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Zoology of Unstable Modes in a Stratified Cylinder Wake

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Abstract

This paper delineates experimental and numerical results concerning the effect of the stratification and the tilt on the 3D instabilities of a cylinder wake led at moderate Reynolds numbers from the 2D von Karman vortex street. For a vertical cylinder, shadowgraph visualisations put forward an unstable mode similar to the mode A well known in homogeneous fluids. When the cylinder is tilted a new unstable mode is observed at moderate Froude numbers, characterized by undulated layers of strong density gradients and axial flow corresponding to Kelvin-Helmholtz billows created by the strong shear present in the critical layer of each tilted von Karman vortex. Eventually, for large tilt angles, two other modes have been observed. For a weak stratification, an instability appears due to the overturning of the isopycnals by the von Karman vortices whereas for a strong stratification, a short wavelength unstable mode can be observed, even in the absence of von Karman vortices.

Introduction

Although the homogeneous dynamics of a cylinder wake has been extensively studied, very few papers have focused on stratified wakes despite obvious applications to geophysical flows and submarine wakes. Given that oceans and the atmosphere are stratified, most environmental flows like island wakes and mountain range wakes are strongly influenced by the mean density gradient. The goal of this paper is to analyse the influence of the stratification on the transition from a laminar to a turbulent wake of a bluff body.

Homogeneous wakes are usually studied by focusing on cylinder wakes, noting that even if the cylinder is a simple bluff body, it leads to the same features than a complex geometry bluff body. For high Reynolds number, the dynamics of the cylinder wake is modified due to the appearance of three-dimensional instabilities. The first unstable mode, called mode A, has been observed experimentally by [2].

Stratified cylinder wakes have received very few attention. Boyer et al. (1989) discovered a large variety of regimes for a horizontal cylinder when the Froude number varies from 0.02 to 13 and the Reynolds number varies from 5 to 4000. Moreover, the stratification tends to prevent the appearance of von Karman vortices, due to the stabilization of shear flows by a stratification. This effect can be quantified by the Richardson criterion, leading to the dependence of the critical Reynolds number with the Froude number (Meunier 2012). However, the dynamics of the wake is strongly modified when the cylinder is no longer horizontal but tilted with respect to the vertical. Indeed, Meunier (2012) showed experimentally and numerically that von Karman vortices can be emitted for strong stratifications at moderate Reynolds numbers.

Experimental Details

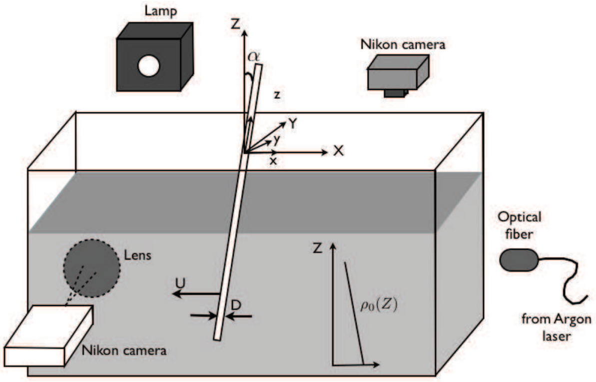

Figure 1: Schematic of the experimental set-up: the stratified cylinder wake is analyzed by dye visualizations on the right side and by shadowgraph method on the left side.

The model experiment used to study the dynamics of a stratified cylinder wake is presented schematically in figure 1. A 150 cm long, 75 cm wide and 50 cm high Plexiglas tank is built in order to make visualizations from all sides. The tank contains a stable linearly stratified fluid in salt up to a water height H = 40 cm obtained by the double bucket method. The typical Brunt-Väisälä frequency defined by $N \simeq \sqrt{(-g/\rho_0)(d\bar{\rho}/dZ)}$ is close to 2 rad/s (with ρ_0 the mean density in the tank).

