

## Irregular Shock Refraction in Magnetohydrodynamics

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### Abstract

Analytical solutions to the magnetohydrodynamic shock refraction problem are used to explore the limiting interface angles for which regular refraction will occur, for a particular choice of problem parameters and varying magnetic field strength. Beyond this limit, irregular shock refraction occurs and no analytical solutions exist at present. Thus, the wave structure resulting from irregular magnetohydrodynamic shock refraction is determined via ideal simulations, allowing its properties to be explored.

### Introduction

Shock refraction occurs when an incident shock ( $I$ ) interacts with an interface, or contact discontinuity ( $CD$ ), separating fluids of different densities. The initial condition for the canonical flow where shock refraction occurs is shown in figure 1. This flow is characterized by the incident shock Mach number  $M$ , the density ratio across the interface  $\eta$ , the angle between the interface and the shock velocity  $\alpha$ , and the ratio of specific heats  $\gamma$ . When all waves resulting from the refraction process meet at a point and are planar, this is known as regular shock refraction. In hydrodynamics, regular shock refraction results in single transmitted ( $T$ ) and reflected ( $R$ ) waves, and the shocked contact discontinuity ( $SC$ ) is a shear layer, as illustrated in figure 2(a).

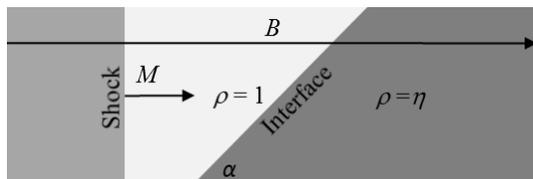


Figure 1: Initial condition for simulations of the MHD shock refraction problem with contour levels indicating density.

Shock refraction can be important any application involving shock waves and variable density flows. These include inertial confinement fusion (ICF) [1], astrophysical phenomena [2], supersonic and hypersonic air-breathing engines, shock-flame interactions and the operation of impulse facilities such as shock tunnels and expansion tubes. In ICF, the surface of a spherical capsule is rapidly ablated, driving a converging shock into deuterium-tritium fuel contained within the capsule [1]. When this shock interacts with the perturbed material interfaces within the capsule, vorticity is deposited on them by the shock refraction process. This is the origin of the Richtmyer-Meshkov instability (RMI) that promotes mixing between the capsule material and the fuel, which limits the possibility of achieving energy break-even or production [3]. In ICF, however, the fluids

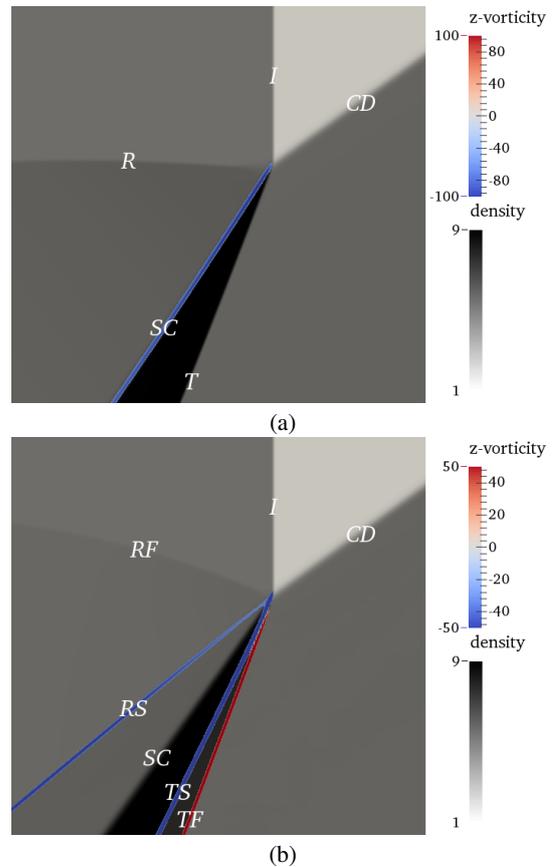


Figure 2: Shock structures resulting from regular refraction of a  $M = 2$  shock at an  $\alpha = 36^\circ$ ,  $\eta = 3$  density interface for  $\gamma = 7/5$  and (a) no magnetic field or, (b) an initially horizontal magnetic field with  $\beta = 3/4$ . Density contours are shown in greyscale. These are overlaid with vorticity contours clipped at low vorticity magnitude.

