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Scaling Analysis of Unsteady Natural Convection Boundary Layers on an Evenly Heated Plate with a Time-dependent Temperature

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Abstract

In this paper, a scaling analysis using a simple three-region structure was conducted for the unsteady natural convection boundary layer (NCBL) of a homogeneous Newtonian fluid with $\mathrm{Pr} > 1$ adjacent to a vertical plate evenly heated with a time-dependent sinusoidal temperature. A series of scalings were developed for the thermal boundary thickness, the viscous boundary thicknesses, the maximum vertical velocity within the boundary layer, and the local and average Nusselt number across the plate, which are the major parameters representing the flow behavior, in terms of the governing parameters of the flow, *i.e.*the Rayleigh number Ra, the Prandtl number Pr, and the dimensionless natural frequency f_n of the time-dependent sinusoidal temperature, at the start-up stage, at the transition time scale which represents the ending of the start-up stage and the beginning of the transitional stage of the boundary-layer development, and at the quasi-steady stage.

Introduction

As a classic fluid mechanics problem, NCBL flow has been widely studied. Earlier studies had focused on experimental and analytical investigations of the steady behavior of the flow, in particularly that on a heated semi-infinite vertical wall and in a rectangular cavity with differentially heated sidewalls. More recent studies have focused on the transient flow behavior. In particular, scaling analysis has proven to be a very effective tool to reveal the transient behavior of such a flow since Patterson and Imberger [1] made a pioneering scaling analysis of the transient NCBL flow in a two-dimensional rectangular cavity with differentially heated sidewalls. This study has inspired many subsequent studies to extend scaling analysis to many different aspects of transient NCBLs under various configurations and flow conditions. The readers are referred to our recent papers (*e.g.*[2]) for a more detailed review of some of these studies.

Unsteady NCBLs on a vertical plate heated by a time-dependent heat flux or temperature are found in many applications, such as in the Trombe wall system of a passive solar house and in a solar chimney for electricity generation. In the Trombe wall case, the wall, which is usually painted in black or with a solar selective coating, absorbs solar radiation and converts it into heat which is then transported to the dwelling by the heated air via NCBL flow in the channel formed by the glazing and the wall. A solar chimney operates in a similar manner. For both cases, the time-dependent solar radiation, which varies sinusoidally under a clear sky condition (only in the first half of the sinusoidal cycle), serves as the heat flux for the NCBL flows. Although there have been numerous studies on NCBLs on a vertical plate heated by a heat flux, the majority of these studies have been on the cases where the applied heat flux is either uniformly constant or spatially varied but not time dependent. Lin and Armfield [3] recently carried out a scaling analysis to develop scalings for the unsteady NCBL of a homogeneous Newtonian fluid with $\mathrm{Pr} > 1$ adjacent to a vertical plate evenly heated with a time-varying sinusoidal heat flux, which were validated and quantified by a series of direct numerical simulations. In the current study, this scaling analysis is extended to the unsteady NCBL of a homogeneous Newtonian fluid with $\mathrm{Pr} > 1$ adjacent to a vertical plate evenly heated with a time-dependent sinusoidal temperature, which, to our best knowledge, has not been done so far, although its fundamental significance and practical application importance.

Scaling Analysis

Under consideration is the unsteady NCBL of a homogeneous Newtonian fluid with $\mathrm{Pr} > 1$ adjacent to a vertical plate evenly heated with a time-dependent sinusoidal temperature in the form of

$$T_w(t) = T_a + T_{w,m}\sin(2\pi f t) \tag{1}$$

where t is time, T_a is the initial fluid temperature at $t = 0$, $T_{w,m}$ and f are the amplitude and the natural frequency of the time-dependent sinusoidal temperature applied to the plate, respectively. The flow is assumed to be two-dimensional and the fluid is initially at rest. The plate lies at $X = 0$ with the origin at $Y = 0$ (X and Y are the horizontal and vertical coordinates, respectively), with the plate boundary conditions

$$U = V = 0,\ T_w(t) = T_a + T_{w,m}\sin(2\pi f t)\ \text{at}\, x = 0 \text{ for } Y > 0$$

where $T_{w,m}$ and f are assumed to be constant for a specific time-dependent temperature condition.

