Modelling Water Wave and Tethered Structure Interactions using 3D Smoothed Particle Hydrodynamics

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Abstract

Knowledge of how a floating offshore structure will react to the rough conditions of the seas, particularly during storms, is critical to their safe and efficient design. This paper describes the first step in validating a Smoothed Particle Hydrodynamics model for analysing and predicting the response of floating structures to rough sea conditions. The validation cases considered are those of a tethered spherical buoy oscillating in the horizontal and vertical directions, and of the same buoy responding to incident wave trains. These cases are compared against experimental data obtained in a wave flume, with results showing the importance of adequate particle resolution.

Introduction

Small amplitude water waves have long been studied by scientists, and their interaction with simple structures can be accurately predicted by a wide range of mathematical approximations [3]. However, when the structures become more complex in design, analytical methods and mathematical approximations can no longer be used to solve the governing equations. Furthermore, when the waves become large in amplitude, or more irregular, nonlinear effects become significant and a model describing the wave-structure interaction becomes increasingly difficult to obtain. Sophisticated numerical techniques are required when analysing these situations which occur frequently in the ocean.

One of the techniques that is capable of modelling the interactions between structures and fluids is Smoothed Particle Hydrodynamics (SPH) [1, 4, 6, 7, 8]. In this paper we utilise SPH to simulate the interaction between fluid and structures. The Lagrangian approach of SPH does not require a mesh to be used, which is advantageous when analysing fluid structure interaction on a free or constrained structure as the expensive remeshing stage is not required.

The SPH technique has previously been used to study the effect of fluid structure interactions, and a number of benchmark tests have been conducted previously. Doring et al. [2] tested the problem of water entry of a wedge, achieving good agreement between experimental and their 2D numerical results. Le Touzé et al. [5], and Rudman and Cleary [8] both conducted 3D SPH simulations of a large wave impacting a structure (a ship and an oil platform respectively), but Le Touzé et al.’s results over predicted the water heights, and Rudman and Cleary did not present a comparison with experiments. Cummins, Sylvester, and Cleary [1] presented a thorough benchmark case for a dam-break problem, measuring the impact of flood water on a fixed square column, achieving good agreement with experiments.

These studies however do not consider structures that are tethered to the ocean floor. Structures such as Tension Leg Platforms and offshore wind turbines are tethered to the ocean floor due to being placed in waters that are too deep to be fixed. In this paper we present two benchmark cases for such scenarios utilising a rising tethered sphere; oscillation in the vertical and horizontal directions, and the motion under the influence of an incident wave train. The results of the comparisons between simulations and experiments will help to determine what simulation parameters are critical to ensure simulation accuracy when modelling tethered structures.

Experimental Setup

The experiments were conducted in a 40 m long, 1 m wide wave tank at Monash University, a schematic of which can be seen in Figure 1. The tank contains a 20.5 m testing section that has a horizontal floor, a 6 m long development region, and an inclined beach at the end of the test section. A piston type wavemaker is located prior to the development region that moves on an incline of 30° to the horizontal. A wave absorbing beach is located at the end of the testing section, rising on an incline of 18°. The depth of the tank is measured in the testing section, and for this study is filled to a depth of 0.8 m.

The coordinate system was chosen with y-axis pointing upwards, the y = 0 plane corresponding to the floor of the test section, and the z = 0 plane corresponding to the mid-plane of the tank. The x-axis was defined in the lengthwise direction of the tank with x = 0 located at the start of the test section. The coordinate system can be seen in Figure 1 by the red arrows with the z coordinate pointing out of the page.

For each experiment a tethered spherical buoy is used to observe the fluid-structure interaction. The sphere is tethered by a cable, spring (stiffness k = 30.88 N/m), and pulley system as depicted by the schematic in Figure 2. The spherical buoy used has diameter of 0.203 m, mass of 1.7451 kg, and centre of mass located 27 mm vertically below the centre of the sphere (towards the cable attachment point). Relative to the drag of the buoy moving through the water, the pulley system provides insignificant damping.

The cable and spring initial lengths were set to ensure that at the rest position half the sphere is submerged. The location of the pulley system is such that the buoy centroid is located at x = 1.15 m. The pulley system is required to ensure that the coil spring used does not extend or contract within the water and cause the spring to be damped.

In each of the experiments the trajectory of the sphere was mea-
The motion recorded in the experiments and simulations is then non-dimensionalised by the following expressions:

\[ x^* = \frac{(x - x_e)kd}{a} \]  
\[ y^* = \frac{(y - y_e)}{a} \]  
\[ t^* = \frac{t - t_0}{T} \]

where \( d \) is the depth, \( a \) is the mean wave amplitude, \( T \) is the period of oscillation, \( x_e \) and \( y_e \) are the equilibrium positions of the buoy in the horizontal and vertical directions respectively, and \( t_0 \) is the reference time. These give natural frequencies of 1.67 Hz for the vertical motion, and 0.18 Hz for the horizontal motion.

Finally, by measuring the time between peaks of the experimental trajectories in Figures 3 and 4, an estimation of the natural frequency of oscillation can be determined for each direction. These give natural frequencies of 1.67 Hz for the vertical motion, and 0.18 Hz for the horizontal motion.