A circular cylinder of diameter D is introduced in the tank and is towed horizontally in the stratified fluid, at a velocity U varying between 0.5 cm/s and 5 cm/s. The diameter varies between 0.3 cm and 1.8 cm. The cylinder can be tilted relative to the vertical, at an angle α in the cross-stream plane as it can be seen on figure 1. The cylinder is mounted on a carriage translated along horizontal rails, entrained by a belt driven by a DC motor coupled to a 1: 90 reducer. As shown on Fig. 1, two visualization methods are used: the shadowgraph method and dye visualizations The stratified wake of the tilted cylinder is characterized by four non-dimensional parameters: the cross-stream tilt angle α, the Reynolds number $Re = UD/\nu$, the Froude number F = U/ND and the Schmidt number Sc. However, the last parameter defined as $Sc = \nu/\kappa = 700$ (κ being the diffusivity of salt in water and ν being the kinematic viscosity) is large in this study, which reduces the problem to 3 main non-dimensional parameters.

Results

Four 3D unstable modes have been experimentally observed for different values of the 3 dimensionless parameters.

For large Froude numbers, the 2D wake is destabilized when the Reynolds number increases above 190. Figure 2 shows the periodic structures created about 5 diameters downstream of the cylinder, created about 5 diameters downstream of the cylinder, which is characteristic of the classical mode A, well-known in a homogeneous fluid and made by counter-rotating vortex pairs perpendicular to the von Karman vortices. Moreover, the wavelength of this mode is close to 4 diameters, which is in excellent agreement with the theoretical value $\lambda_z/D = 3.96 \pm 0.02$ obtained by [3] for the mode A.

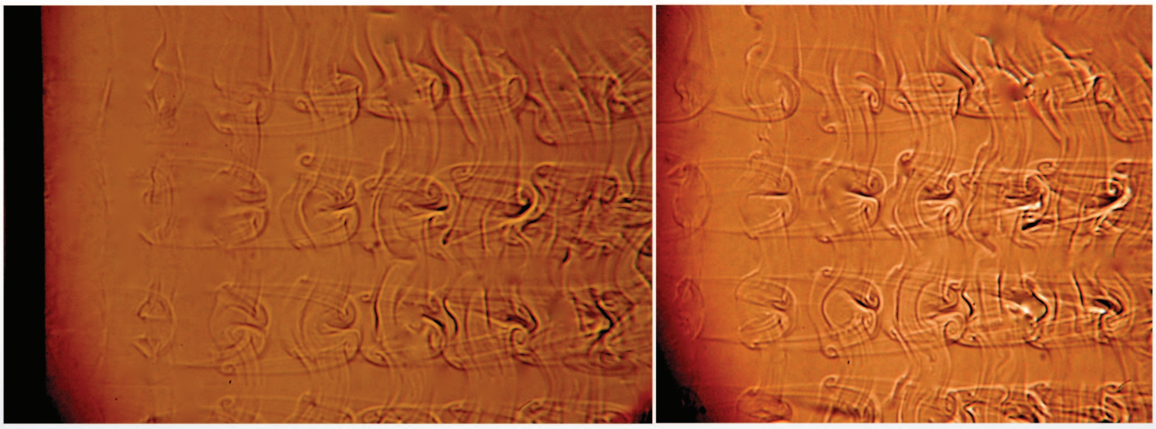

Figure 2: Shadowgraph visualizations of the mode A: the wake behind a vertical cylinder is obtained for Re = 214 and F = 10. The cylinder is going to the left of the picture and there is a time delay of approximatively 3 D/U between both pictures to reveal the temporal evolution of the instability. The field of view is approximately 8 by 15 diameters.

The mode A is the first 3D unstable mode when the cylinder is tilted with an angle smaller than 45°, but the dynamics is strongly modified when the tilt angle α is increased above 45°.