The governing equations of motion are the Navier-Stokes equations with the Boussinesq approximation for buoyancy, which together with the temperature equation can be written in the following two-dimensional forms,

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

$$\frac{\partial U}{\partial t} + \frac{\partial (UU)}{\partial X} + \frac{\partial (VU)}{\partial Y} = -\frac{1}{\rho}\frac{\partial P}{\partial X} + \nu\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{3}$$

$$\begin{aligned}\frac{\partial V}{\partial t} + \frac{\partial (UV)}{\partial X} + \frac{\partial (VV)}{\partial Y} \quad = \quad & -\frac{1}{\rho}\frac{\partial P}{\partial Y} + \nu\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) \\ & + g\beta(T - T_a)\end{aligned} \tag{4}$$

$$\frac{\partial T}{\partial t} + \frac{\partial (UT)}{\partial X} + \frac{\partial (VT)}{\partial Y} = \kappa\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right) \tag{5}$$

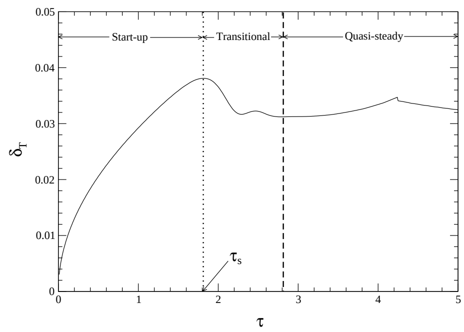

Figure 1. The three distinct stages in the boundary-layer development, seen in the typical numerically simulated time series of the dimensionless local thermal boundary-layer thickness $\delta_T = \Delta_T/H$ at height $Y = 0.5H$ for the specific case $\mathrm{Ra} = 10^8$, $\mathrm{Pr} = 7$ and $f_n = 0.1$, where time τ is made dimensonless by H/V_0, f_n is the dimensionless natural frequency of the time-dependent flux applied to the plate, and τ_s $(= t_s/(H/V_0))$ is the dimensionless transition time scale representing the end of the start-up stage and the beginning of the transitional stage.

where U and V are the X and Y direction velocity components, P is pressure, g is the acceleration due to gravity, β, ν and κ are the thermal expansion coefficient, kinematic viscosity and thermal diffusivity of the fluid at the temperature T_a, respectively. Gravity acts in the negative $Y-$direction.

For the unsteady NCBL flow considered here, the major governing parameters are the Rayleigh number Ra and the Prandtl number Pr, defined as

$$\mathrm{Ra} = \frac{g\beta\Delta T_0 H^3}{\nu\kappa},\ \mathrm{Pr} = \frac{\nu}{\kappa} \tag{6}$$

where

$$\Delta T_0 = \overline{T}_w - T_a \tag{7}$$

and $\overline{T}_w$ is the time-averaged temperature on the plate which is calculated by

$$\overline{T}_w = \frac{1}{t_{total}}\int_0^{t_{total}} [T_a + T_{w,m}\sin(2\pi f t)]dt = \frac{2}{\pi}T_{w,m} + T_a \tag{8}$$

where t_{total} is the total heating time of the time-dependent temperature applied to the plate. In this paper, it is assumed that $2\pi f t_{total} = \pi$, i.e., $f = 0.5/t_{total}$ (hence only the first half, heating cycle is considered). Apparently, Ra defined above is the time-averaged global Rayleigh number for the unsteady NCBL over the duration of heating, calculated in terms of $T_{w,m}$ by

$$\mathrm{Ra} = \left(\frac{2}{\pi}\right)\frac{g\beta T_{w,m}H^3}{\nu\kappa} \tag{9}$$

The scaling analysis is carried out by examining in detail the various balances in the governing equations ([1]) for unsteady NCBL flows and by using the similar procedures by [3, 4, 5], modified appropriately in the context of the time-dependent temperature applied to the plate.

With the initiation of the flow, a vertical boundary layer will be developed adjacent to the plate which will experience a start-up stage dominated by one-dimensional conduction, followed by a transitional stage during which traveling waves, associated with the leading edge effect, are present and a transition to two-dimensional convection occurs, before eventually reaching a quasi-steady stage, with the transition time scale t_s (τ_s is its dimensionless form) separating the start-up stage and the transitional stage. This is illustrated in figure1 where a typical numerically simulated time series of the thermal boundary-layer thickness Δ_T ($\delta_T = \Delta_T/H$ is its dimensionless form) is shown. Δ_T is defined as the horizontal distance between the plate and the location where the fluid temperature reaches $0.01[T_w(t) - T_a]$. Similar behavior is also observed for the other parameters of interest to this work, *i.e.*the maximum vertical velocity within the boundary layer, V_m, the inner viscous boundary-layer thickness, Δ_{vi}, and the local Nusselt number across the plate, Nu_y.