involved become rapidly ionized and hence will interact with magnetic fields. This will also occur in astrophysical settings and attempted magnetohydrodynamic power generation or inlet control in scramjets. Such interactions may be modeled in the framework of ideal magnetohydrodynamics (MHD). In MHD, the presence of a magnetic field  $\mathbf{B}$  greatly alters the regular shock refraction process, as illustrated in figure 2(b): Additional transmitted and reflected waves are generated that transport vorticity away from the shocked interface [4]. This leaves the interface vorticity free, hence suppressing the RMI [5, 6]. Furthermore, recent experiments demonstrate that immersing ICF targets in magnetic fields results in increased ion temperature

and neutron yield during capsule implosion [7], which is partially attributed to electron confinement. This provides further motivation for utilizing magnetic fields in ICF experiments.

Our present understanding of the underlying mechanism by which the MHD RMI is suppressed in the presence of a magnetic field relies on regular shock refraction occurring everywhere along the density interface, facilitating vorticity transport from its vicinity. In hydrodynamics, there are well established limits beyond which regular refraction cannot occur, resulting in what is known as irregular refraction [8]. One shock structure that can result from irregular refraction, known as single Mach reflexion [8], is shown in figure 3: the waves no longer meet at a single point as a Mach stem forms between  $R$  and  $T$ . Crucially, a second shear layer emerges from the triple-point where  $I$ ,  $R$  and the Mach stem intersect. In MHD, however, irregular shock refraction is a mostly unexplored topic. Conditions may exist, which we aim to establish, where irregular MHD shock refraction occurs and the process by which vorticity is transported from the vicinity of shocked interfaces is uncertain. To determine whether the suppression mechanism for the MHD RMI is uniformly valid, we must map the conditions for which irregular MHD shock refraction occurs, and establish whether the wave structure resulting from irregular refraction leaves the vicinity of the interface vorticity free.

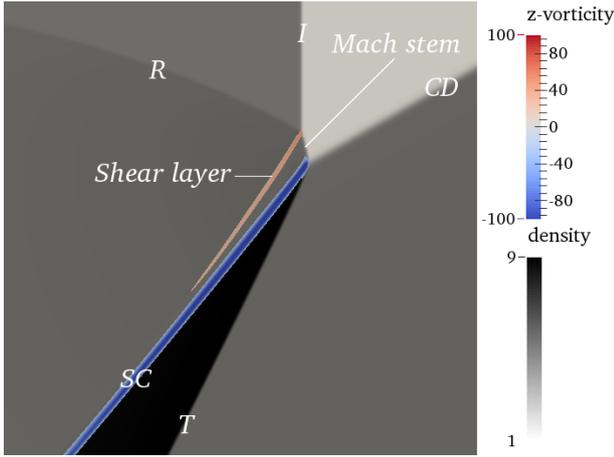


Figure 3: Shock structures resulting from regular refraction of a  $M = 2$  shock at an  $\alpha = 30^\circ$ ,  $\eta = 3$  density interface for  $\gamma = 7/5$  and no magnetic field. Density contours are shown in greyscale. These are overlaid with vorticity contours clipped at low vorticity magnitude.

In this paper, we use analytical solutions to the MHD shock refraction problem to explore the limiting interface angles for which regular refraction will occur, for a particular choice of problem parameters ( $M = 2$ ,  $\eta = 3$ ,  $\gamma = 7/5$ , initial  $\mathbf{B}_0$  aligned with the shock normal). This will be done for a limited range of magnetic field strengths, characterized by  $\beta = 2p_0\mu_0/\mathbf{B}_0 \cdot \mathbf{B}_0$ , where  $\mu_0$  is the permeability and  $p_0$  is the initial thermodynamic pressure, for which only shocks result from the refraction process. Beyond the limit of regular refraction, the result of the process is unknown and no analytical solutions exist at present. Thus, the wave structure resulting from irregular MHD shock refraction is determined via ideal MHD simulations, allowing its properties to be explored.

