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Turbine Blockage in Non-Uniform Flow

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Abstract

Analytical models of the flow passing through a porous actuator disc can be used to estimate the power that can be extracted by a wind or tidal turbine. However, the models that have been developed to date assume that the flow field is uniform upstream of the turbine. In practice this is not the case for both wind and tidal turbines, where a non-uniform sheared velocity profile is more likely. To help overcome this problem, in this paper we extend the traditional actuator disc models to incorporate non-uniform upstream flow. Using the extended model we show that flow non-uniformity alters the power that can be removed by a disc in a confined flow because it alters the 'effective blockage' of the disc. Results from the model compare very well with numerical solution of the Euler equations and have potentially important implications for quantifying blockage effects experienced by wind turbines in tunnels and tidal turbines in tidal channels when the velocity profile is not uniform.

Introduction

Estimating the power that can be extracted by a wind or tidal turbine in uniform flow is complicated because the resistance offered by the turbine acts to divert flow around it. Modelling this flow diversion in its entirety requires analysis of flow structures across multiple scales, including the blade of the turbine, the diameter of the turbine and the length of the turbine wake. Recently computational models have been developed for single turbines which aim to model the flow across most scales (see, for example, [2]), however these computational models require further development to be useful in design of new turbines or arrays of turbines.

A well-known approximation to circumvent the need to predict some of the complicated flow structure around an actual turbine is to replace the turbine with a porous actuator disc having the same diameter of the turbine and able to exert a uniform streamwise resistance to the flow (the resistance being related to the streamwise force associated with the turbine blades). This assumption avoids the need to resolve the flow structure at the blade/turbine scale and, more importantly, allows arguments of mass, energy and momentum to be used selectively to quantify the diversion of flow around the disc and estimate the power extracted by the disc (see, for example, [1]).

For a single disc representing an isolated turbine in uniform flow, first introduced by Lanchester, Betz and Joukowsky [8], it can be shown that, because of flow diversion, at most only 16/27 times the upstream kinetic flux passing through an area equal to that of the disc can be extracted by the disc as useful power. In practice this limit (often referred to as the Betz limit) has proven to be a useful benchmark for the wind industry, whilst the combination of actuator disc theory with a blade element theory (which relates the disc resistance to forces on the turbine blades) has provided a valuable wind turbine design tool [1].

Motivated by the success of the actuator disc assumption for modelling wind turbines, in recent years extensions have been made to model turbines in confined flows [5], rows of closely space turbines [6,7] and arrays of staggered or centred turbines [3]. Each of these extensions has been presented in the context of tidal stream turbines and has provided insight into optimum strategies to arrange turbines. However, in all cases the flow upstream of the turbine(s) has been assumed to be uniform. This assumption is violated in real flows, where a non-uniform sheared velocity profile is present in most scenarios for tidal turbines and wind turbines.

The aim of this paper is to present a preliminary analytical model of a turbine with a non-uniform upstream flow. The resulting model is then explored and compared with numerical solutions of the Euler equations obtained using a spectral Discontinuous Galerkin Finite Element Method. Implications of the model are discussed.

Actuator Disc Model

To extend the traditional actuator disc model we start with the non-uniform velocity profile sketched in figure 1. This profile has the advantage of being simple and is equivalent to that considered previously by Draper and Nishino [3], but for a different application. The velocity profile in figure 1 is completely defined by the streamwise velocity u, the width of the channel A/B (where A is the cross-sectional area of the disc and B defines the blockage ratio equal to disc cross-sectional area divided by channel cross-sectional area) and two additional parameters r and ϕ. In this paper we will assume that $\phi \geq 0$ (i.e. unidirectional flow). We will also assume that $1/B \geq r \geq 1$, so that the flow passing through the disc is locally uniform. In this simple problem, letting $r \rightarrow 1/B$ recovers the classic uniform confined flow problem analysed previously by [5].

Accounting for the non-uniform flow, the assumed flow field is defined by the curved dividing streamlines shown in figure 1. These streamlines separate the core flow through the turbine from the two bypass flow regions having upstream velocity u and ϕu, respectively. Within the different regions of the flow field the parameters α_2, α_4, β_4 and β_5 represent multipliers on the average velocity. These regions coincide with stations situated at different streamwise locations to represent far upstream (location 1), far downstream where the pressure has equalised across the flow (location 4) and immediately either side of the disc (location 2 and 3). The only assumption placed on the multipliers are that $0 \leq \alpha_2 \leq 1$ and $\alpha_4 \leq \alpha_2$.

