Dynamics of a Quasi-Two-Dimensional Wake Behind a Cylinder in an MHD Duct Flow with a Strong Axial Magnetic Field

A. H. A. Hamid\textsuperscript{1, 2}, W. K. Hussam\textsuperscript{1} and G. J. Sheard\textsuperscript{1}

\textsuperscript{1}The Sheard Lab, Department of Mechanical and Aerospace Engineering, Monash University, Victoria 3800, Australia
\textsuperscript{2}Faculty of Mechanical Engineering, Universiti Teknologi MARA, 40450 Selangor, Malaysia

Abstract
A confined laminar viscous flow past a two-dimensional bluff body in the presence of a strong uniform magnetic field is considered. The effects of Reynolds number (Re) and Hartmann number (Ha) on the dynamics of the wake are examined, with a focus on the shedding frequency and the distribution of the wake vortices. These two parameters are of primary interest as they play an important role in determining the mixing and heat transfer properties of the downstream flow. The results indicate that the imposed magnetic field significantly alters the dynamic behaviour of the wake behind a cylinder.

Introduction
It is well-known that beyond a critical \( Re \), the flow around a circular cylinder generates a regular pattern of vortices known as the Kármán vortex street [9]. Analysis of such bluff body wakes are typically divided into three main focus areas: the correlation between drag coefficient, base pressure and shedding frequency; the vortex dynamics, where the formation and re-arrangement process are addressed; and the stability of the mean velocity profile in the wake [10].

For low Reynolds number (50 < \( Re \) < 150), the vortices are shed in a half-wavelength staggered array. Classically the vortex street within this Reynolds number range has been divided into three distinct regions, namely the formation region, the stable region, and the unstable region [12]. The formation region occurs immediately behind the cylinder, where the vorticity dissipates and organizes into a coherent structure [9]. It has been shown in previous studies that the formation process plays a significant role in determining the shedding frequency and other wake behaviours [9, 12]. In general, the increase of turbulence level leads to higher shedding frequency. Succeeding the formation region is the stable region, where [12] found that the wake advection velocity is about 90% of the free stream velocity. As the vortices advected further downstream, the viscous effect has become less dominant and eventually leads to vortex street breakdown. A new, larger vortical structure evolves in the core with velocity \( U_0 \) and high gradients in the vicinity of the lateral walls.

Numerical Method and Validation
In the current investigation a flow of electrically conducting fluid passing over a circular cylinder placed on the centreline of a duct is considered. Figure 1 depicts the numerical domain and the corresponding macro-element mesh. The ratio of cylinder diameter to the duct width (i.e. blockage ratio, \( \beta = d/2L \)) is fixed at 0.1 throughout this study. Also shown in the figure is a typical Hartmann velocity profile, characterized by a flat profile in the core with velocity \( U_0 \) and high gradients in the vicinity of the lateral walls. The length scale is normalized by the half channel width, \( L \). However, for the sake of discussions, the \( Re \) and the geometrical length in the succeeding discussions are presented in cylinder diameter scale, \( d \). The use of two different length scales in an MHD cylinder wake flows is inevitable: the two-dimensional linear braking term is governed by \( Ha \) and \( L \), whereas the \( Re \) and thus the structure of the cylinder wake is governed by \( d \) [4].

When a strong magnetic field is imposed to a conducting fluid, the resulting wake possesses a distinct features as compared to the normal hydrodynamic flows. Typical example of such flows is in fusion power-reactor breeding blankets, where an electrically conducting fluid flows in channels within the blankets under a strong plasma-confining magnetic field. This class of flows are known as magnetohydrodynamic (MHD). The interaction between induced electric currents and the applied magnetic field results in an electromagnetic Lorentz force, which in turn gives a damping effect to the flow [14, 5] and subsequently alters the formation of vortex street.

