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Dissipation in Oscillating Water Columns

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Abstract

The Oscillating Water Column (OWC), the first generation Wave Energy Converter (WEC), is regarded as one of the most effective technologies to extract ocean wave energy. Most OWCs are designed to resonate at incoming wave frequencies, so that efficiency of the device should be the maximum. However, the damping factors, which control the efficiency of the OWCs, are not clearly identified. Therefore, an extensive study of the internal fluid dynamics of an OWC is required to identify those damping factors, and furthermore, to estimate the amount of energy loss due to their presence in OWCs.

Dissipation in reciprocating motion, resulting from the viscous and Reynolds stresses, is modelled both linearly and nonlinearly. A comparison of linear and nonlinear dissipation with Power Take-Off (PTO) damping is presented. The resulting model permits the relative contributions of the internal viscous and turbulent dissipation and the PTO damping to be assessed.

Introduction

A simple OWC device can be explained as a hollow cylinder which is partially submerged in water, with a high-speed unidirectional air tubine installed at the upper end. Waves in the ocean cause the water column to oscillate inside the device, and as a consiquence the trapped air above the internal free surface is driven through the turbine. For a sufficiently narrow device, the water column can be considered as a liquid pendulum [9]. The natural frequency of such a liquid pendulum is $\sqrt{g/L}$, where L is the length of OWC under water, and g is the acceleration due to gravity. In reality, the water column in OWC devices does not behave exactly like a rigid body [5]. However, for the simplicity of performance evaluation, it is preferable to make such assumption.

An OWC was first commercially built in Japan by Yoshio Masuda, in 1965. In 1976, he built a much larger device, named Kaimei, to investigate different features of OWCs and take this technology in an advanced stage. However, the outcome of this testing program was not satisfactory enough, and a lack of theoretical knowledge had been identified as one of the main reasons for the device not being properly successful. The theoretical development of OWCs was set off in the mid 1070s, based on the knowledge from ship hydrodynamics. Remarkable works on theoretical study are presented in Evans [3], Mei [12], etc.

The operating principle of general OWCs is identical to a forced mass-spring-damping system, where the water column works as a mass, and the body force of it (due to gravity) plays the role of the spring restoring force. There are several damping factors those control the power output of an OWC. The major part of the damping comes from the PTO system, which is, in general, the turbine-generator arrangement. Due to the pressure oscillation in the trapped air region, some energy radiates away from the OWC, known as radiation damping. If the device aspect ratio is high, sloshing at the air-water interface causes significant damping. Energy dissipation due to the viscosity at the wall, the presence of turbulence in the flow and the vortex formation at the entrance are other important damping factors.

Generally, OWCs are designed to have a natural frequency that coincides with the incoming wave frequency, so that the device resonates. At resonance, a device can extract enormous amount of energy, unless the damping dominates. Estimating the amount of energy loss due to different damping factors is one of the most important prerequisites of constructing an efficient OWC. Numerous works have been done on estimating PTO and radiation damping in OWCs, and solving the relevant aspects of linear wave theory, such as [3, 4]. However, very few studies are available in the literature that consider the energy loss due to viscosity and vortex formation and try to model them as damping factors [2,11]. Dissipation due to the presence of turbulence in the flow remains neglected.

In the present paper, a fluid-dynamical model of a simple OWC is derived, including the PTO system. The PTO provides a damping term, but that is desirable dissipation, since it represents useful power extracted from the system. The fluid-dynamical dissipation, in contrast, is parasitic and undesirable. All the assumptions needed to derive an ordinary differential equation model of the internal fluid dynamics of an OWC from the Navier-Stokes equations are identified. Prior to [9], literature on OWCs heuristically assumed an oscillator equation modelled the system, without discussing the required assumptions.

The first-order flow created inside the OWC by ocean waves is reciprocating. This means it is sinusoidal in time. Therefore it periodically and completely reverses direction. It is never steady or even quasi-steady. Only a few experimental, theoretical and numerical studies are available, compared with the enormous body of work on steady flows.