Scalings at the Start-up Stage

The forcing for the vertical boundary layer is from conduction of heat through the plate. The ratio of the unsteady term $(\Delta T/t)$ to the dominant convection term $(V\Delta T/H)$ in the temperature equation (5) is $O(H/Vt)$, where

$$\Delta T = T_w(t) - T_a = T_{w,m}\sin(2\pi f t) \tag{10}$$

and for sufficiently small time t this is much larger than 1 (*i.e.*$Vt \ll H$), so the initial balance is between the heat conducted in through the plate (*i.e.*the term $\kappa\Delta T/\Delta_T^2$) and the unsteady term, which leads to the following scaling for the thermal boundary layer thickness Δ_T at the start-up stage,

$$\Delta_T \sim \kappa^{\frac{1}{2}} t^{\frac{1}{2}} \tag{11}$$

Hence, a horizontal gradient in temperature exists from the plate to a distance Δ_T in the ambient fluid.

The buoyancy forces resulting from this heating act to accelerate the flow over the thickness Δ_T only. In this region, the ratio of the inertial term to the viscous term in the vertical momentum equation (4) is $O[(V/t)/(\nu V/\Delta_T^2)] \sim O(\Delta_T^2/\nu t) \sim O(1/\mathrm{Pr})$ as $\Delta_T \sim \kappa^{\frac{1}{2}} t^{\frac{1}{2}}$ as shown in (11). This is much smaller than 1 for $\mathrm{Pr} \gg 1$, so that the balance over Δ_T is between the viscous term, $\nu\partial^2 V/\partial X^2$, and the buoyancy term, $g\beta\Delta T$.

The peak velocity V_m must occur within Δ_T. Suppose V_m is at a horizontal distance Δ_{vi} from the plate. Also for $\mathrm{Pr} > 1$ there will be a region of flow outside Δ_T where there is flow which is not directly forced by buoyancy, but is the result of the viscous diffusion of momentum. Suppose this is Δ_v from the wall. Hence a three-region structure originally proposed by [5, 4], as reproduced in Fig. 2, can be depicted for natural convection boundary layers of $\mathrm{Pr} > 1$ fluids.

In regions I and II, the balance is between viscosity and buoyancy, i.e.,

$$0 \sim \nu\frac{\partial^2 V}{\partial X^2} + g\beta\Delta T \tag{12}$$

However, in region III the balance is between viscosity and inertia, since there is no buoyancy there.

In region I, the balance (12) gives,

$$\nu\frac{V_m}{\Delta_{vi}^2} \sim g\beta\Delta T \tag{13}$$

i.e.,

$$V_m \sim \frac{g\beta\Delta T}{\nu}\Delta_{vi}^2 \tag{14}$$

In region II, the forcing is over the distance $(\Delta_T - \Delta_{vi})$, but the gradient of V is over $(\Delta_v - \Delta_{vi})$. The best way to look at this is to integrate the vertical momentum equation over region II, giving

$$0 \sim \nu\left(\frac{\partial V}{\partial X}\right)_{\Delta_{vi}}^{\Delta_T} + g\beta\int_{\Delta_{vi}}^{\Delta_T}\Delta T\,dX \tag{15}$$

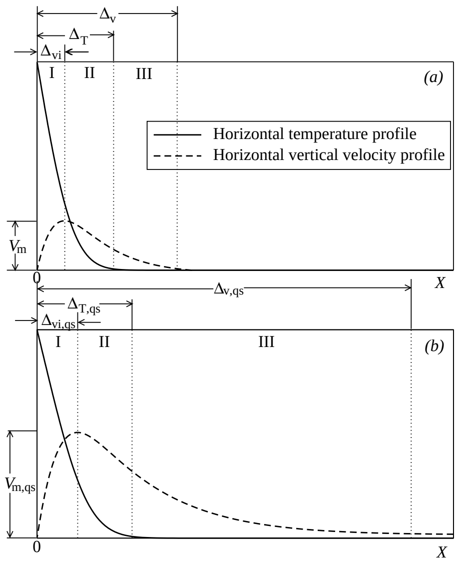

Figure 2. The three-region structure of the unsteady NCBL adjacent to the vertical plate for $\mathrm{Pr} > 1$ fluids [4, 5]: (*a*) at the start-up stage and (*b*) at the quasi-steady stage.