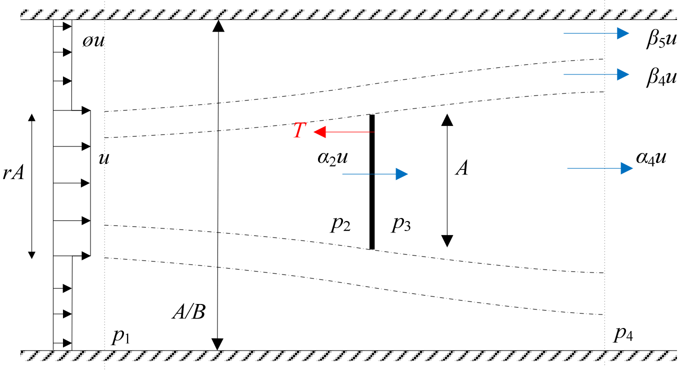

Figure 1. Actuator disc in confined flow with an upstream piece-wise constant velocity profile defined by the parameters r and ϕ.

If we assume that B, r and ϕ are known, together with the velocity multiplier α_4 (representing, essentially, the porosity of the disc) we can solve for the unknown velocity coefficients in the same way as [3] by applying arguments of continuity and conservation of energy and momentum assuming that the flow is inviscid and irrotational everywhere except on the dividing streamlines themselves.

Conservation of mass across the entire flow field can be used initially to write:

$$\alpha_2 = -\frac{\alpha_4(1-\beta_4)}{B(\alpha_4-\beta_4)} - (1-rB)\frac{\beta_4\alpha_4}{B(\alpha_4-\beta_4)}\left(\frac{1}{\beta_4}-\frac{\phi}{\beta_5}\right). \tag{1}$$

Application of the Bernoulli equation along any streamline passing through the disc also leads to:

$$p_2 - p_3 = p_1 - p_4 + \frac{1}{2}\rho u^2\left(1-\alpha_4^2\right). \tag{2}$$

Similarly, application of the Bernoulli equation along streamlines in the flow bypassing the disc leads to:

$$p_1 - p_4 = \frac{1}{2}\rho u^2\left(\beta_4^2 - 1\right) = \frac{1}{2}\rho u^2\left(\beta_5^2 - \phi^2\right), \tag{3}$$

and

$$\beta_5 = \left(\beta_4^2 + \phi^2 - 1\right)^{1/2}. \tag{4}$$

The result in Equation (3) can now be combined with (2) to give:

$$T = \frac{1}{2}\rho A u^2\left(\beta_4^2 - \alpha_4^2\right), \tag{5}$$

where static equilibrium across the disc has been used to write $T = (p_2 - p_3)A$. Finally, conservation of streamwise momentum leads to (after rearranging):

$$T = \frac{A(p_1-p_4)}{B} + \rho A u^2 \alpha_2(1-\alpha_4) - \rho A u^2(r-\alpha_2)(\beta_4-1) - \rho A u^2 \phi\left(\frac{1-rB}{B}\right)(\beta_5-\phi) \tag{6}$$

Combining (1), (2) and (3) with (5), equation (6) can be rewritten as

$$A_1\beta_4^2 - A_2\beta_4 + A_3 = 0, \tag{7}$$

where

$$A_1 = 1 - B, \tag{8}$$

$$A_2 = 2(1-\alpha_4) - 2(1-rB)\left(1-\frac{\alpha_4\phi}{\beta_5}\right), \tag{9}$$

$$A_3 = \left(1 - 2\alpha_4\left(1-\frac{B\alpha_4}{2}\right)\right) - 2(1-rB)\left(1-\alpha_4+\phi(\beta_5-\phi)\right). \tag{10}$$

Equation (7) can be solved easily numerically using the prescribed inputs α_4, B, r and ϕ and Equation (4).

The power removed by the disc is then

$$P = \alpha_2 u \times T = \alpha_2\left(\beta_4^2 - \alpha_4^2\right)\left(\frac{1}{2}\rho A u^3\right) = C_P\left(\frac{1}{2}\rho A u^3\right), \tag{11}$$

where α_2 is calculated from (1) (or from (6) when $B = 0$) and C_P is a power coefficient.

