Multiphase porous media model for heat and mass transfer during drying of agricultural products

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Abstract

Modelling of food processing is complex because it involves sophisticated material and transport phenomena. Most of the agricultural products such fruits and vegetables are hygroscopic porous media containing free water, bound water, gas and solid matrix. Considering all phase in modelling is still not developed. In this article, a comprehensive porous media model for drying has been developed considering bound water, free water separately, as well as water vapour and air. Free water transport was considered as diffusion, pressure driven and evaporation. Bound water assumed to be converted to free water due to concentration difference and also can diffuse. Binary diffusion between water vapour and air was considered. Since, the model is fundamental physics based it can be applied to any drying applications and other food processing where heat and mass transfer takes place in porous media with significant evaporation and other phase change.

Introduction

Food process modelling is a very complex due its intricate physical structure. There are many modelling efforts in food processes such as drying [1, 6, 10], frying [13], microwave heating [14, 17], thawing [2], baking [19], and puffing [16] etc. These models can be classified into two broad categories; empirical models and theoretical model [12]. The empirical models can be rapidly developed and quite effective. But they do not provide physical insight of the process and predictive capability. Whereas, physics based model are preferred for predictive models not only in food drying but also in areas outside food, such as aeronautics. Among the theoretical models, distributed evaporation model, in contrast to sharp boundary models, are becoming popular in food processing. Datta [3] termed this distributed evaporation models as mechanistic models because these models consider heat and mass transfer equation for each phase (solid, liquid, air and vapour) in porous media. Even though the final equations seems simple continuity equations, the rigorous study for the transition from the individual phase at the microscopic level to representative average volume at the macroscopic level provide the fundamental and convincing basis [4]. These models are computationally effective and applied to wide range of food processing such as frying [13], microwave heating [17], puffing [16] baking [19], meat cooking [5] etc. Zhang and Datta [18] also states that, model that includes internal evaporation considering both liquid water and vapour should be used in a heat and mass transfer process. However, those mechanistic models consider an equivalent porosity which is analogous to non-hygroscopic material, where no bound water is present [4]. They assume that all the water is separated from solid matrix and equivalent porosity is equal to the volume fraction occupied by gas and liquid. Thus the porosity remains constant in those models. However, none of this assumption represents the real phenomena in food, since there is both free and bound water presents in almost every food material [9]. Transport mechanism for bound and free water is different. In this article, a multiphase porous media model for-

mulation has been developed considering bound water transport and variable porosity. The transport of free water, bound water, water vapour, and air is considered. The porosity is considered as the volume fraction of pore (free water and gas) in the food. It increases as the bound water leaves the cell resulting in cell shrinkage, therefore, increasing the pore volume.

Mathematical modelling

Assumptions

- Porosity is defined as the volume fraction occupied by the gas and free water.

- Bound water is considered as the intracellular water that includes cell vacuole water and cell wall water, which is about 90 percent of total water content [7].

- Bound water can transport by means of diffusion from cell to cell and evaporate at the surface, and convert to free water depending upon the concentration difference of bound and free water.

- The total volume is assumed to be constant.

Geometry

Since the material studied was an apple slice, a two dimensional axisymmetric geometry as shown in "figure 1" was considered for simulation purpose.

Mathematical formulation

The total volume of the sample can be written as

$$V = V_g + V_{fw} + V_s \tag{1}$$

Where the V_g, V_{fw}, V_s are the volume occupied by gas, free water and solid respectively. The solid phase volume comprised of bound water volume V_{bw} and non-transportable solid matrix (dry mass)V_{dm}. Intracellular water within the cell is also considered as bound water.
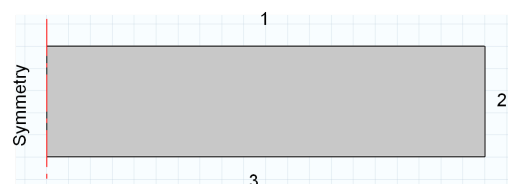
$$V_s = V_{dm} + V_{bw} \tag{2}$$

Figure 1: Axisymmetric geometry for apple slice.

