Wave Refraction on Southern Ocean Eddies

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Abstract
Motivated by differences in direction of propagation between the modelled and measured waves observed at the Southern Ocean Flux Station (SOFS), we investigated the possible effects of wave refraction on a Southern Ocean current field. We implemented a numerical model to describe the refraction of wave trains when propagating over a spatially varying current field and to see how they would be affected as they approach the location of the SOFS.

The model was applied to three typical swell directions observed in the Southern Ocean, coming from south (S), southwest (SW) and west (W). The regions of convergence (increasing energy) and divergence (decreasing energy) of the wave rays are defined for each situation, as well as the relative changes in wave energy, and hence wave height. It is shown that a specific eddy located at southwest from the buoy can have strong influences in the wave propagation and, in the case of westerly and southwesterly swells, can generate a clear divergence of the wave rays that would otherwise have reached the buoy location if there were no currents.

Introduction
The Southern Ocean Flux Station, deployed by the Australian Centre for Weather and Climate Research (CSIRO and Bureau of Meteorology) near -47°S and 142°E, is the first successful long-term mooring deployment in the Southern Ocean (see [5]). The analysis of the wave data showed to have a good agreement with wave models. However the peak direction showed characteristic disagreements at times, as a systematic wave refraction. The literature regarding the theoretical approach of wave refraction on currents is extensive. [2] expressed the changes in wave direction as a function of the current speed for a constant current shear model, in a Snell’s law form. However this model can hardly be applied to realistic situations. [1] presented a simplified solution to obtain a ray tracing model. Despite its simplicity it shows that the main characteristics of the currents which affects the wave propagation is not the current speed, but the horizontal current gradient. Since then, different works have been performed to analyse the refraction effects of a current field on a propagating wave. [3] studied how a current vortex could refract an incoming wave train. An interesting conclusion is that due to the opposite signal of the current gradient on each side of the vortex, the wave rays can create convergence and divergence zones ahead.

Ocean eddies are particularly important in deep ocean. Mesoscale geostrophic vortices are a common feature and numerical models can represent them fairly well. At the SOFS we observed considerably weak current velocities, however the location is surrounded by a complex system of eddies [4], which creates an interesting scenario for studying wave-current interactions.

Methods

Wave Refraction Model
The first effect to be considered when a wave train propagates into a current field is linear refraction. A basic concept in the wave study is the Doppler Shift that occurs in the wave frequency when propagating in a moving medium:

\[ \omega = \sigma + k \cdot U, \] (1)

where \( \omega \) is the absolute frequency or the frequency in a fixed frame of reference and is a function of the wavenumber vector \( k \), the position \( x \) and the spatially varying current, \( U(x,y) \) and \( \sigma \) is the intrinsic frequency or the frequency in a frame moving with the current. Equation (1) has an important influence in the wave field, since it modifies two very important parameters: frequency and wavenumber. Besides this first concept, the geometrical optics approximation can describe the wave packet kinematics and the consequent changes in wavenumber and position when propagating over an inhomogeneous medium (e.g. [7]):

\[ \frac{dx}{dt} = \frac{\partial \omega}{\partial k} - \frac{\partial k}{\partial x}, \quad \frac{dk}{dt} = -\frac{\partial \omega}{\partial x} \] (2)

These two equations represent widely applied laws in wave kinematics. The Doppler shift, caused by the currents presence and represented by equation (1), when applied into equations (2), enables us to infer important changes in wavenumber, frequency and direction of propagation undertaken by waves along the rays.

The first equation in (2) represents the changes in group velocity, i.e. the velocity with which the wave energy is propagated:

\[ \frac{dx}{dt} = \frac{\partial}{\partial k} \left( \sigma + \frac{\partial k}{\partial x} \right), \]

\[ e_g = \frac{c_g}{\partial} + U. \] (3)

Here \( e_g \) is the new group velocity in the presence of currents and \( c_g \) when the currents are absent. It means that the path through which the wave energy is propagated is directly proportional to the currents. The second equation in (2) is the well known law of ‘conservation of waves’ or ‘conservation of crests’ and relates the wavenumber variations in time with the angular frequency changes in space. If we develop this equation further using again equation (1), the changes in wavenumber can be expressed as
Here we can see that the wavenumber changes not as a function of the current itself, but as a function of the current gradient, represented by the last term on the right-hand side of the final equation. For convenience we can express equation (4) in terms of the wave propagation coordinates:

\[
\frac{dk}{dt} = -\frac{\partial}{\partial x} \left( \sigma(k) + k \cdot U \right)
\]

\[= -\frac{\partial \sigma(k)}{\partial x} - k \cdot \frac{\partial U}{\partial x}, \quad (4)
\]

where \(s\) is the coordinate in the direction of the wave propagation and \(d\) is the local depth.

