzles undesirable and hence axisymmetric nozzles became the norm for high-enthalpy flow conditions. In addition, an axisymmetric nozzle is superior to a two-dimensional one by offering more efficient expansion [2] and is thus more advantageous for hypervelocity impulse facilities wherein boundary layer growth is quite substantial.

When it comes to nozzle profiles, conical and contoured nozzles are the most prevalent choices. Contoured nozzles are known for generating uniform parallel exit flow under design condition but can produce non-uniformities at off-design conditions. In contrast, conical nozzles are less sensitive to inflow conditions other than design values [16]. However, the exit flow out of a conical nozzle is diverging and is therefore unsuitable for scramjet testing. Based on the above considerations, an axisymmetric contoured design is preferred.

Design Methodology

The classic approach to contoured nozzle design compensates for viscous effects by adding a computed displacement thickness to an inviscid nozzle profile obtained using the method of characteristics, either directly [5] or in an iterative manner [6]. Due to complex thermochemical processes and strong coupling between the boundary layer and the core flow, hypersonic nozzles designed with this method tend to end up with centreline disturbances and shock waves [6]. With the development of CFD techniques and the increasing capacity of computing resources, CFD-based computational optimisation becomes realistic and produces better nozzle designs [7].

The framework of computational optimisation design method applied in the Centre for Hypersonics of the University of Queensland was first established by Craddock [1] for designing scramjet engine flow paths and shock tunnel nozzles, later used by Scott [11] for designing the Mach 10 nozzle for the X2 expansion tunnel, and with an updated implementation by Chan [17], more shock tunnel nozzles were designed and successfully commissioned.

The design procedure starts by generating an inviscid nozzle contour with the method-of-characteristics code iMOC [12]. The nozzle profile is then fitted with 9 Bézier control points, which are capable of smoothly representing different contour patterns with a small number of variables. These points are approximately evenly distributed along the nozzle axis with the first three slightly clustered toward nozzle entrance to better capture details of the initial expansion. The first two points are fixed, and the differences between the radial coordinates of the neighbouring points are taken as the design variables which resemble nozzle wall slopes and improve convergence [8].

Since multiple CFD simulations are necessary during the optimisation process, a dedicated nozzle flow simulation code NENZF-r [10] is used to save computation time, which drives the Navier-Stokes flow solver Eilmer [13] in space-marching mode. A 5-species air (N_2, N, O_2, O, NO) reaction scheme [15] is adopted. The Nelder-Mead algorithm [4] is employed to navigate the search for the optimum design. The procedure is illustrated in Figure 1. After the optimisation converges, further analysis and assessment is required to confirm the final design, which is detailed in the next section.

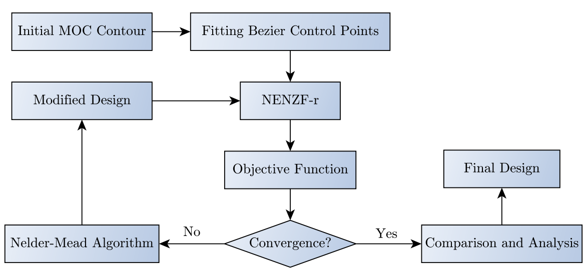

Figure 1. Work flow for the CFD-based computational optimisation design

With smaller nozzle inlet diameters and lower target Mach numbers, desirable optimum designs can be obtained by inputting full-size inviscid method-of-characteristics nozzle contours into the optimisation scheme described above. Given the large inlet size and high exit Mach number of the new X3 nozzle, it is impossible to follow the same practice, as the full-length Mach 12 nozzle would be over 4.5m long. Therefore, a truncated nozzle contour has to be used as design input, wherein the conflict between reduced nozzle length and increased flow angle surfaces, and nozzles with undesirable characteristics tend to be produced more frequently during the optimisation process.