In this definition, free water was considered to be able to flow due to pressure gradient in the pores. The porosity φ is defined as

$$\varphi = \frac{V_g + V_{fw}}{V} \tag{3}$$

Saturation of free water and gas phase was defined as the fraction of pore volume occupied by that particular phase,

$$S_{wf} = \frac{V_{fw}}{V_g + V_{fw}} = \frac{V_{fw}}{\varphi V} \tag{4}$$

$$S_g = \frac{V_g}{V_g + V_{fw}} = 1 - S_{fw} = \frac{V_g}{\varphi V} \tag{5}$$

The saturation for bound water and solid matrix was defined as the volume of solid occupied by the bound water and solid matrix respectively.

$$S_{bw} = \frac{V_{bw}}{V_d m + V_{bw}} = \frac{V_{bw}}{V_s} \tag{6}$$

$$S_{dm} = \frac{V_{dm}}{V_{dm} + V_{bw}} = 1 - S_{bw} = \frac{V_{dm}}{V_s} \tag{7}$$

The mass concentrations of free water, vapour, and air were given by "equation (8 -10)",

$$c_{fw} = \frac{\rho_w}{M_w}\varphi S_{fw} \tag{8}$$

$$c_v = \frac{p_v}{RT}\varphi S_g \tag{9}$$

$$c_a = \frac{p_a}{RT}\varphi S_g \tag{10}$$

Bound water does did not remain in the pores rather it is was contained within the matrix. Therefore the bound water concentration is was given by,

$$c_{bw} = \frac{\rho_w}{M_w}(1-\varphi)S_{bw} \tag{11}$$

Continuity equation to solve for gas pressure

$$\frac{\partial}{\partial t}(\varphi S_g) + \nabla . n_g = R_{evap} \tag{12}$$

$$n_g = \frac{k_g k_{r,g}}{\mu}\nabla P \tag{13}$$

Free water transport

$$\frac{\partial c_{fw}}{\partial t} + \nabla . (-D_{fw}\nabla c_{fw}) + \nabla v_{fw}c_{fw} = -R_{evap} \tag{14}$$

Air and vapour transport

$$\rho\frac{\partial w_v}{\partial t} + \nabla .(-S_g\varphi\frac{C_g^2}{\rho_g^2}M_v M_a D_{eff,g}\nabla w_v) + \nabla v_v c_v = R_{evap} \tag{15}$$

$$w_a = 1 - w_v \tag{16}$$

Bound water transport

$$\frac{\partial c_{bw}}{\partial t} + \nabla .(-D_{bw}\nabla c_{bw}) = -R_{b,f} \tag{17}$$

A portion of bound water was assumed to be converted to free water and was proportional to the concentration difference as shown in the following equation.

$$R_{b,f} = k_{b,f}(c_{bw} - c_{fw}) \tag{18}$$

The value of $k_{b,f}$ can be adjusted to fit the experiment.

Evaporation

A non-equilibrium formulation as described in [14] was considered to calculate the evaporation rate. The following expression was used to model phase change in porous media [8].

$$R_{evap} = k_{evap}\frac{M_v}{RT}\left(p_{v,eq} - p_v\right) \tag{19}$$

The evaporation constant k_{evap} is material and process-dependent parameter signifying the rate of evaporation. The equilibrium vapour pressure is given by the sorption isotherm of apple and vapour pressure is obtained from ideal gas law by the following equations.

$$p_{v,eq} = p_{v,sat}(T)e^{\left(-0.182M^{-0.696}0.232e^{-43.949M}M^{0.0411}ln[p_{v,sat}(T)]\right)} \tag{20}$$

$$p_v = \frac{\rho_v}{RT} \tag{21}$$

Energy conservation equation

$$\rho_{eff}C_{p,eff}\frac{\partial T}{\partial t} + \rho_{eff}C_{p,eff}v.\nabla T = \nabla .(k_{eff}\nabla T) - \lambda R_{evap} \tag{22}$$

The thermo-physical properties of the mixture are were obtained by averaging those of pure components as shown in the following equations.

$$\rho_{eff} = \varphi(S_g\rho_g + S_{fw}\rho_{fw}) + (1-\varphi)(S_{bw}\rho_{bw} + S_{dm}\rho_{dm}) \tag{23}$$

$$C_{p,eff} = \varphi(S_g C_{p,g} + S_{fw}C_{p,fw}) + (1-\varphi)(S_{bw}C_{p,bw} + S_{dm}C_{p,dm}) \tag{24}$$

$$k_{eff} = \varphi(S_g k_g + S_{fw}k_{fw}) + (1-\varphi)(S_{bw}k_{bw} + S_{dm}k_{dm}) \tag{25}$$

Initial and boundary conditions

Heat and mass transfer values were calculated by Nusselt and Sherwood number correlation at drying air temperature 60^0C and air velocity $0.7 m/s$ and found to be $0.01 m/s$ and $20 W/m/K$ respectively[11, 15].

Initial conditions: Initial porosity was considered 0.20. Initial saturations of free water and bound water were considered as 0.20 and 0.90 respectively. The initial concentration was calculated from "equation (8-11)" . The initial condition for continuity equation "equation (12)" was $P = P_{amb}$ [11].

Boundary conditions: The boundary condition at the symmetry boundary was that there was no flux. The boundary conditions in boundary 1, 2 and 3 as shown in "figure 1" were listed below in the "table 1".