Dissipation terms in the momentum conservation equation, for the reciprocating flow system in an OWC, will be identified. Out of all the dissipation factors, the viscous and Reynolds shear stresses, and the PTO system are considered and modelled; and the others are neglected for the time being. A linear and a nonlinear damping model of the shear stresses are presented. The amount of energy loss from the both damping factors are estimated and compared with each other.

Formulation

The flow in an OWC is unsteady and reciprocating, i.e. periodic with zero mean flow. Reciprocating flow has been studied for past few decads to investigate the overall friction coefficients, transition criterion of laminar to turbulent flow and the turbulence structure [1, 7]. Friction coefficients from the literature could be adapted to compute the damping due to viscosity in an OWC. However, the evaluation of energy dissipation due to the turbulence in reciprocating flow has remained untouched. The velocity distribution in a reciprocating flow system shows that, even for a low Reynolds number, the flow becomes extremely turbulent during the deceleration period [1]. Thus, for a significantly high Reynolds number, turbulence would be a major damping factor, and it requires careful attention to be determined properly. In this work, the viscous stress and the Reynolds stress are combined in one term, and then modelled as a linear and nonlinear damping to fit into the governing ordinary differential equation (ODE).

In the literature, the equation of motion for an OWC has been presented as a simplified integral form of the momentum equation, which is basically a combination of a mass, spring, damping and external force terms. This attempt is to get to that simple integral equation from the governing Navier-Stokes equations. For the convenience, first the equations are nondimensionalized. Assumptions required to simplify these complex equations are highlighted.

The mass and momentum conservations in the OWC flow system are respectively given by,

$$\frac{\partial \rho^*}{\partial t^*}+\nabla_j^*(\rho^* u_j^*)=0, \tag{1}$$

$$\frac{\partial(\rho^* u_i^*)}{\partial t^*}+\nabla_j^*(\rho^* u_i^* u_j^*)=-\nabla_i^* p^*+\nabla_j^*\tau_{ij}^*+\rho^* g_i, \tag{2}$$

where τ_{ij}^* is the viscous stress tensor which can be represented for an isotropic fluid as

$$\tau_{ij}^*=\mu\left(\nabla_j^* u_i^*+\nabla_i^* u_j^*-\frac{2}{3}\nabla_k^* u_k^*\delta_{ij}\right), \tag{3}$$

"*" represents dimensional quantities. $u_i{}^*$ is the velocity vector, p^* is the pressure, μ is the dynamic viscosity, assumed constant, ρ^* is the density and g is the acceleration due to gravity.

In a cylindrical co-ordinate system, $\boldsymbol{x}^*=(x^*,r^*,\phi)$, the length scaling is

$$\boldsymbol{x}^*=\mathbf{L}\boldsymbol{x}, \tag{4}$$

where the length-scaling vector, $\mathbf{L}=(L,D,D)$, in which L and D are respectively the length and diameter of the device. The velocity, density and pressure are decomposed into a mean flow or primary flow (presented with the bar), and a fluctuating component (presented with the prime). These variables, along with time (t^*) are scaled as,

$$\begin{aligned} t^* = &\quad \omega^{-1}t, \quad \mathbf{u}^*=\hat{\xi}_F\,\omega\left(\bar{\mathbf{U}}+\mathbf{u}'\right), \\ \rho^* = &\quad \rho_w\,\rho, \quad p^*=\frac{\rho_w gLD}{\hat{\xi}_F}(\bar{P}+p'), \end{aligned} \tag{5}$$

where ω is the wave angular frequency, $\hat{\xi}_F$ is the amplitude of the water column oscillation, and ρ_w is the water density. If the Volume of Fluid (VOF) model is used for tracking the air-water interface in the OWC, then the non-dimensional density is given as,

$$\rho=C+\rho_a(1-C), \tag{6}$$

where C is the water volume fraction and $\rho_a=\rho_a^*/\rho_w$, is the non-dimensional density of the air. The density of the water, ρ_w, is assumed constant, while the air density, ρ_a, varies with time due to the compressibility. Owing to the turbulence in the OWC flow system, there would be a fluctuation of the volume fraction (C) at the interface location, as shown in [15]. Then the C can also be decomposed as,

$$C=\bar{C}+c'.$$

This type of fluctuation can be studied properly, if the flow system is solved numerically. However, the present paper is not dealing with this type of precise calculation; rather highlighting all the possible factors those are essential to capture the complete picture of an OWC flow system.