The generation of shock waves within the nozzle is the least wanted situation, which in effect may make the exit core flow Mach number more uniform but would decrease the core flow size as well. The shock waves are found to be associated with two scenarios. One is the contraction of the nozzle cross section area, and the other is large pressure differential between the boundary layer and the core flow. A second major problem is the pressure disturbances along nozzle centreline, which is commonly observed in contoured nozzles but can be eliminated at least for the design condition.

To deal with these problems, a penalty function is imposed on the objective function comprising different terms, each addressing an above-mentioned situation that is linked to or indicative of unacceptable nozzle flow characteristics. The optimisation problem is thus formulated as below:

- Design Variables: $\vec{X} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7]^T, 0 < x_i < x_{max}$
- Objective Function: $f(\vec{X}) = [f_\theta(\vec{X}) + f_M(\vec{X}) + f_P(\vec{X})]^2$

  $$f_\theta(\vec{X}) = \frac{\phi_\theta^2}{N}\sum_{j=1}^{N}(\theta_j(\vec{X}) - \theta_{design})^2$$

  $$f_M(\vec{X}) = \frac{\phi_M^2}{N}\sum_{j=1}^{N}(M_j(\vec{X}) - M_{design})^2$$

- Penalty Function: $f_P(\vec{X}) = f_{PR}(\vec{X}) + f_{CD}(\vec{X}) + f_{BD}(\vec{X})$

  $$f_{PR}(\vec{X}) = \frac{\phi_P^2}{N}(\bar{P}_N - P_{max})^2$$

  $$f_{BD}(\vec{X}) = \begin{cases} Z_1 & x_i < 0 \\ 0 & 0 < x_i < x_{max} \end{cases}$$

  $$f_{CD}(\vec{X}) = \begin{cases} Z_2 & \text{with centreline disturbance} \\ 0 & \text{without centreline disturbance} \end{cases}$$

  N is the number of cells in the core flow along the radial direction, $\bar{P}_N$ the average pressure value in the core flow, and $\phi_M, \phi_\theta, \phi_P$ are weight coefficients

The objective function $f(\vec{X})$ is composed of terms that measure the deviations of nozzle exit Mach number and flow angle from design objectives as well as penalty function $f_{PR}(\vec{X})$ that aids to curb large pressure gradients at the edge of the core flow, $f_{BD}(\vec{X})$ that is activated when a converging contour is generated and $f_{CD}(\vec{X})$ that takes effect when centreline disturbances are detected. Z_1 and Z_2 are very large numbers imposed to eliminate nozzle contraction and centreline disturbances completely, whereas $f_P(\vec{X})$ varies from iteration to iteration. This is because pressure differential between the core flow and the boundary layer cannot be eliminated and only needs to be contained below a certain level to avoid shock wave generation.

Results and Discussions

Centreline Disturbances

Centreline disturbances, although quite common to axisymmetric nozzles, signal the generation of weak shock waves and should be avoided. However, insufficient analysis and comparison has been dedicated to assessing its influence on nozzle performance. Intermediate nozzle contours with centreline disturbances were created in the optimisation process and with the activation of the associated penalty function term, such flow features were eradicated in nozzle contours produced afterwards. Comparisons of Mach number contours, exit Mach number and

Pitot pressure profiles, as well as flow angle distributions between nozzles with and without centreline disturbances are in Figures 2, 3 and 4.

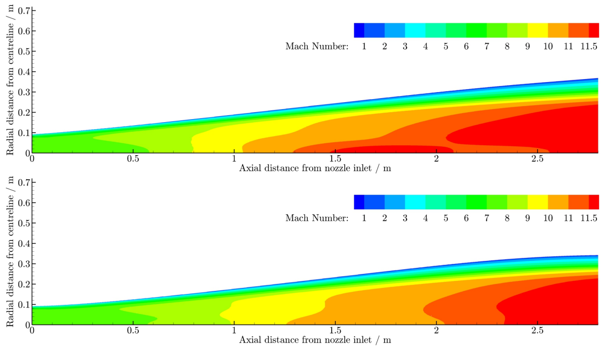

Figure 2. Mach number contours of Mach 12 nozzles with (top) and without (bottom) centreline disturbances