Physics	Boundary conditions
"Equation 12"	P_{amb}
"Equation 14"	$h_{mv}\varphi S_{fw} \frac{(p_v - p_{v,air})}{RT}$
"Equation 15"	$h_{mv}\varphi S_g \frac{(p_v - p_{v,air})}{RT}$
"Equation 17"	$h_{mv}(1 - \varphi) S_{bw} \frac{(p_v - p_{v,air})}{RT}$
"Equation 22"	$h_T (T - T_{air})$

Table 1: Boundary conditions.

Results and Discussion

The moisture content at surface, centre and average moisture content are shown in "figure 1". It was apparent that the surface was dried at a faster rate than the centre.
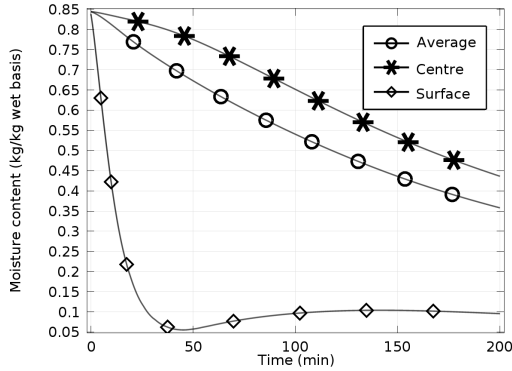
Figure 2: Average, surface and centre moisture content with time.

The saturation of different phases is shown in "figure 2". Free water saturation became almost zero whereas the bound water was only gradually removed.
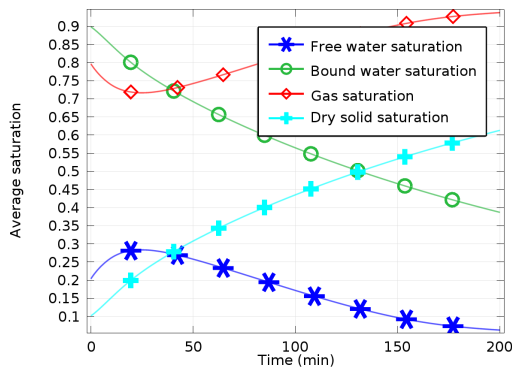
Figure 3: Average saturation of different phase.

According to the definition of saturation in "equation (4 -6)", the summation of saturation of bound water and dry solid was unity, and summation of free water and gas saturation was unity. The saturations obtained in the model conform to the definition.

"Figure 3" shows the temperature at surface and centre of the material as a function of time. Evaporation at the boundary was neglected at this moment. Due to inaccuracy and difficulties in measuring internal temperature of the material, the only simulation shown was in the material centre. The simulation showed that the centre temperature was relatively low for first 60 mins of the drying period. This was because the thermal conductivity was lower. However, the temperature difference was not significant; this may be because of the lowthickness of the material.
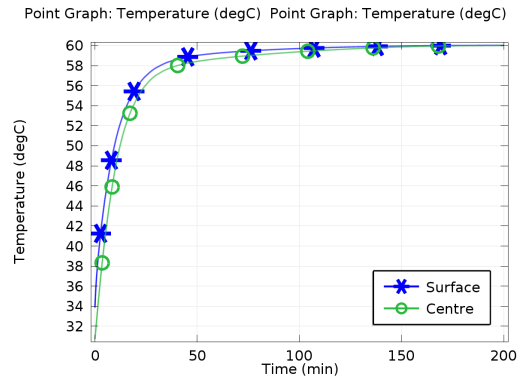
Figure 4: Surface and centre temperature with time.

"Figure 4" shows the saturation vapour pressure , equilibrium vapour pressure from sorption isotherm and vapour pressure in (kPa) with time. The saturation vapour pressure and equilibrium vapour pressure were essentially the same for a given temperature and moisture content. However, the actual vapour pressure depended upon the amount of evaporation occurring inside the material. The evaporation constant k_{evap} was significantly influenced by the vapour pressure. Higher value of k_{evap} increased the vapour pressure to make this value closer to equilibrium vapour pressure.
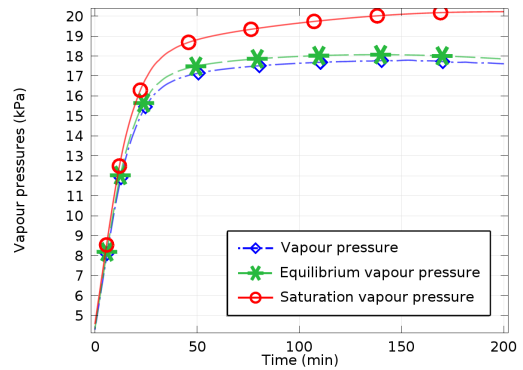
Figure 5: Saturation, equilibrium and actual vapour pressure (kPa) with time.