After the scalings and ensemble averaging of equations (1)-(3), the momentum conservation equation becomes,

$$\begin{aligned} &\frac{\partial}{\partial t}\left[\bar{U}_i\bar{\rho}+(1-\rho_a)(\overline{u_j'c'})\right]+\frac{\hat{\xi}_F}{D}\nabla_j\left[\bar{\rho}\bar{U}_i\bar{U}_j+\bar{\rho}(\overline{u_i'u_j'})\right. \\ &\qquad \left.+(1-\rho_a)\bar{U}_j(\overline{u_i'c'})+(1-\rho_a)\bar{U}_i(\overline{u_j'c'})\right] \\ &=-\frac{1}{\hat{\mathrm{Fr}}}\nabla_i\bar{P}+\frac{1}{\mathrm{Re}_\omega}\nabla_j\left(\nabla_j\bar{U}_i+\nabla_i\bar{U}_j-\frac{2}{3}\nabla_k\bar{U}_k\delta_{ij}\right)+\frac{\bar{\rho}g_i}{\xi_F\omega^2}, \end{aligned} \tag{7}$$

where $\bar{\rho}=\bar{C}+\rho_a(1-\bar{C})$ and $\boldsymbol{\nabla}=\frac{D}{L}\frac{\partial}{\partial x}\vec{e}_x+\frac{\partial}{\partial r}\vec{e}_r+\frac{1}{r}\frac{\partial}{\partial\phi}\vec{e}_\phi$. In the non-dimensional momentum equation, the Froude number, $\hat{\mathrm{Fr}}=\left(\omega^2\,\hat{\xi}_F^2\right)/(gL)$ and the kinetic Reynolds number, $\mathrm{Re}_\omega=\left(\rho_w\omega D^2\right)/(\mu)$.

Momentum Equation for the Water and Air-Water Interface Zones

For sufficiently high aspect ratio (D/L), sloshing in the OWC becomes a major damping factor, which can be investigated by understanding the air-water interface dynamics. This requires soluton of the equation (7) numerically, without any simplification. However, the present paper assumes an interface of zero thickness to avoid its complex dynamics.

Unlike the interface, it is possible to simplify the momentum equation for the water zone. In the water region, $\bar{C}=1$ and $c'=0$. The x-component of the reduced version of equation (7) is,

$$\begin{aligned} &\frac{\partial\bar{U}}{\partial t}+\frac{\hat{\xi}_F}{D}\left[\frac{D}{L}\bar{U}\frac{\partial\bar{U}}{\partial x}+\bar{V}\frac{\partial\bar{U}}{\partial r}+\frac{\bar{W}}{r}\frac{\partial\bar{U}}{\partial\phi}\right]=-\frac{1}{\hat{\mathrm{Fr}}}\frac{D}{L}\frac{\partial\bar{P}}{\partial x} \\ &\quad+\frac{1}{\mathrm{Re}_\omega}\left[\frac{D^2}{L^2}\frac{\partial^2\bar{U}}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial\bar{U}}{\partial r})+\frac{1}{r^2}\frac{\partial^2\bar{U}}{\partial\phi^2}\right] \\ &\quad-\frac{\hat{\xi}_F}{D}\left[\frac{D}{L}\frac{\partial\overline{u'^2}}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}(r\,\overline{u'v'})+\frac{1}{r}\frac{\partial(\overline{u'w'})}{\partial\phi}\right]+\frac{g_x}{\xi_F\omega^2}. \end{aligned} \tag{8}$$