Functionally, we can now write $C_P(r, \phi, B, \alpha_4)$. However, for convenience we will replace the parameter α_4 with a different coefficient, k, defined as a local thrust coefficient for the turbine so that:

$$T = \frac{1}{2}k\rho A(\alpha_2 u)^2. \tag{12}$$

Hence $k = (\beta_4^2 - \alpha_4^2)/\alpha_2^2$, which can be easily determined numerically via iteration. Arguably k gives a more physically appealing description of disc porosity. Adopting this change we can now write $C_P(r, \phi, B, k)$.

For reference, in the limit $r \to 1/B$ it can be easily confirmed that (7) drops back to the same equations presented by Garrett and Cummins [5] for a turbine in a uniform flow. In that special case the maximum power coefficient for a given blockage ratio occurs when $k = 2(1+B)^3/(1-B)^2$ and is given by [5]:

$$C_{P,\max}^{U} = \frac{16}{27}\frac{1}{(1-B)^2}. \tag{13}$$

To explore the power coefficient in the case of a non-uniform flow, figure 2 presents the power coefficient computed as a function of disc resistance k, for a disc blockage of $B = 1/5$ and three different velocity profiles. This figure shows clearly how the velocity profile can impact directly the power extracted by the porous disc. Essentially, when the bypass flow is slower (faster) than the flow which intercepts the disc, a lower (higher) pressure difference can be achieved across the disc in the streamwise direction and this reduces (increases) the power extracted by the disc.

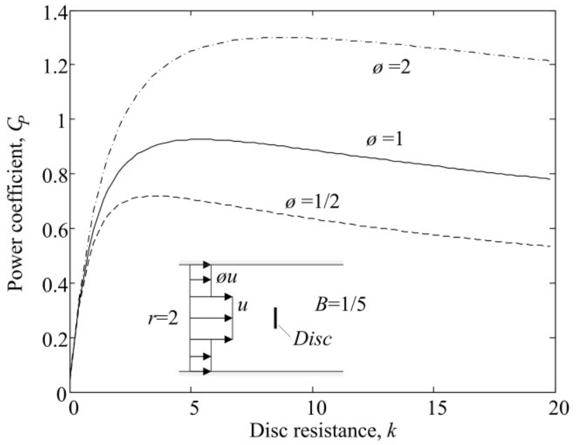

Figure 2. Power coefficient as a function of disc resistance for three different velocity profiles.

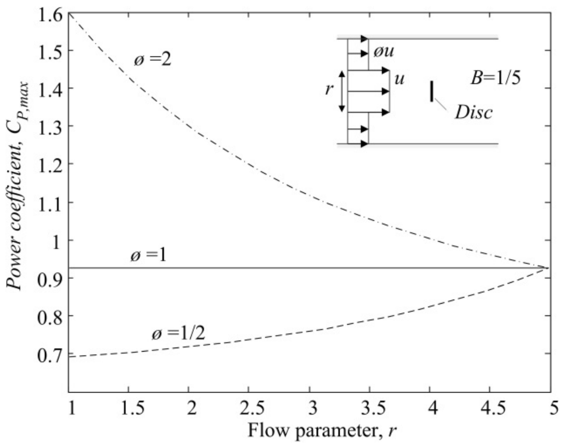

Figure 3. Variation in maximum power coefficient with flow velocity parameter r.

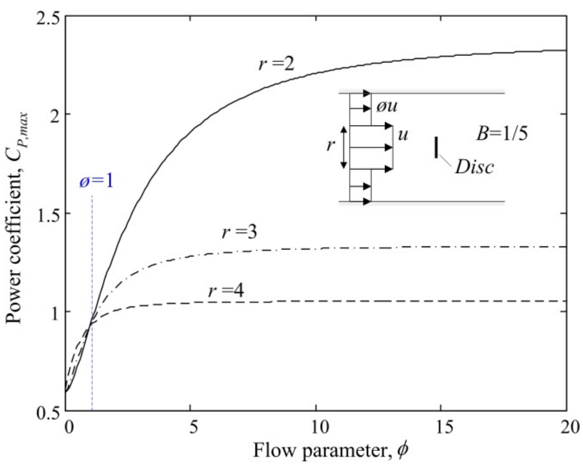

Figure 4. Variation in maximum power coefficient with flow velocity parameter ϕ.

