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Thermodynamic analysis of the energy separation in a counter-flow vortex tube

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Abstract

The phenomenon of temperature separation within a vortex tube is a complex result of different factors, including pressure gradient, flow stagnation and mixture due to the internal flow structure, energy transfer between the different flow layers and heat transfer between the tube and the ambient air. There has not been a well accepted explanation for the phenomenon due to its complex internal flow condition. This paper reports analytical results of different expansion models, effect of different working fluids on the cooling performance and energy balance in the tube. Supports for pervious published hypothesis and recommendation for further research are concluded from these analytical results.

Introduction

The vortex tube is a thermal device that can generate two exhausts at different temperature from a single injection. The typical structure of a counter-flow vortex tube and the proposed flow pattern within the tube are presented in Figure 1. The working medium (yellow), which is generally air or other compressible gas, is injected into the tube through a tangential nozzle. The low temperature stream (blue) is discharged through the central exit near the injection and the high temperature stream (red) is exhausted from the peripheral exit at the far end of the tube.

Since its invention in 1933 by Ranque [7], a number of investigations on the vortex tube have been published, which concern the exploration of the mechanism within the tube, theoretical analysis of the separating process, optimisation of the tube performance, and numerical simulation of the working process within the tube. However, the mechanism of the temperature separation within a vortex tube still remains unclear and there has not been a well accepted explanation. This manuscript reports a theoretical analysis of the temperature change in different expansion processes accompanying with the comparison between the calculated and experimental temperature profiles. The cooling performance of the vortex tube using different gases in previous experimental investigations are analysed. A relationship between the gas properties and cooling performance is proposed based on the analysis of calculated and collected data. Moreover, application of the thermodynamic laws within a vortex tube is validated and recommendations for further investigation are concluded.

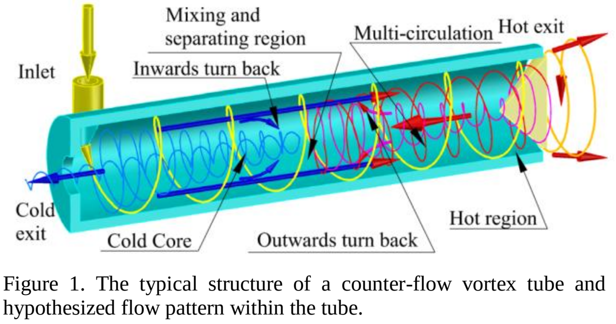

Figure 1. The typical structure of a counter-flow vortex tube and its hypothesized flow pattern within the tube.

Thermodynamic analysis of different expansion models

Due to the pressure gradient within a vortex tube, a consideration of expansion as the major factor for the temperature drop inside a vortex tube, has been reported by many researchers. Hence, different expansion models, including adiabatic expansion, free expansion and Joule-Thomson expansion, are analysed here to clarify the nature of the sudden expansion.

For an adiabatic and isentropic expansion, the variations in temperature, T, pressure, P, and entropy, S, can be related as:

$$\left(\frac{\partial T}{\partial P}\right)_S = -\frac{\left(\frac{\partial S}{\partial P}\right)_T}{\left(\frac{\partial S}{\partial T}\right)_P} = \frac{\alpha V T}{C_P} \tag{1}$$

Here, α is the volume expansivity, V is the gas molar volume and C_P is the specific heat at constant pressure.

For a free expansion, the internal energy of a system keeps constant in the process, hence we have $dU = 0$. Therefore, the temperature of an ideal gas in a free expansion would keep constant, while for a real gas, its temperature varies. And this is the reason here to conduct the temperature variation of a real gas in a free expansion. Similarly to the above process, the relationship between the temperature, pressure and internal energy in a free expansion process is given by:

$$\left(\frac{\partial T}{\partial P}\right)_U = \frac{-T\left(\frac{\partial V}{\partial T}\right)_P - P\left(\frac{\partial V}{\partial P}\right)_T}{C_P - P\left(\frac{\partial V}{\partial T}\right)_P} \tag{2}$$

To enable quantitative analysis of the temperature variation in the free expansion process, the Virial equation of state is used in this analysis. Take the first two components of the Virial equation and make $A = RT, B = b - \frac{a}{RT}$, it reads:

$$\left(\frac{\partial T}{\partial P}\right)_U = \frac{aT}{C_P R T^2 - R^2 T^2 - Pa} \tag{3}$$

Here, a and b are coefficients of the Van der Waals equation of state, R is the specific gas constant, P is the pressure.