It has been shown in previous studies that the wake stability is influenced by the near-wake behaviour [7] and the vortex trajectories [9]. While a great deal of attention has been paid to wake vortices in hydrodynamic (HD) flow, less attention has been paid to magnetohydrodynamic (MHD) counterparts. This is despite the potential application of bluff-body turbulence promoters for enhancement of heat transport in ducts within magnetic-confinement fusion reactor blankets [6]. An example of such work in MHD flows is by [3], where they did parametric study of the cylinder wake evolution at high \( Re \) and \( Ha \) regime. They found that for a given \( Re \), wake formation length and shedding frequency increases as \( Ha \) is decreased. In the current work, particular attention is given to the low \( Re \) regime, motivated by the typical low velocity operating condition in fusion reactor application [16]. Furthermore, the low \( Re \) regime means that the wake behaviour under the effect of magnetic field can be directly compared with the hydrodynamic counterpart cases. To the best of the authors’ knowledge, no such comparison has been made. The findings of the current work will furnish valuable information for promoting heat transfer in fusion-related duct flows.
shedding frequency data from [11] is compared with the results obtained from the current simulations and the simulated results match well with the experimental data (refer figure 2). Further validation of the code can be found in [6, 5].

A grid independence study for spatial resolution has been performed by varying the element polynomial degree across a wide range, while keeping the macro element distribution unchanged. The mesh near the lateral walls and the cylinder was refined to resolve the anticipated high gradients, especially with the flows involving strong magnetic fields (refer figure 1). The monitored parameters (i.e. the pressure and viscous component of the time-averaged drag coefficient, $C_{D,P}$ and $C_{D,visc}$ respectively, and the shedding frequency, $St$) revealed good convergence when the polynomial order increases. Polynomial degree 7 produced at most a 0.1% error, and is therefore used hereafter.

**Result and Discussion**

In all simulations, two basic regions of wake vortices are apparent; a formation region in which the vorticity evolved from cylinder boundary-layers organises into a vortex street, and a stable region in which the shed vortices convect downstream in a periodic laminar manner. This section presents the results of shedding frequency analysis and vortex distributions.

**Shedding Frequency Analysis**

In the current investigation, the effect of axial magnetic field on shedding frequency is of interest. The dependencies of $Ha$ and $Re$ on the shedding frequency are illustrated in figure 3. It is to be noted that $H = 0$ correspond to hydrodynamic flows. The dimensionless frequency is represented by the Strouhal number, $St = fD/U_0$, where $f$ is shedding frequency, calculated from the fluctuating lift force imparted on the cylinder due to the near-wake flow unsteadiness.

As figure 3 indicates, the Strouhal number is dependent on both $Ha$ and $Re$. In the range of the $Ha$ and $Re$ considered in this study, $St$ increases with increasing $Ha$ at a given $Re$. This observation can be attributed to the fact that the imposed magnetic field tends to stretch the shear layer at the near wake, and hence mass conservation requires that the wake advection velocity, $U_0$ is increased. It can be seen in figure 4 that stronger magnetic field intensity produces a narrower wake, thus extending the formation region behind the cylinder before the shear layer roll up into a vortex street.

The accelerating effect of the magnetic field on the wake advection is illustrated in figure 5. The increase in $U_0$ as $Ha$ is increased explains the increase in $St$. It is also interesting to note that at a given $Ha$, $St$ increases but $U_0$ decreases with increasing $Re$. In this case, the increase in $St$ cannot be associated with an increase of $U_0$, but rather to the increased vorticity production rate as $Re$ is increased. At high $Re$, the vorticity attached to the cylinder quickly organises into a coherent structure due to the high rate of vorticity supply, thus the duration associated with the formation of vortex shedding becomes shorter, leading to a shorter formation length and a higher $St$.

In order to verify this argument, the formation length, $L_F$, has been measured and the results are presented in table 1. The formation length is defined as a distance from the rear of the cylinder to the position of first shed vortex peak strength [9]. The measurement error, $\epsilon_{L_F}/L_F$ presented in table 1 is associated with the phase gap between two consecutive snapshots, i.e. 0.25$\pi$ (which corresponds to nine snapshots per shedding cycle). The maximum error per wavelength is 11%, where the wavelength is calculated by employing the Taylor hypothesis (which is valid since the present case is in laminar flow regime), i.e. $\lambda = U_0/f$. It is found that the distance taken for the vorticity to complete its formation decreases as $Re$ is increased, which supports the aforementioned statement. This trend is also in agreement with the experimental results of the HD counterpart [12, 15]. Furthermore, the decrease in the formation length is more prominent at higher $Ha$ value.