For the simplification, we neglect the flow developing regions and assume a fully developed flow throughout the water zone. This assumption is only valid if the ratio D/L is small enough, which is also compatible with the assumption of zero interface thickness. Additionally, a furthur assumption of axisymmetric flow reduces the equation (8) to

$$\frac{\partial\bar{U}}{\partial t}=-\frac{1}{\hat{\mathrm{Fr}}}\frac{D}{L}\frac{\partial\bar{P}}{\partial x}+\frac{1}{r}\frac{\partial(r\tau_t)}{\partial r}+\frac{g_x}{\xi_F\omega^2}, \tag{9}$$

where the total shear stress (Reynolds stress and viscous stress) is given by

$$\tau_t=-\frac{\hat{\xi}_F}{D}\overline{u'v'}+\frac{1}{\mathrm{Re}_\omega}\frac{\partial\bar{U}}{\partial r}. \tag{10}$$

The cross-sectional area average of equation (9) is

$$\frac{\mathrm{d}\bar{U}_b}{\mathrm{d}t}=-\frac{1}{\hat{\mathrm{Fr}}}\frac{D}{L}\frac{\mathrm{d}\bar{P}}{\mathrm{d}x}-4\tau_t+\frac{g_x}{\xi_F\omega^2}, \tag{11}$$

where $\bar{U}_b$ is the mean bulk velocity and henceforth, it will be expressed by $\dot{\xi}_0$ (time derivative of the displacement ξ_0). Integrating eqution (11) with respect to x from 0 to S_0 (length of the water column) gives,

$$\ddot{\xi}_0=-\frac{1}{\hat{\mathrm{Fr}}}\frac{D}{S_0}(\bar{P}_a-\bar{P}_e)-4\tau_t-\frac{2g_x}{\omega^2 S_0}\xi_0, \tag{12}$$

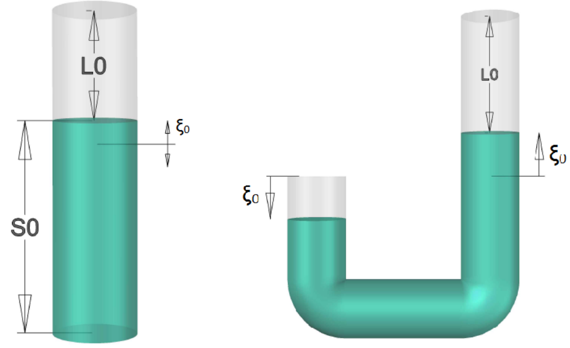

Figure 1: Two different models of OWC device; (left) the most general type OWC, and (right) a U-shaped OWC.

where $\bar{P}_a$ is the pressure in the air chamber and $\bar{P}_e$ is the pressure at the OWC entrance. The pressure from a half sinusoid above the sea level is

$$\bar{P}_w^* = \rho_w g H_{tc}/2\sqrt{2},$$

where H_{tc} is the height from trough to crest, as explained in [14]. Assuming that this is the pressure at the OWC entrance, i.e, the change of pressure is constant between the sea level and OWC entrance, gives

$$\bar{P}_e = \frac{H_{tc}\hat{\xi}_F}{2\sqrt{2}LD}.$$

Now, equation (12) can be rewritten as

$$\ddot{\xi}_0 + 4\tau_t + \frac{2g_x}{\omega^2 S_0}\xi_0 + \frac{1}{\hat{\mathrm{Fr}}}\frac{D}{S_0}\bar{P}_a = \frac{1}{\hat{\mathrm{Fr}}}\frac{\hat{\xi}_F H_{tc}}{2\sqrt{2}S_0 L}. \tag{13}$$

As mentioned earlier, reciprocating flow has been studied for various purposes, however, the energy loss due to the shear stress in such flow system is not yet clearly understood. Both experimental and numerical work are required to understand the role of the shear stress as damping. In this work, we tried to gather the required information about the shear stress in reciprocating flow from the existing literature and replace it in the above-mentioned equation by a damping term.

Non-Linear Damping Model for Total Shear Stress, τ_t

The experimental study by Hino [8] shows that, in reciprocating flow, the transition from laminar to turbulent flow occurs at $\mathrm{Re}_\delta = \bar{U}_b\delta/\nu = 550$, where $\delta = \sqrt{2\nu/\omega}$. In a prototype OWC, the approximate range of Re_δ is 1000 to 2000, which is far above that critical number. Another experiment on reciprocating flow by Jensen [10] shows that the flow becomes fully turbulent at $\mathrm{Re} = \hat{\xi}_F^2/\omega\nu \approx 1\times 10^6$. Generally, in an OWC, the Re is above this limit and therefor, it is reasonable to presume that, the flow in an OWC is fully turbulent.