In a Joule-Thomson expansion process, the variation between the temperature and pressure can be expressed as:

$$\left(\frac{\partial T}{\partial P}\right)_H = \frac{1}{C_P}\left(\frac{2a}{RT} - b\right) \tag{4}$$

To simplify the quantitative analysis of the temperature variation using different gases in the vortex tube, the mathematical equation for an ideal gas undergoing an adiabatic process is used here as:

$$P^{1-\gamma} T^{\gamma} = C \tag{5}$$

Here, P and T represent the pressure and the temperature of the system respectively, γ is the heat capacity ratio of different gases. The accuracy of the calculated temperature using this equation is ensured by the calculated compressibility factor for different gases used in the vortex tube.

	Z	T_{Free} (K)	T_{JT} (K)	$T_{JT\ factor}$ (K)	$T_{Adiabatic}$ (K)
Air	0.99	-0.268	-0.256	-0.232	-53.001
O_2	0.99	-0.271	-0.277	-0.28	-52.57
He	1.05	-0.0111	0.099	0.0625	-71.45
N_2	0.99	-0.268	-0.249	-0.221	-53.001
CO_2	0.97	-0.520	-0.688	-1	-42.46
Ar	0.99	-0.444	-0.377	-0.372	-71.45
NH_3	0.97	-0.648	-0.879	-2.1	-44.627

Table 1. Compressibility factors of different gases and temperature variation in different expansion models

It can be seen from the table that the temperature variation of different gases in a free expansion process is always negative and less than 1 in magnitude (negative number represents the temperature drop). Note the temperature of the ideal gas system keeps constant in a free expansion, and these small magnitude temperature variations are generated by real gases. Similarly, in a Joule-Thompson process, the systems of different gases have negligible temperature changes. The last column of the table is the temperature drop of different gases in an adiabatic expansion, which show the ability of adiabatic expansion in reducing the temperature. Considering the temperature drop in the lower pressure region within a vortex tube system, i.e., generally 10 to 30 degree at different conditions, only the adiabatic expansion could be considered to be involved in the temperature change within a vortex tube.

Moreover, the velocity profile within a vortex tube has been reported by many researchers [1-4]. Considering the dimension of a commercial vortex tube (generally in order of 100 mm), the magnitude of velocity in the low pressure region near the injection, which generally varies from 10 m/s to 50 m/s, indicates the mechanism occurs in an extremely short time, so that there is no opportunity for significant heat exchange. Hence, the consideration of adiabatic expansion in the vortex tube is further supported. The flow mechanism near the injection of a vortex tube was observed as a force vortex with a large magnitude centrifugal acceleration [1-4] (in order of 10^6 rad/s in commercial tubes). Therefore, because of this large centrifugal acceleration, the central fluid performs compression on the peripheral fluid. Energy is transferred towards the periphery via the work done by the central fluid, which contributes to the temperature variation within a vortex tube. These comparison an analysis provide further support for the proposed hypothesis of the temperature separation within a vortex tube [5].

The cooling performance of the vortex tube using different gases

The cooling performance or temperature drop of a vortex tube is a function of different parameters, including the geometry of the tube, inlet and outlet properties, and the properties of the working fluid. This function can be expressed as:

$$\Delta T_c = f(G_{vt}, P_{in}, P_{out}, F) \quad (6)$$

Where, G_{vt} is the geometrical parameters of the vortex tube system, P_{in} and P_{out} represent the parameters at inlet and outlet of the vortex tube respectively, and F is the property of the working fluid. In order to identify the difference in vortex tube cooling performance that caused by the different working gases, experimental data is collected from published articles [6-8] which used different gases in the vortex tube. The maximum temperature drops from the vortex tube using different gases and the theoretical temperature drop in an adiabatic expansion are collected in table 2. It can be seen clearly from the table that the maximum temperature drop from the vortex tube using different gases varies at different inlet pressure. The better cooling performance of the vortex tube with helium and argon are observed from these data. However, the temperature variation profile from the numerical simulation [8] does not show similar tendency to the experimental results, especially the temperature drop using helium. Therefore, together with the discussion in the introduction about the paper [8], the temperature profile will not be included in further analysis.