This argument also explains the contradicting trend of $St$ at higher $Re$ regime as reported by [3]. They found that at a given $Re$, the $St$ decreases with increasing $Ha$. At a higher $Re$, the rate of vorticity production is increased, thus reducing the ac-
Vortex Distributions

Vortex trajectories along the computed domain have been recorded based on the locations of peak vorticity. In all cases, the trajectories of positive and negative vorticity show a symmetric profile with respect to the wake axis. From these trajectories, lateral spacing ($l$) and longitudinal spacing ($L$) of the vortices have been computed and normalized by the cylinder diameter. The lateral spacing is defined as the vertical distance between peak vorticity of adjacent opposite-sign vortices, or twice the distance between a peak vorticity of either a positive or negative-signed vortex and the wake centerline (for a symmetric vortex shedding). The longitudinal spacing is defined as the distance between successive vortices of the same sign.

The spatial evolution of lateral spacing is presented in figure 6. For brevity, only cases with $Ha = 0$ and cases with highest $Ha$, i.e. $Ha = 150$ are shown. A very distinctive wake pattern was observed between the two classes of flows. When the magnetic field is absent, the lateral spacing first decreases to zero at a distance about 130-160% of its respective wavelength (shown by the dotted lines). At this position, an inversion of the wake takes place, where vortices cross the wake centreline to the side opposite to that on which they formed. After this the lateral spacing increases along the computed domain. By referring to figure 4(a), it is apparent that the inversion mechanism is the entrainment of vorticity from the free stream into the wake, which tends to push the vortex towards the inverted position. It is also important to note that the distance at which the wake inverts from the cylinder decreases when Re is increased, which is in agreement with the previous findings [2]. However, in the presence of a magnetic field, no wake inversion takes place, at least along the length of the computed domain. By inspecting the decay of wake vortices, it is expected that inversion will never take place for MHD flow at this particular $\beta$. In contrast, wake inversion will eventually occur in $Ha = 0$s flow even for a low $\beta$ [2]. This can be explained due to the fact that in the presence of magnetic field, the Lorentz force tends to flatten the incoming flow velocity profile, which eliminates vorticity from the core flow. As a result, the wake vortices advect downstream in an almost parallel manner (as indicated by the small gradient of the MHD data in figure 6) without interaction with the free stream vorticity. While there is no appreciable difference in $h$ for all Re in HD flows, $h$ decreases as Re is increased in MHD flows. As Re is increased, the rate of vorticity supply to the shear layer attached to the cylinder increases. As a result, the tip of the shear layer tends to roll up further towards the wake axis, narrowing the wake. Furthermore, it can be seen that the wake is wider under the influence of a magnetic field than the corresponding non-MHD wake, regardless of the flow Re. This observed widening is supported by the previous 3D simulation by [8].

On the other hand, the longitudinal spacing is almost constant for a given $Ha$ and Re, except in the formation region, where $l$ increases until the vortex stabilizes. Here, the local value of $l$ is not important, and therefore only its mean values are presented in table 1. It is important to note that the local $l$ was determined from the phase diagram, and that the average value was calculated over the stable region. These average values are comparable with the wavelength calculated previously, which strengthens the previous argument that the wake flow is