Wave Train Tests

The interaction between the buoy and an incident wave train is the second benchmark considered in this work. Wave trains with periods of 1 and 3 seconds are considered, with two different amplitudes. The smaller amplitude tests are denoted as Experiments 1A and 3A, while the tests with larger amplitudes are denoted as Experiments 1B and 3B. Table 1 summarises the mean wave amplitudes used in each of the experiments.

The motion recorded in the experiments and simulations was compared to simulations. Figure 3 shows a comparison between the experimental vertical trajectory (obtained from the mean of 7 experiments) and the trajectories observed in simulations with different resolutions. The test with 10 mm particle spacing achieves a good estimate of the amplitude of the first crest, however the amplitude thereafter and the period of oscillation are poor. Refining the particle spacing to 5 mm results in a trajectory that is much closer to the experimental results. However the amplitude of the troughs and crests beyond the first crest are still under estimated, and the period of oscillation is too long.

In the horizontal oscillation test, the buoy was displaced by 0.5 m in the x direction only and released to return to the rest position. The trajectories in each direction were measured, and a comparison between experiments and simulations was performed. Figure 4 shows the horizontal trajectory of the buoy in both the experiments and a simulation with particle spacing of 10 mm. The period of oscillation in the horizontal direction appears to be in good agreement between the experiments and simulation, however the amplitude of motion is severely damped in the simulation. Again only a small amount of data has been obtained for the 5 mm resolution case, however the data collected thus far is in good agreement with the experimental data.

Finally, by measuring the time between peaks of the experimental trajectories in Figures 3 and 4, an estimation of the natural frequency of oscillation can be determined for each direction. These give natural frequencies of 1.67 Hz for the vertical motion, and 0.18 Hz for the horizontal motion.
wave period, \( k \) is the wave number, \((x_e, y_e)\) is the equilibrium position of the buoy, and \( t_0 \) is the time at which the buoy is atop the first full sized crest. There is very little motion in the \( z \) direction so the \( z \) motion of the buoy is not considered.

Two SPH simulations with mean particle spacings of 10 mm were then performed, one creating a 1 second period wave train and the other creating a 3 second period wave train using a fluid control routine based on the shallow wave equations. The wave amplitudes of the simulations were 4.5 cm and 6.0 cm respectively, chosen as approximate midpoints of the experimental wave amplitudes.

Figure 5 shows a number of side views of the buoy in a 1 second period wave train, both experimental (case 1A) and simulated. The position of the buoy relative to the wave in each frame are in good agreement between the experiments and simulations.

For the 1 second case, Figure 6 the experimental heave motions of the buoy are in good agreement after \( t^* = 0 \), and the simulated peaks are also in good agreement. Figure 7 shows a large amplitude modulation of the trajectory is observed in the surge, the amplitude of which appears to depend on the amplitude of the wave train. The same modulation is also observed in the simulation, however the amplitude is much smaller. At this resolution heavy damping was observed in the horizontal oscillation simulation test and this is likely to also reduce the amplitude of the modulation. Figures 8 and 9 show the frequency spectra of the motion in the surge and heave motions respectively. The dashed vertical lines in each indicate the natural frequency in the respective direction of oscillation, as determined in the Oscillation Tests. As expected the frequency spectra contain clear peaks at 1 Hz, the frequency of the wave train, however in the surge frequency spectra, peaks can be observed at the natural frequency, indicating that the frequency of the modulation is at the natural frequency.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Wave Amplitude (cm)</th>
</tr>
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<tbody>
<tr>
<td>1A</td>
<td>3.2</td>
</tr>
<tr>
<td>1B</td>
<td>5.8</td>
</tr>
<tr>
<td>3A</td>
<td>4.7</td>
</tr>
<tr>
<td>3B</td>
<td>7.1</td>
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</tbody>
</table>

Table 1. Mean wave amplitudes used in the Wave Train tests

In the 3 second period case, the crests of the buoy heave motion (Figure 10) are in very good agreement with each other, however the troughs have some disagreement. The disparity at the bottom of the troughs is due to the cable that tethers the buoy going into slack for the 3B and simulation cases but not in the 3A case. Since the cable can only provide a tension force, the troughs in 3B and the simulation have no tethering force, while there is some tethering force in 3A. The experimental results for the surge motion (Figure 11) are in good agreement, however the simulation results are inaccurate at the troughs and the surge motion is out of phase with the experiments. Similar to the 1 second case, this could be due to over damping predictions in SPH at such a coarse resolution.

In both of the Oscillation tests the resolution was observed to be an influential parameter in determining the accuracy of the simulation. In these cases a refined resolution can be expected to return numerical predictions that more closely match the experimental results.

Conclusion

In this paper the interactions between a tethered buoy and both static water and wave trains were studied numerically and compared to experimental results. The simulation technique, SPH, achieved good results in predicting the motion of the buoy under the various effects. However the resolution of the study has a significant impact on the accuracy of the simulation. Using too coarse a resolution caused the period of oscillations in the vertical direction to be overpredicted, and the surge motion becomes significantly damped. When a finer resolution is used there is closer agreement between the experiment and simulation results.

This paper has shown us that with adequate resolution, SPH becomes a powerful tool in predicting the consequences of a
fluid and tethered structure interaction. Future continuation of this work will explore the impact of a breaking wave over the buoy in a bid to show that SPH can be used to model rough sea conditions effectively.

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References