Moreover, in [1], the phase variation of the total shear stress (τ_t), measured in a reciprocating flow, has been compared with the theories developed for steady turbulent flow and the laminar pulsatile flow. The experimental Reynolds number was within the transition region. Thus, during the acceleration period, the measured τ_t shows well agreement with the Uchida's solution for laminar ocsillatory flow. However, during the decelaration period, when the flow behaved as fully turbulent, the experimental result of τ_t agrees with the result from the Blasius correlation,

$$\frac{\tau_t^*}{\rho_w} = 0.03325\,\bar{U}_b^{*2}\left(\frac{2\nu}{D\bar{U}_b^*}\right)^{-0.25}, \tag{14}$$

which applies to steady turbulent flows in smooth pipes.

Since this correlation works fine at the turbulent flow regime in reciprocating flow, and we are presuming turbulent flow throughout the OWC, we can use this correlation to substitute τ_t in equation (13). Now, after scaling, and introducing the kinetic Reynolds number Re_ω to equation (14), we have,

$$\tau_t = 0.0275\,\bar{U}_b^{2.25}\left(\frac{\hat{\xi}_F}{D}\right)^{1.25}(Re_\omega)^{0.25}. \tag{15}$$

Substituting τ_t in equation (13) by (15) gives,

$$\ddot{\xi}_0 + 0.11\,(Re_\omega)^{0.25}\left(\frac{\hat{\xi}_F}{D}\right)^{1.25}|\dot{\xi}_0|^{1.25}\,\dot{\xi}_0 + \frac{2g_x}{\omega^2 S_0}\xi_0 + \frac{1}{\hat{\mathrm{Fr}}}\frac{D}{S_0}\bar{P}_a = \frac{1}{\hat{\mathrm{Fr}}}\frac{\hat{\xi}_F H_{tc}}{2\sqrt{2}S_0 L}. \tag{16}$$

Linear Damping Model for Total Shear Stress, τ_t

Modelling τ_t with a linear damping term in equation (13) is complicated, since the flow in the OWC is turbulent. Here we use the theory developed by Ogawa in [13], which studied the reciprocating flow in a U-tube, both experimentally and theoretically. A linear correlation between the velocity gradient in the transverse direction, and the ratio of the velocity to radius has been presented. The velocity distribution of the Bingham plastic flow is assumed to establish this correlation. Interestingly, the correlation works fine in both the laminar and turbulent flow regime.

Now, the total shear stress according to Ogawa's theory is

$$\frac{\tau_t^*}{\rho_w} = \nu_t\frac{\mathrm{d}\bar{U}^*}{\mathrm{d}y^*} = \nu K_\nu\frac{\bar{U}_b^*}{R^*}, \tag{17}$$

where the velocity factor,

$$K_\nu = 25\,D\left[1 + \frac{4.5\times 10^{-9}}{D^4}\right]K_\nu'\ (D:m),$$

and $K_\nu' = \dfrac{Re}{8.75 + 0.0233\,Re}$; $Re = \sqrt{\dfrac{2g}{S_0}}\,\dfrac{\hat{\xi}_F D}{\nu}$.

After scalling equation (17), and then replacing the τ_t in equation (13) by it, gives

$$\ddot{\xi}_0 + \frac{8K_\nu}{Re_\omega}\dot{\xi}_0 + \frac{2g_x}{\omega^2 S_0}\xi_0 + \frac{1}{\hat{\mathrm{Fr}}}\frac{D}{S_0}\bar{P}_a = \frac{1}{\hat{\mathrm{Fr}}}\frac{\hat{\xi}_F H_{tc}}{2\sqrt{2}S_0 L}. \tag{18}$$

Dissipation Due to the Power-Take-Off (PTO) System

Air in the air chamber gets compressed when the water column moves upward, and it expands when the water column moves downward. This tells us along with the equation (13) that, the air chamber pressure, $\bar{P}_a$ works as a negative force or damping in the equation of motion. This air pressure is highly dependent on the turbine geometry.