Since $(\partial V/\partial X)_{\Delta_{vi}} = 0$ (that is where the maximum is) and approximating $(\partial V/\partial X)_{\Delta T} \sim V_m/(\Delta_v - \Delta_{vi})$ and $\int_{\Delta_{vi}}^{\Delta_T} \Delta T\, dx \sim \Delta T(\Delta_T - \Delta_{vi})$, this gives

$$\nu \frac{V_m}{\Delta_v - \Delta_{vi}} \sim g\beta\Delta T(\Delta_T - \Delta_{vi}) \tag{16}$$

Hence,

$$V_m \sim \frac{g\beta\Delta T}{\nu}(\Delta_T - \Delta_{vi})(\Delta_v - \Delta_{vi}) \tag{17}$$

Matching this with (14) obtained above for V_m gives

$$(\Delta_T - \Delta_{vi})(\Delta_v - \Delta_{vi}) \sim \Delta_{vi}^2 \tag{18}$$

so that

$$\Delta_T\Delta_v - (\Delta_T + \Delta_v)\Delta_{vi} + \Delta_{vi}^2 \sim \Delta_{vi}^2 \tag{19}$$

which leads to

$$\Delta_{vi} \sim \frac{\Delta_T\Delta_v}{\Delta_T + \Delta_v} \tag{20}$$

In region III, as there is no buoyancy force, the flow is driven solely by diffusion of momentum, meaning that the unsteady term balances the viscous term, giving

$$\frac{V}{t} \sim \nu\frac{V}{\Delta_v^2} \tag{21}$$

This leads to

$$\Delta_v \sim \nu^{\frac{1}{2}} t^{\frac{1}{2}} \sim \mathrm{Pr}^{\frac{1}{2}}\Delta_T \tag{22}$$

which is the scaling for Δ_v at the start-up stage. Hence, the scaling (20) becomes

$$\Delta_{vi} \sim \frac{\mathrm{Pr}^{\frac{1}{2}}}{1+\mathrm{Pr}^{\frac{1}{2}}}\Delta_T \sim \frac{1}{1+\mathrm{Pr}^{-\frac{1}{2}}}\kappa^{\frac{1}{2}} t^{\frac{1}{2}} \tag{23}$$

This is the scaling for Δ_{vi} at the start-up stage.

So the scaling (14) for V_m becomes

$$V_m \sim \frac{g\beta\Delta T}{\nu}\Delta_{vi}^2 \sim \frac{g\beta\Delta T}{\nu}\left(\frac{1}{1+\mathrm{Pr}^{-\frac{1}{2}}}\right)^2 \Delta_T^2 \tag{24}$$

Since $\Delta_T \sim \kappa^{\frac{1}{2}} t^{\frac{1}{2}}$, with Eqs. (9) and (10), this leads to

$$V_m \sim \frac{\kappa^2 \mathrm{Ra}}{H^3}\left(\frac{1}{1+\mathrm{Pr}^{-\frac{1}{2}}}\right)^2 \sin(2\pi f t)t \tag{25}$$

which is the scaling for V_m at the start-up stage.

The local Nusselt number across the plate at height Y at the start-up stage is then,

$$Nu_y \sim \frac{(\partial T/\partial X)_{X=0}}{\Delta T_0/H} \sim \frac{H}{\Delta T_0}\frac{\Delta T}{\Delta_T} \tag{26}$$

which, with (7), (10), and (11), leads to

$$Nu_y \sim \frac{\sin(2\pi f t)}{\Delta_T/H} \sim \frac{H\sin(2\pi f t)}{\kappa^{1/2}t^{1/2}} \tag{27}$$

This scaling shows that at the start-up stage, the local Nusselt number is in fact independent of height Y and hence the average Nusselt number over the whole plate at the start-up stage has the same scaling as (27).

