Reynolds Number Influence on the Vortex-Induced Vibration Critical Point of a Pivoted Cylinder

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Abstract
The aim of the present investigation was to examine the Reynolds number dependence of the critical vortex-induced vibration (VIV) point for cylinders in a pivoted system. For translational systems, the existence of a critical mass ratio for cylinders undergoing vortex-induced vibration has been well established. At mass ratios below this critical point, the reduced velocity at VIV lock-out tends to infinity, resulting in the resonance forever condition as described by the authors first reporting this phenomenon. For translating cylinder VIV, a Reynolds number dependence of the critical mass ratio has been noted. A variation of the critical mass ratio from 0.36 at low Reynolds number, tending toward 0.54 at higher values has been observed. To the author’s knowledge, no such Reynolds number dependence has previously been reported for pivoted cylinders. The approach adopted in the present investigation involved measuring the VIV response of a positively buoyant pivoted cylinder being towed at very high reduced velocity at different Reynolds numbers. High reduced velocity was attained by establishing a very low system natural frequency and the vibration frequency to the system natural frequency, is

frequency equation may be obtained by substituting harmonic forces since

The structural mass, $m$, includes any enclosed fluid, but excludes the hydrodynamic mass. Note that the mass ratio is equivalent to the magnitude of the ratio of the weight, $W$, and buoyancy, $B$, as

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The mass ratio parameter influences both the amplitude and frequency response of the cylinder. With higher mass ratios (e.g. a cylinder vibrating in air, with a mass ratio $O(100)$), changes in added mass are relatively insignificant due to the low density of the fluid. The natural frequency then remains relatively unchanged throughout the lock-in range. When the fluid medium under consideration is much denser (e.g. a cylinder vibrating in water), distinct changes in the natural frequency are observed. The increasing natural frequency observed with increasing reduced velocity is directly attributable to the decreasing added mass throughout the lock-in range [18, 21]. An overview of the characteristics of low mass damping VIV is given in the review paper by Gabbai & Benaroya [2].

Since the hydrodynamic mass variation is largely responsible for synchronisation of the shedding and vibration frequencies, typically much wider lock-in regions are experienced at low mass ratio [17, 21]. The limit of this trend is found at the critical mass ratio of around 0.54 [5], below which there exists no lock-out point and VIV occurs at all velocities above the initial lock-in. This is the resonance forever condition as described by the authors first reporting this phenomenon.

Using the approach adopted by Khalak and Williamson [10] a frequency equation may be obtained by substituting harmonic force and response expressions in the forced linear oscillator equation of motion. The resulting frequency equation, the ratio of vibration frequency to the system natural frequency, is

Change in hydrodynamic mass, as demonstrated in the experiments of Vikested [21]. The range of reduced velocity over which this synchronisation occurs is known as the lock-in range. Mostly, the ensuing vibrations are undesirable, resulting in increased fatigue loading and component design complexity to accommodate these motions. The transverse vibrations also result in higher dynamic relative to static drag coefficients. With decreasing mass ratio, an increase in the amplitude response is generally evident [17]. Also, the smaller the mass ratio, the larger the relative influence of the hydrodynamic mass on the vibration response of the structure.

Various definitions for the mass ratio are widely employed. In this work, the mass ratio is defined as the ratio of the structural mass, $m$, to the displaced fluid mass, $m_d$, as

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The investigation by Horowitz and Williamson [8] where the VIV motions of a rising and falling cylinder were examined yielded a critical mass ratio of 0.54. This arrangement, despite allowing the cylinder multiple degrees of freedom, produced results in close agreement with previous experiments. A system free to vibrate inline and transverse to the flow has the potential at low mass ratio to display a super-upper response branch (e.g. Pesce and Fujarra, [14] and Stappenbelt & Lalji, [16]) rather than the upper response branch observed in transverse only experiments. By extrapolation of their data, the two degree of freedom system experiments by Jauvtis and Williamson [9] revealed a critical mass ratio value of 0.522.

Few published studies have examined the critical point for a pivoted cylinder. The studies by Leong and Wei [12] and Voorhees, Dong, Atsavapranee, Benaroya and Wei [22] attempt to apply the concept of the mass ratio to a rotational system. Insufficient information is provided in these papers to ascertain the mass moment of inertia of the cases covered. The former study presents only partial response curves for limited mass ratios and the latter only provides experimental results above the critical point.

In the pivoted cylinder study by Flemming and Williamson [1] the mass moment of inertia ratio is surmised, using the approach by Khalak and Williamson [10], to be the governing parameter for the VIV critical point. Adopting the mass moment of inertia ratio as the governing parameter appears to be the logical choice as it is the rotational analogy of the mass ratio in a translational system. In the investigation by Flemming and Williamson [1], three mass moment of inertia ratio cases were covered, ranging from $I^*/I^\text{crit} = 0.59$ to $I^*/I^\text{crit} = 1.03$. These experiments, although all performed above the critical point, yielded effective added inertia coefficient ($C_{\text{Ed}}$) data that allowed a prediction of a critical point value of $I^*/I^\text{crit} < 0.5$. It must be noted however that the $C_{\text{Ed}}$ values obtained were not constant in the lower response branch (as was the case for $C_{\text{Ed}}$ values for the translating cylinders), rather, they were continually decreasing until the de-coherence region.