The result in figure 2 indicates that the power coefficient can be very sensitive to flow non-uniformity. To explore this sensitivity further figure 3 and figure 4 present the maximum power coefficient as a function of r and ϕ, respectively, for a disc with fixed blockage. From figure 3 it is evident that as the width of the central part of the velocity profile increases, the maximum power coefficient, $C_{P,max}$, increases (decreases) when the outer flow is slower (faster) than the central flow velocity. Figure 4 displays this same result a different way, and indicates that the peak coefficient varies monotonically with ϕ.

Interpretation of changes in maximum power coefficient

An obvious first question is: does the power coefficient normalised in terms of the bulk velocity (i.e. cross-sectional average velocity of the upstream flow profile) rather than simply the velocity, u, collapse the results given in figures 2, 3 and 4? Based on figure 3, the answer to this question is no. For instance, the peak coefficient for $\phi = 0.5$ and 2 are different by a factor of ~2 on figure 3. However, the ratio of the bulk velocity cubed for each of these examples is closer to 4.

Given the inability of the bulk velocity to explain the results, an alternative and informative way to interpret them is to define an effective blockage ratio, which is the blockage ratio at which a uniform flow would give the same power coefficient as that for the non-uniform flow. This power coefficient is therefore calculated, noting (13), as:

$$B_{eff} = 1 - \left(\frac{16}{27 C_P} \right)^{1/2}, \tag{14}$$

where the superscript implies non-uniform flow.

For $B = 0.2$, figure 5 presents contours of effective blockage as a function of the parameters r and ϕ. This figure illustrates some important limits. Firstly, for $\phi \gg 1$ it can be seen that $B_{eff} \to 1/r$, indicating that the fast outside flow now acts to essentially confine the slower central flow in the same way as impermeable walls. Secondly, for $\phi \ll 1$ we have $B_{eff} \to 0$ irrespective of r. This limit suggests that for ocean currents, for example, which may be representative of a small passage of fast flowing water surrounded by slower moving water, no blockage effects are realised due to the finite width of the current. Finally, for $\phi = 1$ it can be seen that the effective blockage equals the actual blockage of 0.2, since then the flow is uniform.

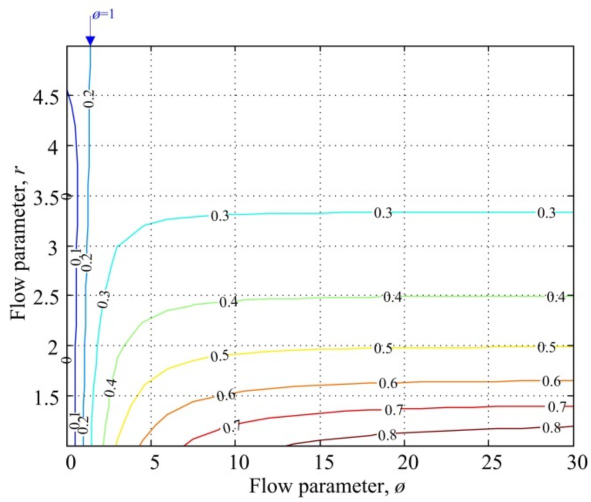

Figure 5. Contours of effective blockage B_{eff} based on (14) for a turbine with $B = 0.2$.

Continuously varying velocity profile

The piecewise constant velocity profile is perhaps the simplest idealisation of a non-uniform flow. However, the solution presented above can be adapted easily to continuous velocity profiles. For example, if we assume the central part of the velocity retains a width rA and velocity u, but we allow the velocity to reduce linearly to zero outside this width, then r is the only parameter to define the profile and it can be shown that (1), (7) and (11) are still valid provided that (see [4]) ϕ =0.5, $\beta_5 = \phi/[\beta_4 - (\beta_4^2 - 1)^{1/2}]$ and A_3 is calculated as

$$A_3 = \left(1 - 2\alpha_4\left(1 - \frac{B\alpha_4}{2}\right)\right) - 2(1 - rB)(1 - \alpha_4) - \frac{2}{3}(1 - rB)\left[1 - \beta_4^3\left(1 - \left(1 - \beta_4^{-2}\right)^{3/2}\right)\right]. \tag{15}$$