Scalings at the Transition Time Scale t_s

The boundary layer will continue to grow until convection of heat carried away by the flow balances the conduction of heat transferred in from the plate, The start-up stage is then complete and the transitional stage starts. At a height Y, this happens at

$$\frac{V_m\Delta T}{Y} \sim \kappa\frac{\Delta T}{\Delta_T^2} \tag{28}$$

i.e.

$$\frac{\kappa^2\mathrm{Ra}}{H^3 Y}\left(\frac{1}{1+\mathrm{Pr}^{-\frac{1}{2}}}\right)^2 \sin(2\pi f t)t \sim \kappa\frac{1}{\kappa t} \tag{29}$$

This leads to

$$t^2 \sim \frac{H^3 Y}{\kappa^2\mathrm{Ra}}\frac{(1+\mathrm{Pr}^{-\frac{1}{2}})^2}{\sin(2\pi f t)} \tag{30}$$

which gives the following scaling for the transition time scale t_s which represents the ending of the start-up stage and the beginning of the transitional stage at height Y,

$$t_s \sim \frac{H^2(Y/H)^{1/2}(1+\mathrm{Pr}^{-\frac{1}{2}})}{\kappa\mathrm{Ra}^{\frac{1}{2}}[\sin(2\pi f t_s)]^{1/2}} \tag{31}$$

The corresponding scaling for the maximum velocity scale at height Y at the transition time scale t_s, from (25), is

$$V_{m,s} \sim \frac{\kappa\mathrm{Ra}^{1/2}}{H}\frac{[\sin(2\pi f t_s)]^{1/2}}{(1+\mathrm{Pr}^{-1/2})}\left(\frac{Y}{H}\right)^{1/2} \tag{32}$$

The scalings for the thermal boundary layer thickness, inner viscous boundary layer thickness, and whole viscous boundary layer thickness at height Y at t_s, from (11), (23), and (22), are respectively,

$$\Delta_{T,s} \sim \kappa^{1/2}t_s^{1/2} \sim \frac{H}{\mathrm{Ra}^{1/4}}\frac{(1+\mathrm{Pr}^{-1/2})^{1/2}}{[\sin(2\pi f t_s)]^{1/4}}\left(\frac{Y}{H}\right)^{1/4} \tag{33}$$

$$\Delta_{vi,s} \sim \frac{H}{\mathrm{Ra}^{1/4}}\frac{1}{(1+\mathrm{Pr}^{-1/2})^{1/2}[\sin(2\pi f t_s)]^{1/4}}\left(\frac{Y}{H}\right)^{1/4} \tag{34}$$

$$\Delta_{v,s} \sim \frac{H}{\mathrm{Ra}^{1/4}}\frac{\mathrm{Pr}^{1/2}(1+\mathrm{Pr}^{-1/2})^{1/2}}{[\sin(2\pi f t_s)]^{1/4}}\left(\frac{Y}{H}\right)^{1/4} \tag{35}$$

The scaling for the local Nusselt number across the plate at height Y at t_s, from (27), is

$$Nu_{y,s} \sim \frac{\mathrm{Ra}^{1/4}}{(Y/H)^{1/4}} \frac{[\sin(2\pi f t_s)]^{5/4}}{(1+\mathrm{Pr}^{-1/2})^{1/2}} \tag{36}$$

Hence, the scaling for the average Nusselt number over the whole plate at t_s is

$$Nu_s \sim \frac{1}{H}\int_0^H Nu_{y,s} dY \sim \frac{\mathrm{Ra}^{1/4}[\sin(2\pi f t_s)]^{5/4}}{(1+\mathrm{Pr}^{-1/2})^{1/2}} \tag{37}$$

Scalings at the Quasi-steady Stage

The mechanisms governing the behavior of the boundary layer during the transitional development stage become quite complicated due to travelling waves caused by the leading edge effect, and it is speculated that no simple scalings can be developed for this stage. Subsequent to the passage of the leading edge waves the boundary layer is at the quasi-steady stage. At any time instant t, the convection of heat carried away by the flow again balances the conduction of heat transferred in from the plate, indicating that at a height Y the balance represented by (28) still holds in the quasi-steady stage.

In the vertical momentum equation (4), over Δ_T, buoyancy balances viscosity. Hence, in region I, the balance represented by (13) and the corresponding scaling for V_m, *i.e.*(14), both apply here as well. In region II, this buoyancy-viscosity balance again leads to (15) to (19), which eventually lead to the same relation (20) as developed in Section .