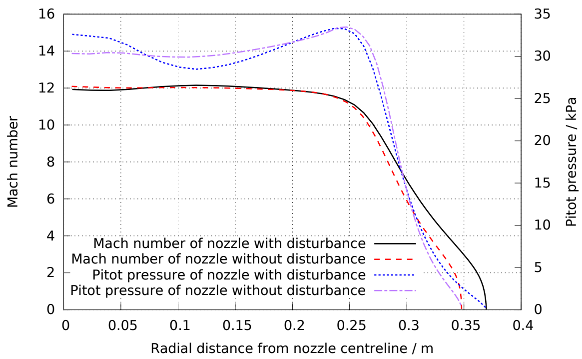

Figure 3. Exit Mach number and Pitot pressure distributions for Mach 12 nozzles with and without centreline disturbances

As is revealed in Figure 2, recompression occurs from about 2.1m to 2.5m along the centreline of the nozzle on the top. Although the two nozzles have similar exit Mach number profiles in Figure 3, the one without centreline disturbances features a more uniform Mach number distribution across the core flow. A larger difference exists between them in terms of exit Pitot pressure uniformity. For the nozzle with centreline disturbances, it drops by about 20% of the average value from the centreline to half way through the core flow and then rises to approximately the centreline value at the core flow edge. It indicates that the inner core flow does not expand sufficiently compared to the rest of the core flow, due to the influence of centreline compression upstream. In contrast, the core flow Pitot pressure of the nozzle without centreline disturbances increases gradually and moderately from the centre axis to the edge.

As is presented in Figure 4, the flow angle for the nozzle with disturbances rises more steeply than its counterpart at the start, then reaches a plateau and stabilises until it increases again from about halfway to the core flow edge, whereas it increases fairly steadily for the disturbance-free nozzle. The flow angle for the disturbance-free nozzle is lower within the 200mm diameter core flow, where flow angle matters the most.

In summary, the disturbance-free nozzle provides not only better exit Mach number and Pitot pressure uniformity, but also smaller flow divergence within the inner core flow. Besides, its exit diameter is also smaller, leading to a decrease in axial vacuum force by 12.7%. Note that nozzles with centreline disturbances were not eliminated until the inclusion of the associated penalty function, which indicates that the optimisation algorithm used is subjected to local minimum convergence, a common drawback of simplex algorithms. In the future, mixed optimisation algorithms should be explored to better achieve global optimum.

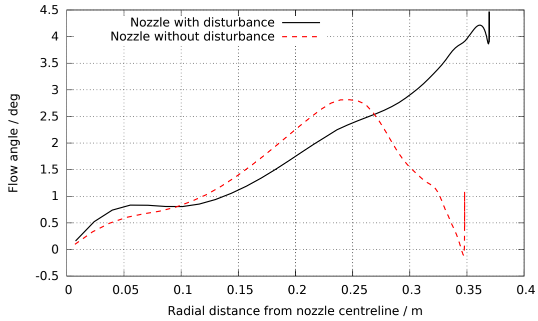

Figure 4. Comparison of exit flow angle distributions for Mach 12 nozzles with and without centreline disturbances

Further Truncation

The optimum Mach 12 nozzle contour obtained is quite close to the one shown at the bottom of Figure 2, but with improved uniformity of exit flow properties and smaller flow angle. It's noted that from about 2.3m downstream the nozzle inlet, the slope of the nozzle profile becomes insignificant, providing certain levels of straightening to the flow. However, with the boundary layer thickness steadily growing without the nozzle profile expanding proportionally, the limited benefit of that additional flow-straightening is readily shadowed by smaller core flow size and higher material and manufacturing cost. With further flow expansion in the dump-tank considered, it is feasible to truncate the 2.8m nozzle by 0.5m and work out how far downstream the nozzle would the flow properties be equivalent to that at the original nozzle exit.