The convection drying process is very slow and the pressure gradient may not be not significant, therefore, the pressure driven flow i.e. convection was neglected. A limitation of this study was that the total volume was assumed to be constant, which was not realistic. We are conducting further research to

validate our findings and include variable volume into an improved model.

Conclusions

This paper presents a food drying model which considered bound water separately in a multiphase porous media model during food drying. Detailed formulations were used to predict porosity and shrinkage from the multiphase transport of the food materials during drying. The present study provided an insight into transportation of bound (intracellular) and free (intercellular) water transport separately. Future studies should focus on incorporation of variable volume and porosity in the model.

Acknowledgements

References

[1] Barati, E. and Esfahani, J.A., Mathematical modeling of convective drying: Lumped temperature and spatially distributed moisture in slab, Energy, 2011, **36** (4),2294-2301.

[2] Chamchong, M. and Datta, A.K., Thawing of foods in a microwave oven: II. Effect of load geometry and dielectric properties, Journal of Microwave Power and Electromagnetic Energy, 1999a, **34**(1),22-32.

[3] Datta, A.K., Porous media approaches to studying simultaneous heat and mass transfer in food processes. I: Problem formulations, Journal of Food Engineering, 2007a, **80**(1),80-95.

[4] Datta, A.K., Porous media approaches to studying simultaneous heat and mass transfer in food processes. II: Property data and representative results, Journal of Food Engineering, 2007b, **80**(1),96-110.

[5] Dhall, A. and Datta, A.K., Transport in deformable food materials: A poromechanics approach, Chemical Engineering Science, 2011, **66**(24),6482-6497.

[6] Diamante, L.M., Ihns, R., Savage, G.P. and Vanhanen, L., Short communication: A new mathematical model for thin layer drying of fruits, International Journal of Food Science and Technology, 2010, **45**(9),1956-1962.

[7] Halder, A., Datta, A.K. and Spanswick, R.M., Water transport in cellular tissues during thermal processing, AIChE Journal, 2011, **57**(9),2574-2588.

[8] Halder, A., Dhall, A. and Datta, A.K., An Improved, Easily Implementable, Porous Media Based Model for Deep-Fat Frying: Part I: Model Development and Input Parameters, Food and Bioproducts Processing, 2007, **85**(3),209-219.

[9] Joardder, M.U.H., Karim, M. and Kumar, C. Better understanding of food material on the basis of water distribution using Thermogravimetric analysis. in International Conference on Mechanical, Industrial and Materials Engineering (ICMIME2013). 2013.

[10] Kumar, C., Karim, A., Joardder, M.U.H. and Miller, G. Modeling heat and mass transfer process during convection drying of fruit. in The 4th International Conference on Computational Methods (ICCM2012). 2012.

[11] Kumar, C., Karim, A., Saha, S.C., Joardder, M.U.H., Brown, R.J. and Biswas, D. Multiphysics modelling of convective drying of food materials. in Proceedings of the Global Engineering, Science and Technology Conference. 2012. Dhaka, Bangladesh.

[12] Kumar, C., Karim, M.A. and Joardder, M.U.H., Intermittent drying of food products: A critical review, Journal of Food Engineering, 2014, **121**(0),48-57.

[13] Ni, H. and Datta, A.K., Moisture, Oil and Energy Transport During Deep-Fat Frying of Food Materials, Food and Bioproducts Processing, 1999, **77**(3),194-204.

[14] Ni, H., Datta, A.k. and Torrance, K.e., Moisture transport in intensive microwave heating of biomaterials: a multiphase porous media model, International Journal of Heat and Mass Transfer, 1999, **42**(8),1501-1512.

[15] Perussello, C.A., Kumar, C., de Castilhos, F. and Karim, M.A., Heat and mass transfer modeling of the osmo-convective drying of yacon roots (Smallanthus sonchifolius), Applied Thermal Engineering, 2014, **63**(1),23-32.

[16] Rakesh, V. and Datta, A.K., Transport in deformable hygroscopic porous media during microwave puffing, AIChE Journal, 2013, **59**(1),33-45.

[17] Rakesh, V., Seo, Y., Datta, A.K., McCarthy, K.L. and McCarthy, M.J., Heat transfer during microwave combination heating: Computational modeling and MRI experiments, AIChE Journal, 2010,**56**(9),2468-2478.

[18] Zhang, J. and Datta, A.K., Some Considerations in Modeling of Moisture Transport in Heating of Hygroscopic Materials, Drying Technology, 2004, **22**(8),1983-2008.

[19] Zhang, J., Datta, A.K. and Mukherjee, S., Transport processes and large deformation during baking of bread, AIChE Journal, 2005, **51**(9),2569-2580.