In this paper we assume that, there is no phase difference between the water column oscillation and the air flow through the turbine, which gives the mass flow rate as

$$\dot{m}^* = -\frac{\mathrm{d}(\rho_a^* V^*)}{\mathrm{d}t^*}, \tag{19}$$

where the volume of air in the air chamber, $V^*(t) = V_0^* - A_a\xi_0(t)$; V_0 is the volume of the air chamber when there is no oscillation, and the cross-sectional area of the air chamber, $A_a = \pi D^2/4$. We also assume that the turbine maintain a linear relationship between the mass flow rate and the air pressure,

$$\dot{m}^* = \frac{K d_0^*}{N^*} \bar{P}_a^*, \tag{20}$$

where K is the turbine coefficient which is fixed for a given turbine geometry, d_0^* is the diameter of the turbine rotor, and N^* is the rotational speed of the turbine. Assuming air as an ideal gas, and considering isentropic compression and expansion in the air chamber, from equation (19) and (20) after linearization we get,

$$\frac{V_0^*}{\rho_a^* c^{*2}} \frac{\mathrm{d}\bar{P}_a^*}{\mathrm{d}t^*} + \frac{K d_0^*}{\rho_a^* N^*} \bar{P}_a^* = q^*, \tag{21}$$

where the volume flow rate, $q^* = A_a \dot{\xi}_0$, and c^* is the speed of the sound in air. After scalling, equation (21) becomes as

$$\frac{\mathrm{d}\bar{P}_a}{\mathrm{d}t} + c^2 \frac{K d_0}{N V_0} \left(\frac{\hat{\xi}_F^2}{DL} \right) \bar{P}_a = \frac{c^2 \rho_a}{V_0} \left(\frac{\pi \hat{\xi}_F^4 \omega^2}{4DL^2 g} \right) \dot{\xi}_0. \tag{22}$$

Results

The average power generated by an OWC is calculated using the linear and nonlinear damping models, as shown in Figure (2). Power calculated from the nondimensional ODEs are translated to dimensional units by multiplying it with $\rho g L \omega D^3$. First, equation (13) and (22) are solved by neglecting the damping due to the shear stress for a device of 70 m long and 5 m dia. Although, such a long and norrow OWC has not yet reached prototype level, the next generation deep water devices may have such dimensions, as proposed in [6].

Figure (2) shows this OWC generates more than 500 KW power when it resonates. The inclusion of linear damping terms into the governing ODE reduces the power output, but not much. Whereas, the power output drops very significantly (to 2100 W), if the nonlinear damping model is used.

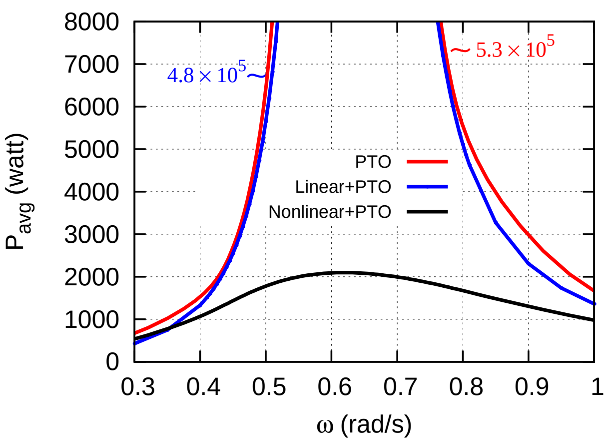

Figure 2: Average power (P_{avg}) generated by the OWC at different wave frequency (ω).

Conclusion

Understanding the different damping factors and estimating their contribution to the energy loss in an OWC is one of the most important design prerequisites. So far, damping due to internal turbulent shear stress has not received much attention in the design of OWCs. The present results show that internal turbulent damping may play an important role when the device is long. The shear stress in this case causes very significant damping of the overall OWC power extraction. In addition, the energy losses due to the vortex generation at the entrance and the sloshing inside the OWC remain unevaluated.

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