Maximum temperature drop (K)						
	Theoretical calculation	Pin=1.5 bar	Pin=2 bar	Pin=3 bar	Pin=7 bar	Pin=6 bar
Air	53.001	10.5	16.5	26	39.5	26
O_2	52.57	10.5	16	27	39	25
He	71.45	13	21.5	34.5	------	72
N_2	53.001	------	------	------	40	26.5
CO_2	42.46	------	------	------	------	28
Ar	71.45	------	------	------	53	------
NH_3	44.627	------	------	------	------	28

Table 2. Maximum temperature drop profile from the vortex tube (theoretical, experimental and numerical results)

It is also noticed that different geometry of the vortex tube and different inlet parameters are used in these investigations. Therefore, to focus on the effect of fluid properties, a non-dimensionalized parameter, i.e., temperature drop ratio *(γ_F)*, is used in this analysis, which is defined as the ratio of the actual temperature drop ($\Delta T_{c,i}$) over the temperature drop using air in the same situation ($\Delta T_{c,air}$).

$$\gamma_F = \frac{\Delta T_{c,i}}{\Delta T_{c,air}}$$

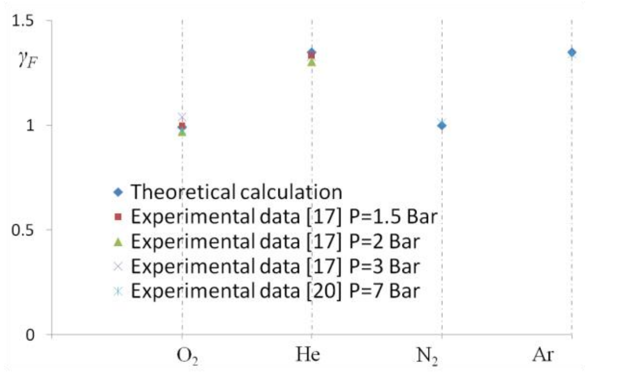

Figure 2. Temperature drop ratio of different gases in an adiabatic expansion and vortex tubes

The remarkable characteristic of this ratio is that the temperature drops in the equation are recorded when different gases are used in the same vortex tube and at same operational conditions. This means the parameters "G_{vt}, P_{in} and P_{out}" for the recorded temperature drops are exactly the same, hence can be removed from the function by this division. Then, only the effect of the fluid property on the cooling performance of a vortex tube is indicated by this ratio. Using the theoretical calculation and experimental data, the temperature drop ratio is calculated and presented in figure 2. As presented in the figure, the temperature drop ratio calculated based on the experimental data agree well with each other. Hence, it indicates the reliability of the non-dimensionalized temperature drop ratio in this analysis. Furthermore, an agreement between the experimental and theoretical temperature drop ratio is also observed from the figure, which shows the strong correlation between the adiabatic expansion model and the cooling performance of a vortex tube. This correlation also support the proposed hypothesis [5] that the expansion or pressure gradient within the vortex tube is the dominant factor in the temperature drop.

Energy balance analysis within a vortex tube

The investigation of the vortex tube always concerns the energy balance between the cold and hot streams. The first law of thermodynamics has been used to identify the energy balance with the assumption of negligible velocity of the exhausts at the exits and isolated system from the ambient surrounding [2, 9-13]. However, the acceptance of the ignorance of the velocity component and heat transfer between the tube and the ambient air has not been validated. Therefore, experimental data is collected and used to evaluate the assumptions used in the energy balance within a vortex tube.