The force moment ratio, $M^*$, is defined as the ratio of the moment about the point of rotation due to the weight force acting on the structural mass to that acting on the displaced fluid mass as

$$M^* = \frac{M}{M_d}.$$  

Note that equation 5 is equivalent to the magnitude of the ratio of moments due to the structural weight (i.e. $W r_s \sin \theta$ where $r_s$ is the distance of the centre of gravity (CoG) to the centre of rotation) and buoyancy (i.e. $B r_b \sin \theta$ where $r_b$ is the distance of the centre of buoyancy (CoB) to the centre of rotation) forces in the plane of transverse oscillations,

$$i.e. M^* = \frac{|W r_s \sin \theta|}{|B r_b \sin \theta|} = \frac{|W r_s|}{|B r_b|}.$$  

The study by Stappenbelt and Johnstone [19] showed the existence of a critical point for a pivoted cylinder which was governed by the force moment ratio rather than the mass moment of inertia ratio. The aim of the present study is to examine the dynamics of a pivoted cylinder to determine the Reynolds number dependence of the critical point for rotational systems.

**Methodology**

The present investigation consists of an experiment in which a pivoted cylinder as illustrated in the sketch of figure 2 was towed along a 32.5m water tank. Inline vibrations were restrained and transverse vibrations were measured by the use of laser displacement transducers. Figure 2 also acts as a parameter definition sketch. The angular displacement relative to the initial position of the cylinder in the plane of transverse oscillation is designated $\theta$. It is the angular position relative to the vertical.
Table 1 details the parameter values for the experiment. The force moment ratio was experimentally controlled by the addition of lump masses inside the end of the cylinder. The experiment was conducted over a Reynolds number range from $5.3 \times 10^3$ to $6.5 \times 10^4$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force moment ratio range</td>
<td>$M^*$</td>
<td>0.315-0.762</td>
</tr>
<tr>
<td>Reynolds number range</td>
<td>$Re$</td>
<td>$5.3 \times 10^3$-$6.5 \times 10^4$</td>
</tr>
<tr>
<td>Cylinder length</td>
<td>$L$</td>
<td>842 mm</td>
</tr>
<tr>
<td>Pivoted length</td>
<td>$L_p$</td>
<td>1256 mm</td>
</tr>
<tr>
<td>Cylinder diameter</td>
<td>$D$</td>
<td>48 mm</td>
</tr>
<tr>
<td>Structural damping ratio</td>
<td>$\zeta$</td>
<td>0.006</td>
</tr>
<tr>
<td>Minimum reduced velocity</td>
<td>$U_r$</td>
<td>$O(1000)$</td>
</tr>
</tbody>
</table>

The non-dimensionalised freestream flow velocity (i.e. the reduced velocity) is given by

$$U_r = \frac{u}{n_B}.$$  \hspace{1cm} (7)

The natural frequency, $f_n$, is proportional to the square root of the angular restoring force coefficient, $k_\theta$. Considering the sum of the moments about the cylinder pivot point it may be shown that

$$k_\theta = W_{rW} - B r_B + 2 k r_k.$$  \hspace{1cm} (8)

To test for resonance as $U_r \to \infty$, the restoring force coefficient, $k_\theta$ must tend to zero. From equation 8 it may be seen that this condition is approached as

$$k r_k \to (W_{rW} - B r_B)/2.$$  \hspace{1cm} (9)

The spring rate and distance of the point of restoring force action from the pivot were utilised to ensure that the reduced velocity was always of the order of 1000. Some restoring capability was of course necessary to maintain the centralising tendency of the cylinder whilst undergoing flow induced vibration.

## Table 1 Experimental parameter values.

The angular response of the cylinder at various force moment ratios at a Reynolds number of $3.16 \times 10^4$ are presented in figure 4. Each data point in these plots represents the averaged half peak to peak angular response of between 60 and 85s of steady-state data collected, representing between 52 and 58 oscillation periods. The angular displacement amplitude is normalised as

### Results

Two time series samples are provided in figure 3. The angular displacements in these plots are normalised by the angle $\theta_0$ equal to half the angle subtended in the arc defined by a chord and radius equal to the diameter and length from the cylinder tip to pivot point respectively. The time series obtained in this study typically displayed a low frequency drift from the equilibrium position due to the low restoring tendency of the system with $k_\theta \to 0$.

![Figure 3. Time response above (a) $M^*$=0.33 and below (b) $M^*$=0.70 the critical point: $Re$=1.2 x 10^4.](image)

It is clear from figure 3 that a transition from low amplitude forced vibration response to high amplitude resonant response occurs indicating the critical point. Figure 3a is an example of the nature of the resonant vibrations below the critical point displaying large amplitude regular oscillations. This is in contrast with figure 3b which is an example of the forced vibration response of the pivoted cylinder above the critical point.

![Figure 4. Response amplitude as a function of the force moment ratio at $Re$=3.16 x 10^4.](image)

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The transition from resonant to forced vibration appears to occur at $M^*=0.504$ implying the occurrence of the critical point in this region. This process was repeated at 24 different Reynolds numbers ranging from $5.3 \times 10^3$ to $6.5 \times 10^4$. The critical points, identified through the forced to resonant amplitude transitions, at these Reynolds numbers are plotted in figure 5.

\[
\theta^* = \frac{\theta_{\text{max}}}{\theta_{\text{p}}}
\]

(10)

The asymptotic behaviour of the critical point at higher Reynolds number noted in the translating cylinder case is also observed in the present case. The critical force moment ratios for pivoted cylinders tend to 0.53 with increasing Reynolds number, corresponding reasonably well with the translating cylinder critical point values reported.

**Acknowledgements**

The authors gratefully acknowledge the contribution to this work by Mr. Matthew Vassallo and Mr. Ashley Heath during their undergraduate research project at the University of Wollongong.

**References**