### Solution Technique for Regular MHD Shock Refraction

Shock refraction problems are generally solved analytically by determining the wave structure that will satisfy a set of matching conditions across the  $SC$ . For the hydrodynamic problem,

only two conditions must be met: the pressure and flow angle on each side of the  $SC$  must be matched. Two degrees of freedom, the angles of  $T$  and  $R$  when regular refraction is possible, are sufficient to allow these conditions to be met. In MHD on the other hand, additional constraints are placed on the  $SC$  (see e.g. [9]): the velocity, magnetic field and pressure must all be continuous across the contact if the magnetic field is not parallel to it (this special case cannot occur for the parameters considered here), otherwise the ideal MHD equations are not balanced and the flow would evolve. These five conditions (in two dimensions) are not all independent, as the MHD equations are subject to the constraint  $\nabla \cdot \mathbf{B} = 0$ . Thus if both components of the velocity, the pressure and the magnetic field magnitude are matched across the  $SC$  by utilizing four degrees of freedom, the divergence constraint ensures that the direction of the magnetic field is also matched. To supply these four degrees of freedom, four waves of unknown angle are required. For simplicity, here we consider parameters where all of these waves are shocks. Multiple reflected and transmitted shocks are possible since in MHD, several types of shock exist.

Shocks are governed by the Rankine-Hugoniot relations. Solving the MHD Rankine-Hugoniot relations for the normal velocity ratio across the shock,  $r = u_{n2}/u_{n1}$ , yields four roots. Aside from the trivial root  $r = 1$ , the remaining three roots may each correspond to entropy increasing shocks, depending on the upstream conditions. In MHD, there are three characteristic wave speeds: the Alfvén velocity,  $c_A = \mathbf{B}/\sqrt{\mu_0\rho}$ , and the fast and slow magnetosonic speeds, denoted  $c_F$  and  $c_{SL}$ , respectively. The component of the Alfvén speed normal to a shock is known as the intermediate characteristic speed,  $c_I = B_n/\sqrt{\mu_0\rho}$ . Thus there are four regimes for the velocity normal to a shock; 1:  $u_n > c_F$ , 2:  $c_F > u_n > c_I$ , 3:  $c_I > u_n > c_{SL}$ , and 4:  $c_{SL} > u_n$ . If the state upstream of a shock is in regime 1, three different regime transitions may occur across the shock: 1  $\rightarrow$  2, known as a fast shock; 1  $\rightarrow$  3 and 1  $\rightarrow$  4, which are both types of intermediate shock. These transitions correspond to the three non-unity roots of the Rankine-Hugoniot relations, which in this case are all real and entropy increasing. From other upstream regimes, 2  $\rightarrow$  3 and 2  $\rightarrow$  4 transitions are also intermediate shocks, while 3  $\rightarrow$  4 transitions are known as slow shocks. A further type of discontinuity propagates at  $c_I$  and has no effect on the thermodynamic state of the fluid, but does rotate the tangential magnetic field and generate a tangential velocity jump, hence it is known as a rotational discontinuity ( $RD$ ). Note that while fast and slow shocks are accepted as being physically realistic, there is debate over the admissibility  $RD$ s in strongly planar flow (where  $u_z = 0$  and  $B_z = 0$ ) and of intermediate shocks in non-strongly planar flow, as summarized in [4] and [9].