If we consider that the intrinsic frequency is not a function of the local depth and consequently does not depend on the spatial coordinate, i.e. assuming deep water, the rate of change of wavenumber becomes

\[
\frac{dk}{dt} = -k \cdot \frac{\partial U}{\partial s}. \quad (6)
\]

Using this relation we can also deduce the changes in wave direction. Since the direction of propagation is given by the two components of wavenumber \((k_x, k_y)\) in the form

\[\theta = \arctan \frac{k_y}{k_x}, \quad (7)
\]

the change in wave direction caused by changes in wavenumber is hence

\[
\frac{d\theta}{dt} = \frac{d}{dt} \left( \arctan \frac{k_y}{k_x} \right). \quad (8)
\]

Using (4), one obtains

\[
\frac{d\theta}{dt} = -\frac{1}{\sigma} \left[ \frac{\partial \sigma}{\partial m} + k \cdot \frac{\partial U}{\partial m} \right]. \quad (9)
\]

where \(m\) is the coordinate perpendicular to the wave propagation. For deep water, (9) becomes:

\[
\frac{d\theta}{dt} = -\frac{1}{k} \cdot \frac{\partial U}{\partial m}. \quad (10)
\]

**Numerical Model and Wave Rays Simulation**

A numerical model to solve equations (2) was implemented using the Runge Kutta 4th order method. This model can simulate the wave ray path and calculate the changes in wavenumber and direction. In order to represent a wave train one thousands of rays were propagated from the west and south grid boundaries, totalling 2000 rays. The wave period for all simulations was 12 seconds as it was the mean period observed from the SOFS data. Equation (9) shows that changes in direction depend also on the wavenumber, more specifically the shorter the period the higher is the refraction and the wave ray is more deviated.

Three swell directions were simulated, corresponding to typical conditions observed in the Southern Ocean: coming from south, southwest and west. These directions were the most observed in the wave data analysis, with a high predominance of southwesterly swells.

The relative change in energy was estimated by the rays counting in regular grid cells. The grid domain was divided in 20x20 cells and the relative energy was defined as the the number for January 22, 2011. This day represents an example of the clear differences found between observed and modelled wave peak direction, and it was therefore chosen for this study. It is interesting to notice the strong eddy located at southwest from the buoy location (marked by the symbol ‘+’). This direction is where the main wave systems come from and thus can potentially influence waves at the SOFS. In the same figure, panel (b) highlights the eddy.

The grid showed in figure 1 was chosen in order to isolate the highlighted eddy and to be able to study how this specific eddy can modify the wave rays propagation that would eventually reach the location of the mooring.

![Figure 1](image)

**Current Model**

The current field used as input for the refraction model was from the CSIRO Bluelink ReANalysis (BRAN v. 3p5) [4], a data assimilation model which uses an Ensemble Optimal Interpolation (EnOI) called BODAS (Bluelink Ocean Data Assimilation System). Among the physical ocean variables are the three components of velocity. To represent the surface currents we used the 12 metres depth output, as it is about accurate as from altimetry [4].

Figure 1 shows the velocity vector field for the Southern Ocean and for the grid used for the refraction model (red rectangle) and for January 22, 2011. This day represents an example of the clear differences found between observed and modelled wave peak direction, and it was therefore chosen for this study. It is interesting to notice the strong eddy located at southwest from the buoy location (marked by the symbol ‘+’). This direction is where the main wave systems come from and thus can potentially influence waves at the SOFS. In the same figure, panel (b) highlights the eddy.

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of rays in a cell divided by the corresponding number of rays of the original incoming wave train (or in the absence of currents), which gives us an estimation of the degree of convergence and divergence of wave rays. Figure 2 shows an example of a wave ray propagating over the domain and the cells in the grid through which the ray will be counted for the energy estimation.

![Figure 2: Grid cells applied to calculate the relative changes in energy. A ray path is shown as example.](image)

**Results**

**Swell from South**

The rays refraction for a southerly swell is shown in figure 3. This figure shows a smaller number of rays than the actual simulation in order to clearer demonstrate the rays pattern.