Comparison to Numerical Solution of Euler Equations

Figure 7 presents results from a numerical solution to the Euler equations using a discontinuous Galerkin Finite Element method (explained in more detail in [4]) for a disc having blockage $B = 1/6$ and k=2.06. Figure 7a represents symmetric non-unfiorm flow, whilst figure 7b represents asymmetric non-uniform flow. In both cases the flow profile is the same as that described in the previous subsection; uniform flow over a width rA, reducing linearly to zero outside this width. The actuator disc theory presented in this paper is indifferent to these two profiles.

Also shown in figure 7 are the power coefficients for the discs computed numerically. These results can be compared with the actuator disc theory presented in the previous subsection, which gives $C_P = 0.66$. This analytical result is only 3% lower than the numerical values given in figure 7. For comparison, ignoring the non-uniformity in the flow, and applying the theory of [5] directly gives $C_P^U = 0.77$. This is 14% higher than that in figure 7. The theory presented herein therefore appears to explain the results in figure 7 much better than previous work.

Finally, it is interesting to note in figure 7 that asymmetry has little effect on the results. Flow asymmetry may become more relevant where viscous effects are more dominant, but for the inviscid limit considered here it is of limited influence.

Conclusions

In this paper we have shown that the classical actuator disc model can be applied to problems involving non-uniform upstream flow. In confined flow, the effect of flow non-uniformity has been explored for the case of an inviscid shear flow, where the shear does not intercept the disc directly. The more general case without this restriction is considered in [4].

The present analysis indicates that a non-uniform flow can affect the power extracted by the disc. In particular, the disc power coefficient decreases below (increases above) that expected in uniform flow if the turbine is moved into a relatively high (low) velocity region of the flow. These results cannot be explained in terms of the bulk flow velocity, but they may be interpreted well as a change in the effective blockage of the disc.

The results have implications for existing theoretical models of arrangements of turbines in shallow flows (e.g. [9]), which presently ignore flow non-uniformity. They are also important for consistent interpretation of blockage effects for wind turbines in wind tunnels when the flow is non-uniform.

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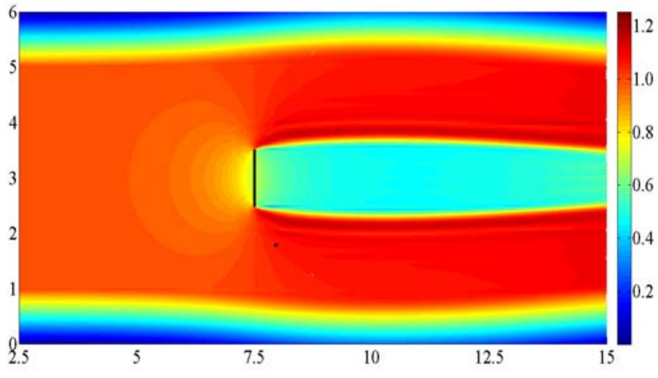
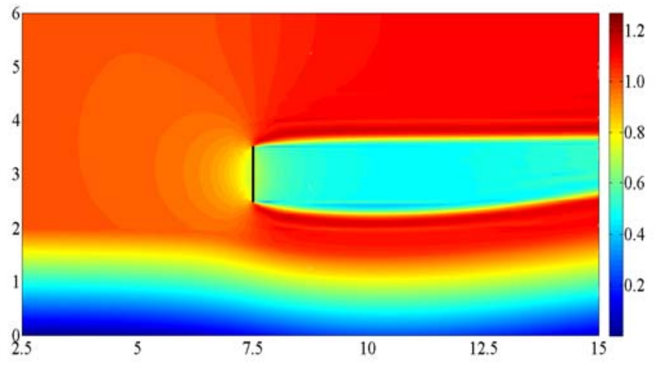


(a) C_P =0.680



(b) C_P =0.677

Figure 7. (a) Streamwise velocity contours for symmetric non-uniform flow; (b) Streamwise velocity contours for asymmetric non-uniform flow. Axes are normalised by disc length. Contours represent velocity normalised by upstream velocity u. r =4, B =1/6.