Ignoring the velocity component and heat transfer, the application of the energy balance within the vortex tube system appears as:

$$E_{in} = E_{out}$$

Where, E_{in} and E_{out} are the total energy of the streams at eh inlet and outlet, respectively. Applying the first law on the system, it follows that

$$\dot{m}_{in} c_p T_{in} = \dot{m}_{in} \epsilon c_p T_c + \dot{m}_{in} (1-\epsilon) c_p T_h \tag{7}$$

Reformatting the equation and taking

$$\Delta T_c = T_{in} - T_c, \quad \Delta T_h = T_h - T_{in}$$

The following relationship can be derived,

$$\dot{E} = \dot{m}_{in} \epsilon c_p \Delta T_c - \dot{m}_{in} (1-\epsilon) c_p \Delta T_h \tag{8}$$

Here, T_{in}, T_c and T_h are the temperature of inlet, cold flow and hot flow respectively, $\dot{E}$ is the transporting rate of the internal energy of the vortex tube system to the ambient, and equals zero under the abovementioned assumptions. $\dot{m}_{in}$ is the mass flow rate at the inlet, ε is the cold mass fraction, c_p is the specific heat ratio of the working fluid. Therefore, to evaluate the energy balance within the vortex tube, a non-dimensionalized energy change rate $(\dot{\theta})$ is defined here as:

$$\dot{\theta} = \frac{\dot{E}}{\dot{m}_{in} c_p} = \epsilon \Delta T_c - (1-\epsilon) \Delta T_h \tag{9}$$

Experimental temperature profiles are collected from previous publications [2, 4, 13-15] as the function of the cold mass fraction. The temperature drop from the vortex tube increases with an increase of the cold mass fraction, until it reaches the maximum temperature change at the cold mass fraction equals 0.3. Then the temperature of the cold flow from the vortex tube increases with further increase in the cold mass fraction. Similarly, the temperature of the hot exit flow increases until reaching its maximum value at ε=0.8, and then drops at ε=0.9. Substituting the temperature profiles as the function of cold mass fraction into the energy change rate, figure 3 is observed. It is concluded form the figure that the positive energy change rate indicates there is energy transferred outwards from the vortex tube to the ambient air. When we look at the temperature change of the two exhausted flow, this energy change also show that the heat released from the temperature drop of the cold flow is more than the heat required to generate the hot exit flow. This is caused by the heat transfer from the wall of the vortex tube to the ambient air, due to the high temperature of the rear part of the tube. With an increase of the cold mass fraction, the temperature of the hot flow increases, and indicates an increase in the wall temperature as well. Therefore, more heat is transferred to the ambient from a higher temperature wall of the vortex tube, which leads to the increase of the energy change rate as shown in figure 3. When the cold mass fraction keeps increasing, the cold region within the vortex tube enlarges as discussed in [5] and results in a higher temperature of the tube wall in a smaller region. The reduction of the high temperature area of the wall is the reason for the decrease in the energy change rate when the cold mass fraction increases from 0.5 to 0.9. Moreover, the heat absorbed from the ambient air through the low temperature region of the vortex tube, i.e., the cold exit of the tube, further contributes to the decrease of energy change rate.

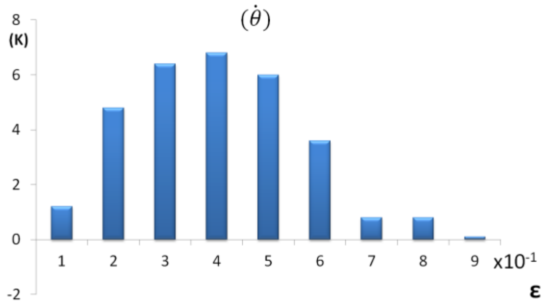

Figure 3. Energy change rate as a function of cold mass fraction after [13]

The maximum energy change rate is about 7 degree at ε=0.4. This means the temperature variation from a vortex tube can be further incrassated by at least 7 degree, if the heat transfer can be prevented. The magnitude and variation of the energy change rate indicate the necessity of careful consideration in the energy analysis of a vortex tube. Similarly, further analysis based on the collected experimental data is performed and presented in figure 4. Series 1-4 are conducted after Shannak [13], while series 5-8 are conducted after Gao [2], Xue[4], Stephan [14] and Promvonge [15], respectively. All the series are presenting the energy change rate as a function of cold mass fraction form small to large. Therefore, it is seen from the figure that all the cases have dominating positive values of the energy change rate. Hence, the vortex tubes lost heat to the ambient air during the experiments. The difference in the magnitude and variation of this parameter is caused by the different inlet pressure and different geometry of the tubes. For example, large scale vortex tubes were employed in the investigations for series 5 [2] and series 6 [4], while relative small vortex tubes were used for the other series. For the negative values in series1 and 2 when the cold mass fraction is close to 1, an overall absorbance of heat from the ambient air should be caused by its geometrical parameters. Therefore, the conclusion drawn from the figure is that the heat transfer between the vortex tube and the ambient air must be counted in the energy analysis of the separating phenomenon within the vortex tube.