This dynamics is strongly modified when the tilt angle α is increased above 45°. Indeed, a new unstable mode appears in a region of moderate stratification $(1.5F3.5)$. Figure 3 shows side shadowgraph views of this unstable mode, characterized by dark and bright undulated layers, revealing the presence of co-rotating vortices. A 2D numerical study, coupled to the theoretical structure of critical layers inside each von Karman vortex, has shown that these co-rotating vortices are simply Kelvin-Helmholtz billows of the strong shears generated by the axial velocity of the 2D flow. This is similar to the tilt-induced instability of a solitary vortex tilted with respect to the stratification [1]. It can be noted that they appear once dark and once white probably because the von Karman vortices are alternate. Above the critical Reynolds number, these straight lines begin to undulate. The undulations grow with time describing S-shape structures far from the cylinder. This is why this mode has been called mode S.

The case of very large tilt angle $\alpha \in [60°, 85°]$ and large Froude number is studied and for nearly horizontal cylinders, a new instability appears at weak stratification. Top shadowgraph views are presented in Fig.4 for Re $\simeq$ 180 and F $\simeq$ 8.5. Under the threshold, straight layers can be seen whereas above the threshold these layers are deformed by counter-rotating vortices, creating rolls, similar to the plumes appearing in Rayleigh-Taylor instabilities. This is why this mode has been called mode RT. This mode is characterized by counter-rotating vortices behind the cylinder as was observed for the mode A. But this new instability present one difference: the tails of the vortices seem to be stretched in the opposite direction (i.e. downstream for this mode and upstream for the mode A). Then, these counter-rotating vortices are very far from the cylinder: they appear about 15 diameters downstream of the cylinder whereas the mode A used to appear 4 diameters downstream.

For large tilt angles and for strong stratifications, the dynamics is strongly modified since three-dimensional instabilities can appear even in the absence of von Karman vortices, i.e. for a stationary flow. This mode, shown in Fig. 5, is due to the presence of lee waves emitted by the cylinder, which contain some axial velocity if the cylinder is tilted. It leads to a strong shear on the centerline downstream of the cylinder which creates streamwise Kelvin-Helmholtz billows. The wavelength of this instability is dramatically smaller compared to the previous modes since it is of the order of 0.3 diameters which is 10 times smaller than the wavelength of mode A. This mode has been called mode L (for lee waves mode).

Figure 3: Shadowgraph sideview visualizations of the mode S: the wake behind a cylinder is tilted at $\alpha = 45°$ for Re = 170 and F = 2.5. The cylinder is going to the left of the picture and there is a time delay of approximatively 7 D/U between both pictures to reveal the temporal evolution of the instability. The field of view is approximately 8 by 15 diameters.