<table>
<thead>
<tr>
<th>$Ha$</th>
<th>$Re_d$</th>
<th>$L_f$</th>
<th>$\epsilon_{L_f}$</th>
<th>$\lambda$</th>
<th>$\epsilon_{L_f}/\lambda(%)$</th>
<th>$l_{avg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>160</td>
<td>0.80</td>
<td>0.17</td>
<td>4.80</td>
<td>3.6</td>
<td>4.80</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>0.78</td>
<td>0.14</td>
<td>4.70</td>
<td>3.0</td>
<td>4.69</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.68</td>
<td>0.12</td>
<td>4.61</td>
<td>2.7</td>
<td>4.61</td>
</tr>
<tr>
<td>100</td>
<td>160</td>
<td>1.48</td>
<td>0.26</td>
<td>4.85</td>
<td>5.5</td>
<td>4.86</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>1.18</td>
<td>0.18</td>
<td>4.71</td>
<td>3.9</td>
<td>4.72</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.94</td>
<td>0.19</td>
<td>4.62</td>
<td>4.1</td>
<td>4.62</td>
</tr>
<tr>
<td>125</td>
<td>160</td>
<td>1.81</td>
<td>0.29</td>
<td>4.87</td>
<td>6.0</td>
<td>4.87</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>1.45</td>
<td>0.22</td>
<td>4.72</td>
<td>4.6</td>
<td>4.73</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>1.13</td>
<td>0.20</td>
<td>4.64</td>
<td>4.3</td>
<td>4.64</td>
</tr>
<tr>
<td>150</td>
<td>160</td>
<td>2.09</td>
<td>0.52</td>
<td>4.89</td>
<td>10.6</td>
<td>4.91</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>1.67</td>
<td>0.35</td>
<td>4.74</td>
<td>7.4</td>
<td>4.74</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>1.33</td>
<td>0.20</td>
<td>4.65</td>
<td>4.4</td>
<td>4.65</td>
</tr>
</tbody>
</table>

Table 1: Vortex shedding formation length and longitudinal length. The formation length is measured from the rear of the cylinder and both length are non-dimensionalized by the cylinder diameter.
laminar. Also to be noted is that $l_{\text{avg}}$ increases as $Ha$ is increased and $Re$ is decreased, and that $l_{\text{avg}}$ is more sensitive to changes in $Re$ than $Ha$. To understand this, one need to realize that the non-dimensional shedding frequency is given by $St = (U_0 - V)/l = U_0/1$ [11]. Although both $St$ and $U_0$ increase as $Ha$ is increased, the increment of $U_0$ is more prominent compared to $St$, which explains the increase in $l$ for increased $Ha$. A similar argument applies to the HD flows, where the reduction in $St$ is more significant compared to $U_0$ when $Re$ is decreased. The explanation of the increased $l$ as $Re$ is decreased in MHD flows is more straightforward: reduction in $Re$ leads to lower $St$ and higher $U_0$.

Conclusion

The present study has investigated the characteristics of wakes behind a circular cylinders in a rectangular duct under a strong axial magnetic field using a spectral-element method. It is found that the formation of vortex shedding and the direction of the imposed magnetic field play significant roles in determining the shedding frequency. The present investigation reveals that an axial magnetic field tends to appreciably increase the shedding frequency. The present investigation reveals that an imposed magnetic field play significant roles in determining the formation of vortex shedding and the direction of the wake vortices and thus the wake inversion phenomenon. In the presence of a magnetic field, two nearly parallel rows of vortices is also a strong function of both $Ha$ and $St$, whereas $U_0$ is only weakly dependent on $Re$ for hydrodynamic flows. An increase in $Ha$ leads to higher $U_0$. It follows then that the longitudinal spacing of wake vortices is also affected since the wake parameters are interdependent. In addition, the incoming flow vorticity plays important role in the lateral dispersion of wake vortices and thus the wake inversion phenomenon. In the absence of a magnetic field, the wake inverted at about 6 - 8 diameter downstream from the centre of the cylinder. However, in the presence of a magnetic field, two nearly parallel rows of vortices in a staggered arrangement resembling an ideal Kármán vortex street was observed.

Acknowledgements

This research was supported by ARC Discovery Grant DP120100153, high-performance computing time allocations from the National Computational Infrastructure (NCI) and the Victorian Life Sciences Computation Initiative (VLSCI), and the Monash SunGRID. A. H. A. H. is supported by the Malaysia Ministry of Education and the Universiti Teknologi MARA, Malaysia.

References