![Figure 3: Rays refraction pattern for a southerly swell.](image)

In this case we can observe that the sides of the eddy produce different refraction patterns. The right side creates a convergence while the left one, a rays divergence. By the eddy right side, there is another current feature characterized by strong velocities and current gradients running to west and another eddy just above this jet stream, less intense and with opposite vorticity compared with the main one. These two features also refract the rays and generate a high convergence zone around the location of the SOFS.

Figure 4 shows the relative changes in energy caused by the rays refraction, where we can clearly see the convergence zone around the buoy location. The relative energy, here considered in terms of the number of rays, reaches 2.5 times (250%) the original value.

**Swell from Southwest**

The swells coming from southwest are the most common in the Southern Ocean. The mean direction obtained from the SOFS data analysis is 230°. It makes the refraction analysis of a southwest swell the most interesting in this study.

Figures 5 and 6 show the refraction of the incident rays and the corresponding relative energy distribution, respectively. Figure 5 again shows less number of rays than the simulation performed, to be visually more comprehensible.

![Figure 5: Rays refraction pattern for a southwesterly swell.](image)

The energy decreases to 0.2, i.e. 20% of the original incoming swell (Figure 6). On the upper right side of the grid, we can also see a clear convergence zone, characterised by the red coloured area in figure 6. This energy convergence is generated both by the refracted rays from the left side of the main eddy and from the strong current jet located at the lower left side of the grid.

![Figure 6: Relative changes in energy for a southwesterly swell.](image)
This region has an increase in energy of up to 2 times.

**Swell from West**

Swell from west are similarly affected by the main eddy of the grid domain. The same pattern of convergence and divergence for the different sides of the vortex occurs, however they do not affect considerably the SOFS location. The divergence point caused by the eddy is located below the mooring. Nevertheless we can see that the '*' symbol representing the measurements location is again in a region with less rays than the original wave train (Figure 7).

From figure 7 we can see that what generates this divergence is another eddy located at west from the buoy, less intense, however with a velocity gradient strong enough to refract the wave rays. The refraction produces maximum divergence before reaching the buoy. Notwithstanding it presents 90% of the original energy.

It is interesting to notice the vertical alternating pattern of divergence and convergence zones, typical of wave refraction on current vortices [3].

![Figure 7: Rays refraction pattern for a westerly swell.](image)

![Figure 8: Relative changes in energy for a westerly swell.](image)

**Conclusions**

To summarise the influence of the ocean circulation on the wave refraction at the Southern Ocean Flux Station, table 1 shows the relative changes in energy at the SOFS compared with the original incoming swell.

In this study the linear effects of refraction are shown to be extremely important in the wave field in the Southern Ocean. Different surface current features, including eddies, can be seen through the BRAN data assimilation model. The strong gradients which characterise these features can refract considerably the wave path and change not only the direction and wavenumber, as predicted by the refraction theory, but also the energy distribution by convergence and divergence of the rays.

<table>
<thead>
<tr>
<th>Swell Direction</th>
<th>South</th>
<th>Southwest</th>
<th>West</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rays Pattern</td>
<td>Convergence</td>
<td>Divergence</td>
<td>Divergence</td>
</tr>
<tr>
<td>Relative Changes in Energy</td>
<td>250%</td>
<td>20%</td>
<td>90%</td>
</tr>
</tbody>
</table>

Table 1: Summary of the refraction effects in terms of swell direction at the SOFS location.

The most common wave direction observed in the Southern Ocean, from southwest, is the most affected by refraction near the location of the SOFS, and as a result the relative changes in energy can reach up to 20% of that from the original incoming swell. We observed that the main responsible for this result was a strong eddy located at southwest from the mooring. We could also see that this eddy is seen during a long period of the year, but changing its shape and position. Therefore the observed differences between model and observed wave parameters can be potentially related to these variants. However further analysis need to be made.

This study aims to demonstrate and exemplify how the wave refraction on currents is important in the Southern Ocean and a precise forecast must take into account the current field. Nevertheless it is noteworthy that considering only the refraction of a single wave train is not enough to describe the disagreements between the model and measured wave peak direction found in the SOFS data. A full spectrum must be propagated as the wave components are affected differently throughout the current field. Furthermore it is necessary to include the nonlinear wave-wave interaction to fully describe the wave-current interaction, as it was shown to be an important term in the maintenance of the spectral stability [6].

**References**