Distributions of Mach number, Pitot pressure and flow angle at two nozzle exits as well as at varied locations downstream the truncated nozzle exit are presented in Figures 5, 6 and 7. It it obvious that the level of expansion experienced by the 2.3m nozzle exit flow over the length of 0.5m downstream is in effect comparable to that within the last 0.5m of the 2.8m nozzle in terms of Mach number, Pitot pressure, flow angle, and core flow size. The possibility of test flow with even higher Mach numbers is also justified since flow properties further downstream still remains desirable. While cutting the cost of material and fabrication by a fair bit, the truncation also reduces the axial vacuum force by 14.3% without compromising flow quality.

Conclusions

Computational optimisation of a truncated Mach 12 nozzle for the X3 expansion tunnel was conducted, and the penalty functions applied in the scheme were able to eliminate nozzle contour contraction, shock wave generation, and centreline flow disturbances. Comparison between nozzles with and without centreline disturbances reveals that apart from slightly better exit Mach number uniformity and flow angularity, the disturbance-free nozzle exhibits noticeably more consistent exit Pitot pressure distributions and is therefore preferable, not to mention the smaller exit diameter and lower axial vacuum force that follows. Further shortening of the optimum Mach 12 nozzle was confirmed by simulating nozzle exit flow into the dump-

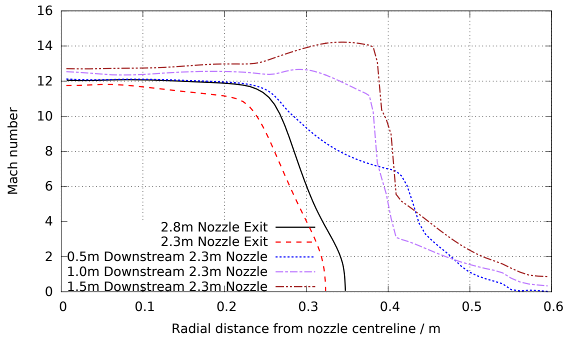

Figure 5. Mach number distributions at different nozzle exit stations and downstream locations in the dump-tank

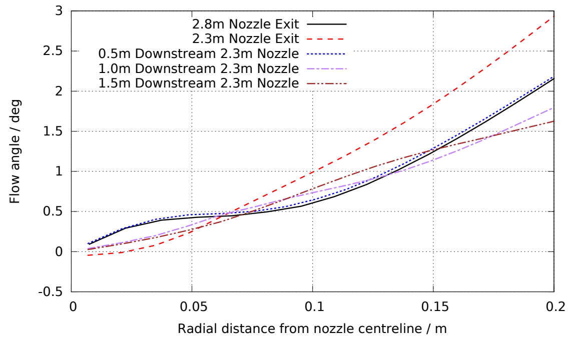

Figure 7. Flow angle distributions at different nozzle exit stations and downstream locations in the dump-tank

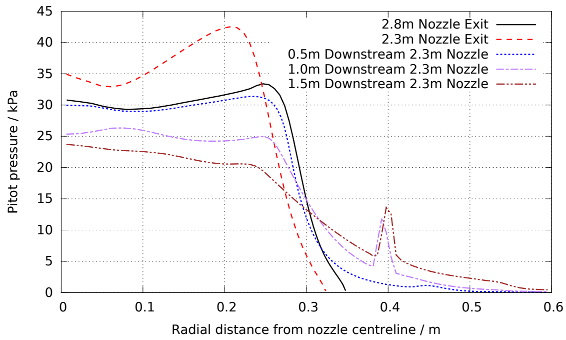

Figure 6. Pitot pressure distributions at different nozzle exit stations and downstream locations in the dump-tank

tank. Equivalent flow conditions were achievable by placing the model downstream of the nozzle with the same length cut from the optimum nozzle. This again brought down material and manufacturing cost as well as axial vacuum force.

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