The full solution technique for the MHD regular shock refraction problem, including conditions where rarefactions are produced, is documented in [4]. A brief overview of the shock-only solution technique is presented here to aid interpretation of the results: We first postulate a wave structure containing four plane shocks that radiate from the intersection point at unknown angles. Mathematically this corresponds to selecting which root of the Rankine-Hugoniot relations will be used to compute the jumps across each shock. The unknown shock angles are then found iteratively using a secant method. This will only converge if the postulated wave structure is valid. In order to map out the regimes of validity for each shock structure, this iterative solution technique was encapsulated in a marching algorithm that sought solutions for varying problem parameters utilizing initial guesses for the shock angles extrapolated from existing solutions. Transitions in shock types were automatically detected and made by monitoring when two roots of the Rankine-Hugoniot relations approached each other during

marching. Where a converged solution could not be achieved for any combination of wave types, we suspect regular refraction is not possible. Note that solutions to the MHD shock refraction problem are in general not unique, and the non-unique solutions differ in the types of waves that occur [4]. As an example, within the parameter ranges considered here, for any solution that includes an intermediate shock, there exists another solution where the intermediate shock is replaced by an RD followed downstream by a slow shock.

### Limit of Regular Refraction

The algorithm discussed in the previous section was used to compute solutions to the MHD shock refraction problem characterized by  $M = 2$ ,  $\eta = 3$  and  $\gamma = 7/5$ . Here, we present results for the solution branch that includes intermediate shocks as the magnetic field weakens. We map the solution regimes for relatively strong magnetic fields with  $\beta < 5/2$ , and for low interface angles ( $\alpha < 4/5$ ), as in hydrodynamics irregular refraction occurs when  $\alpha$  becomes too small. Where regular solutions exist in this parameter space, they are quintuple points, as shown in figure 2(b). These consist of the hydrodynamic incident shock,  $I$ , transmitted and reflected fast shocks,  $TF$  and  $RF$ , and transmitted and reflected sub-fast shocks,  $TS$  and  $RS$ , which may be intermediate or slow. The computed regimes of validity of these solutions are presented in figure 4

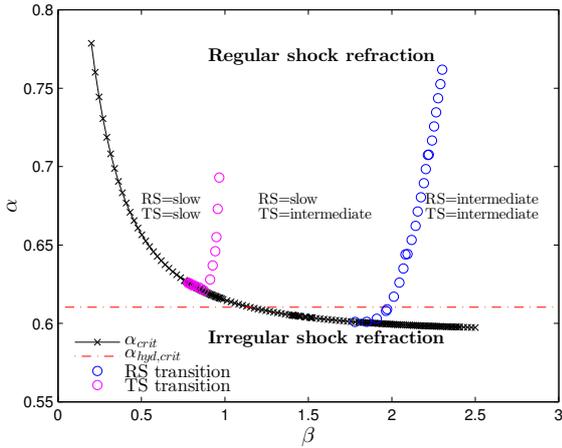


Figure 4: Solution regimes for the MHD shock refraction problem for  $M = 2$ ,  $\eta = 3$ ,  $\gamma = 7/5$  and varying  $\alpha$  and  $\beta$ .

For strong magnetic fields (low  $\beta$ ),  $RS$  and  $TS$  are slow shocks in the regular solutions. As  $\beta$  increases, first  $TS$  then  $RS$  transition to intermediate shocks. These transitions are seen to be delayed with increasing interface angle  $\alpha$ . The proposed low  $\alpha$  limit of regular refraction,  $\alpha_{crit}$ , is revealed to be highly dependent on the strength of the magnetic field. Strong magnetic fields appear promote irregular refraction, with the limiting angle for regular refraction rising steeply as  $\beta$  tends to zero. On the other hand weaker magnetic fields, with  $\beta > 1.15$  up to the maximum value studied here, appear to suppress irregular refraction: the proposed transition to irregular MHD refraction occurs at a lower interface angle than the hydrodynamic limit of  $\alpha_{hyd,crit} \approx 35.5^\circ$  for these conditions. It remains to be determined whether irregular refraction does indeed occur beyond the limit where we are able to compute regular solutions. This will be investigated by conducting numerical simulations for  $\alpha < \alpha_{crit}$ .