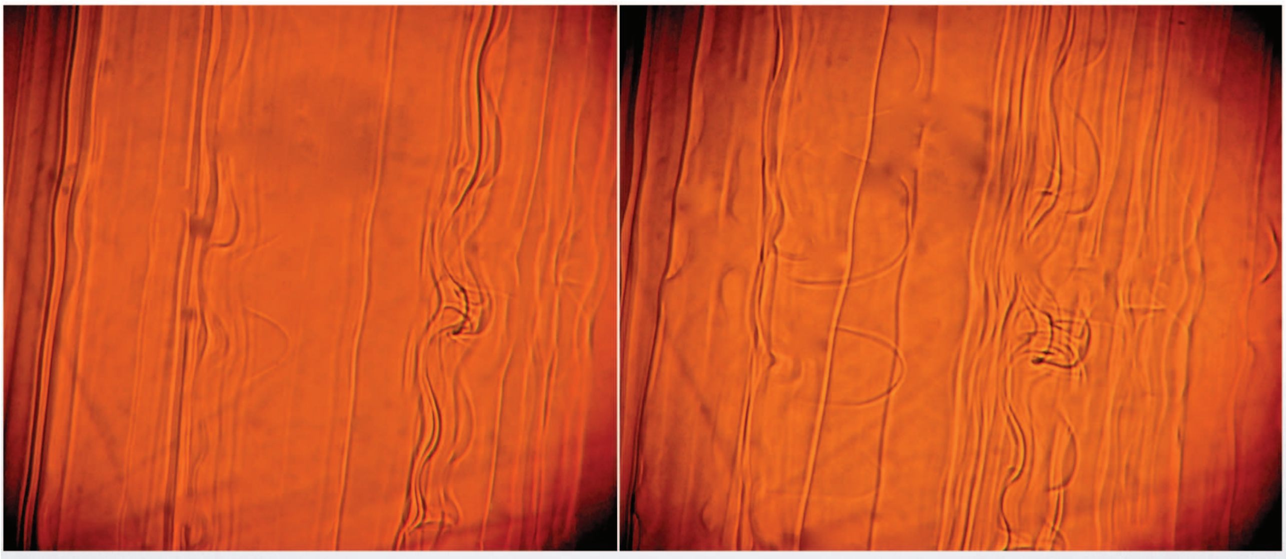

Figure 4: Shadowgraph visualizations of the mode RT: the wake behind a vertical cylinder is obtained at the same Reynolds number Re = 170 and Froude number F = 10. In these side views , the cylinder is going to the left and there is a time delay of approximatively 3 D/U between both pictures to reveal the temporal evolution of the instability. The field of view is approximately 8 by 15 diameters.

Figure 5: Dye visualization of the mode L. The picture is a 45° tilted side view taken at a tilt angle $\alpha = 75°$ for Re = 194 and Froude number F = 0.65. The field of view is approximately 8 by 15 diameters.

Figure 6 presents the regions of (α,F) where the wake becomes three-dimensional. The mode A is the first unstable mode for a large range of Froude numbers (0.1F10) but only for small tilt angles ($0° < α < 60°$).

For large tilt angles ($65° < α < 85°$), two other unstable modes have been discovered (see Fig. 6). For a weak stratification ($3 < F < 10$), shadowgraph visualisations reveal the presence of counter-rotating vortices which seem to be convective rolls which is the signature of a Rayleigh-Taylor instability. For large tilt angles and for strong stratifications, the dynamics is strongly modified and the mode called L is observed for small Froude numbers ($0.4 < F < 2$) and nearly horizontal but tilted cylinders, as shown inn Fig. 6.

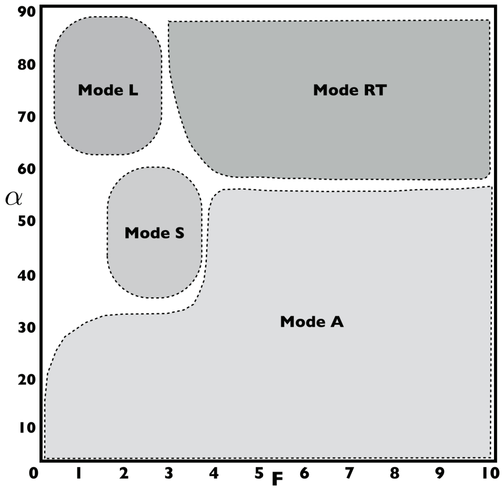

Figure 6: Mapping of the appearance of the 3D unstable mode as a function of the tilt angle α and the Froude number F. Greyscale contours indicate the first unstable mode.

Conclusions

This paper highlights the fact that the stratification strongly modifies the transition from a 2D to a 3D flow in a cylinder wake, with the presence of 3 new unstable modes. The tilt of the cylinder with respect to the vertical plays also a major role which is particularly interesting for geophysical applications where stratified wakes have been usually restricted to 2D horizontal bluff bodies.

To go further, a new bigger tank is being built in our laboratory, which will allow to study a flow closer to geophysical applications (i.e half-cylinder on a tilted plane) but also the boundary layer of a tilted plane in a background stratification. The 2D and the 3D structures will be studied and we will compare the wake led by an undulated plane to a straight plane in order to model the effect of the oceanic topography.

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