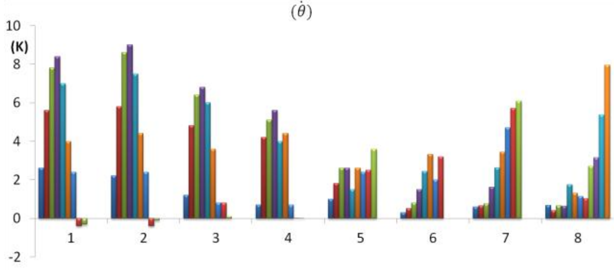

Figure 4. Energy change rate as a function of cold mass fraction

Moreover, further analysis is conducted focusing on the heat transfer between the vortex tube and ambient air. Once a vortex tube is insulated from its surroundings, a constant total internal energy of the system should be observed. Therefore, the temperature separation within a vortex tube should be dominated by the heat transfer between the different layers of flow. To validate this assumption, experimental results are collected from [14], substituted into the energy change rate, and presented in figure 5. The energy change rate displayed in the figure shows negative supports for the assumption. The small magnitude of the energy change rate indicates the effect of insulation, which was also presented by the temperature difference with or without the insulation of the same vortex tube [14]. Due to its magnitude and variation, the negative energy change rate cannot be considered as errors generated during the experimental investigation. According to the definition of energy change rate, its negative value indicates there is some heat absorbed by the vortex tube system during the operation and the total energy of the system increases. However, as the vortex tube is insulated from the environment, there will not be any significant heat transfer during the experiments. This conflict clearly indicates the inappropriate assumption taken in the energy analysis, i.e., the ignorance of the velocity component. Moreover, it is not reasonable to state the heat transfer between different layers of flow is the governing factor in the temperature separation.

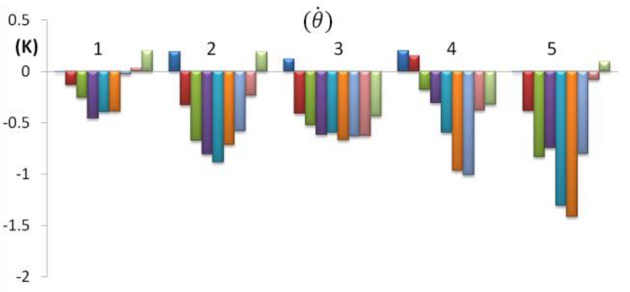

Figure 5. Energy change rate as a function of cold mass fraction in a insulated vortex tube

Conclusions

This manuscript reports theoretical analysis of different expansion models relating to the cooling performance of the vortex tube. Adiabatic expansion is found has the great capability to generate the temperature drop, and hence is considered as a main factor in the temperature separation within a vortex tube. Different cooling performance of the vortex tube using different gases are noticed and collected from previous investigations. A non-dimensionalized temperature drop ratio is defined to compare the difference in the cooling performance of different vortex tubes using different gases. Good agreements of the temperature drop ratios calculated from the experimental results are presented. The good agreement between the theoretically calculated temperature drop ratio and that based on the experimental data also provides positive support for the effect of adiabatic expansion. The analysis of the energy balance within a vortex tube suggests that the heat transfer between the vortex tube and ambient air should be included in the analysis. Analysis in an insulated vortex tube indicates further investigation is required due to the unbalanced energy profiles from the vortex tube. Moreover, the velocity component has more significant effect in the energy analysis, and the unbalanced total energy profile leads to the consideration of increased entropy of the system. Quantitative analysis of the energy balance, effect of the pressure gradient and mixture due to the flow structure are recommended, aiming to provide detail information about the energy profiles, hence identify the dominating factor in the temperature separation within a vortex tube.

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