### Simulation Methodology

The initial condition utilized for shock refraction simulations

is depicted in figure 1. A constant inflow condition is applied on the left boundary, and an extrapolated outflow condition is applied on the right. Symmetry conditions are applied on the top and bottom boundaries. The ideal MHD equations (see e.g. [9]) are solved numerically using a second-order accurate, compressible finite-volume code described in detail in [10]. A dimensionally unsplit upwinding scheme with a Roe-type flux solver is utilized, and a projection method is used to enforce the solenoidal property of the magnetic field. An adaptive mesh refinement scheme of the Berger-Collela type is used under the Chombo framework. The base mesh is a uniform  $256^2$  Cartesian grid and three levels of refinement with refinement ratios of 2 are utilized. The criterion for refinement is  $|\nabla \rho| < \rho/50$ , resulting in an effective resolution of  $2048^2$  in the vicinity of the shocks and contact discontinuities.

### Irregular MHD Shock Refraction Structure

To investigate the flow structure that arises from MHD shock refraction below  $\alpha_{crit}$ , we simulate a shock refraction problem with  $M = 2$ ,  $\eta = 3$ ,  $\gamma = 7/5$ ,  $\alpha = 30^\circ$  and  $\beta = 3/4$ . This value of  $\beta$  was chosen since it yields only fast and slow shocks when regular refraction is possible, hence the solutions, and consequently the value of  $\alpha_{crit}$ , are unique. For  $\beta = 3/4$ , we compute that  $\alpha_{crit} \approx 35.98^\circ$ , thus irregular refraction should occur for  $\alpha = 30^\circ$  and the  $\alpha = 36^\circ$  case shown in figure 2(b) is close to the low  $\alpha$  limiting regular solution.

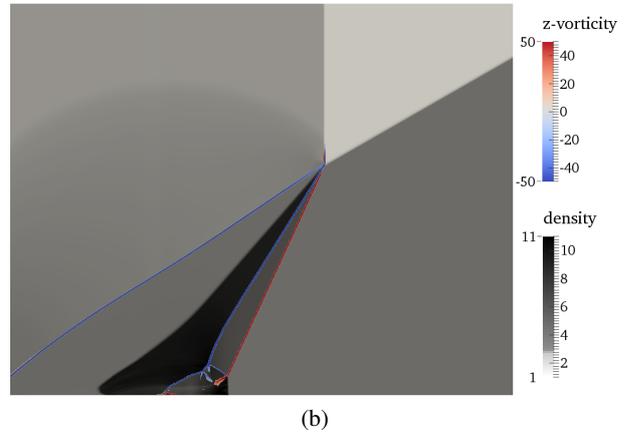
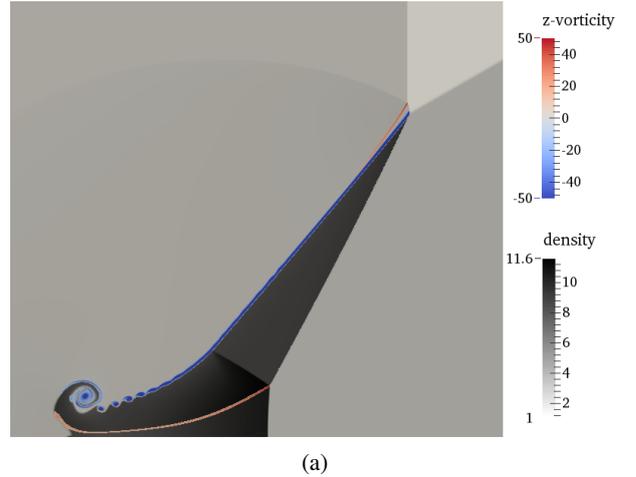


Figure 5: Numerical solutions featuring irregular refraction of a  $M = 2$  shock at an  $\alpha = 30^\circ$ ,  $\eta = 3$  density interface for  $\gamma = 7/5$  and (a) no magnetic field or, (b) an initially horizontal magnetic field with  $\beta = 3/4$ . Density contours are shown in greyscale. These are overlaid with vorticity contours clipped at low vorticity magnitude.