Using (23), the scaling (14) becomes,

$$V_m \sim \frac{g\beta\Delta T}{\nu} \frac{\Delta_T^2}{(1+\mathrm{Pr}^{-\frac{1}{2}})^2} \tag{38}$$

Combining scalings (28) and (38) gives,

$$\frac{\kappa Y}{\Delta_T^2} \sim \frac{g\beta\Delta T}{\nu} \frac{\Delta_T^2}{(1+\mathrm{Pr}^{-\frac{1}{2}})^2} \tag{39}$$

which, with (10) and (9), leads to the following scaling for the thermal boundary-layer thickness at any time t in the quasi-steady state,

$$\Delta_{T,qs} \sim \frac{H}{\mathrm{Ra}^{1/4}} \frac{(1+\mathrm{Pr}^{-1/2})^{1/2}}{[\sin(2\pi f t)]^{1/4}} \left(\frac{Y}{H}\right)^{1/4} \tag{40}$$

With (40), the scaling (38) leads to,

$$V_{m,qs} \sim \frac{\kappa \mathrm{Ra}^{1/2}}{H} \frac{[\sin(2\pi f t)]^{1/2}}{(1+\mathrm{Pr}^{-1/2})} \left(\frac{Y}{H}\right)^{1/2} \tag{41}$$

which is the scaling for the maximum vertical velocity within the boundary layer at any time in the quasi-steady state. The scaling for the inner viscous boundary layer thickness at height Y at any time t in the quasi-steady stage, from (23), is

$$\Delta_{vi,qs} \sim \frac{H}{\mathrm{Ra}^{1/4}} \frac{1}{(1+\mathrm{Pr}^{-1/2})^{1/2}[\sin(2\pi f t)]^{1/4}} \left(\frac{Y}{H}\right)^{1/4} \tag{42}$$

The scaling for the whole viscous boundary layer thickness at height Y at any time t in the quasi-steady stage, from (22), is

$$\Delta_{v,qs} \sim \frac{H}{\mathrm{Ra}^{1/4}} \frac{\mathrm{Pr}^{1/2}(1+\mathrm{Pr}^{-1/2})^{1/4}}{[\sin(2\pi f t)]^{1/4}} \left(\frac{Y}{H}\right)^{1/4} \tag{43}$$

The scaling for the local Nusselt number at height Y at any time t in the quasi-steady stage, from (27), is

$$Nu_{y,qs} \sim \frac{\sin(2\pi f t)}{\Delta_{T,qs}/H} \sim \frac{\mathrm{Ra}^{1/4}}{(Y/H)^{1/4}} \frac{[\sin(2\pi f t)]^{5/4}}{(1+\mathrm{Pr}^{-1/2})^{1/2}} \tag{44}$$

and the scaling for the average Nusselt number over the whole plate at any time t in the quasi-steady stage becomes

$$Nu_{qs} \sim \frac{1}{H}\int_0^H Nu_{y,qs} dY \sim \frac{\mathrm{Ra}^{1/4}[\sin(2\pi f t)]^{5/4}}{(1+\mathrm{Pr}^{-1/2})^{1/2}} \tag{45}$$

It should be noted that, although the scalings (40)–(45) at the quasi-steady stage are in the same form as their respective counterparts at the transition time scale t_s, *i.e.*(33), (32), (34)–(37), these scalings apply for any time t in the quasi-steady stage, whereas the scalings (32)–(37) are only valid at t_s.

Conclusions

In this paper, it was found that the transient and quasi-steady flow behavior of the unsteady NCBL of a homogeneous Newtonian fluid with $\mathrm{Pr} > 1$ adjacent to a vertical plate evenly heated with a time-dependent sinusoidal temperature is controlled by Ra, Pr, and f_n of the sinusoidal temperature and is well represented by parameters such as the thermal boundary-layer thickness, the viscous boundary-layer thickness, the maximum vertical velocity within the boundary layer, and the local and average Nusselt number across the plate. Scalings were developed for the different development stages of the flow, *i.e.*the start-up stage, the transition time scale which represents the end of the start-up stage and the beginning of the transitional stage of the boundary-layer development, and the quasi-steady stage, by using a simple three-region structure proposed by [4, 5]. All these scalings are validated by direct numerical simulation results, which will be detailed in the presentation at the conference.

Acknowledgements

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