Figure 5 compares the zero field (hydrodynamic) and  $\beta = 3/4$  solutions to the shock refraction initial value problem described above. Both solutions are seen to be irregular, confirming that irregular shock refraction does occur in MHD. Note that the hydrodynamic case has been run for longer to allow the Mach stem to grow, since its growth rate is substantially lower than in the MHD case. The irregular shock structure produced in the  $\beta = 3/4$  case appears to be the MHD equivalent of the single Mach reflexion that occurs with zero magnetic field: the reflected fast shock  $RF$  no longer intersects interface and is connected to the transmitted shocks and  $RS$  by the MHD equivalent of a Mach stem. The changes in flow properties across the MHD Mach stem are consistent with it being a fast shock: figure 6 shows that it compresses the flow and increases the magnitude of the tangential magnetic field without changing its sign. The MHD Mach stem is more oblique to the flow than its hydrodynamic equivalent, and grows more rapidly with time. Interestingly, there does appear to be a further, weak compression of the flow above  $RS$  in figure 6.

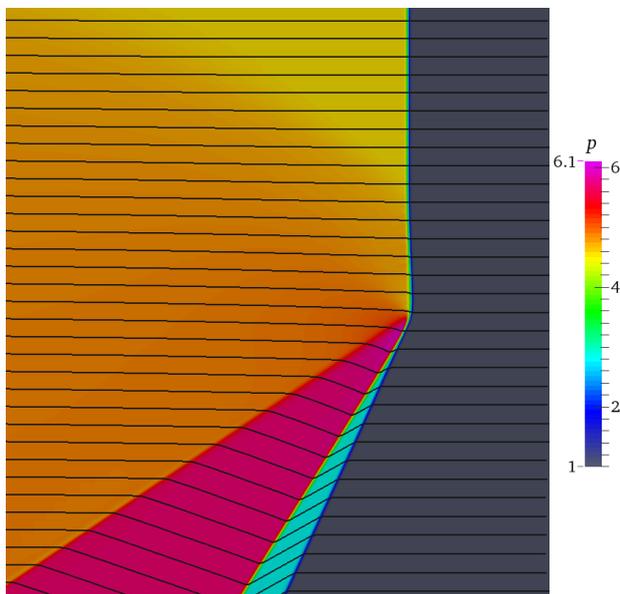


Figure 6: Shock structure produced by irregular refraction of a  $M = 2$  shock at an  $\alpha = 30^\circ$ ,  $\eta = 3$  density interface for  $\gamma = 7/5$  and an initially horizontal magnetic field with  $\beta = 3/4$ . Pressure contours overlaid with magnetic field lines are shown.

Crucially, the irregular MHD shock refraction structure does not generate a shear layer emanating from the triple-point between  $I$ ,  $RF$  and the Mach stem. Thus all vorticity generated by the shock refraction process is transported from the vicinity of the shocked contact by the MHD shocks, as in the case of regular refraction. This prevents the shear instability that causes the roll-up of the interface in the hydrodynamic case, an effect that is clearly demonstrated by comparing figure 5(a) to 5(b). These results indicate that the mechanism by which a magnetic field suppresses the MHD RMI is valid independent of whether regular or irregular refraction occurs at the density interface.

## Conclusions

Making use of an existing analytical solution technique, the limiting interface angles for which regular MHD shock refraction occurs were investigated for a range of magnetic field strengths. It was found that strong magnetic fields promoted the transition to irregular refraction, while weaker magnetic fields delayed it compared to the hydrodynamic case. A numerical

simulation confirmed that irregular MHD shock refraction did indeed occur beyond the computed limiting interface angle. The resulting shock structure resembled an MHD variant of single Mach reflexion, with the reflected fast shock connected to the density interface by a Mach stem, which was classified as a fast shock. The triple-point between the incident shock, reflected fast shock and the Mach stem did not produce a shear layer, indicating that all vorticity was transported from the vicinity of the density interface, preventing its instability.

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