Onset of Transition in Mixed Convection of a Lid-Driven Trapezoidal Enclosure Filled with Water-$Al_2O_3$ Nanofluid

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Abstract

A numerical study is carried out to investigate the transition from laminar to chaos in mixed convection heat transfer inside a lid-driven trapezoidal enclosure. In this study, the top wall is considered as isothermal cold surface, which is moving in its own plane at a constant speed, and a constant high temperature is provided at the bottom surface. The enclosure is assumed to be filled with water-$Al_2O_3$ nanofluid. The governing Navier–Stokes and thermal energy equations are expressed in non-dimensional forms and are solved using Galerkin finite element method. Attention is paid in the present study on the pure mixed convection regime at Richardson number, $Ri = 1$. The numerical simulations are carried out over a wide range of Reynolds ($0.1 \leq Re \leq 10^3$) and Grashof ($0.01 \leq Gr \leq 10^8$) numbers. Effects of the presence of nanofluid on the characteristics of mixed convection heat transfer are also explored. The average Nusselt numbers of the heated wall are computed to demonstrate the influence of flow parameter variations on heat transfer. The corresponding change of flow and thermal fields is visualized from the streamline and the isotherm contour plots.

Introduction

Mixed convection heat transfer commonly arises in many industrial applications, where both the natural and the forced convection play equally a vital role. Analysis of mixed convective flow in a lid-driven enclosure finds applications in cooling of electronic devices, lubrication technologies, heating and drying technologies [1], food processing, float glass production [2], flow and heat transfer in solar ponds, thermal hydraulics of nuclear reactors, dynamics of lakes, crystal growing, metal coating, reservoirs and cooling ponds, materials processing and among others.

Mixed convective flow in a lid-driven enclosure is generally influenced by the variation of Reynolds, Grashof and Prandtl numbers. It is expected that the heat transfer in an enclosure is naturally increased with increasing Reynolds, Grashof or Prandtl number when its respective Reynolds, Grashof or Prandtl number is kept at constant. Another important governing parameter in mixed convection is the Richardson number $(Ri)$ which is the ratio of Grashof number $(Gr)$ and the square of Reynolds number $(Re)$. The Richardson number can be varied by either changing $Gr$ or $Re$ and thus keeping one of these two parameters constant. In general, the natural convection is negligible for $Ri < 0.1$ whereas the forced convection becomes insignificant when $Ri > 10$ and thus mixed convection is considerable within $0.1 \leq Ri \leq 10$. The region at $Ri = 1$ is considered here as the pure mixed convection regime which can be expanded from conduction dominated region to fully-developed turbulent convective flow region by varying both Grashof and Reynolds numbers simultaneously.

A numerous studies of mixed convection in lid-driven enclosures with rectangular or square shapes have been reported extensively in the literature. However, in practical applications, e.g., attic spaces in building, green houses or sun drying of crops, etc., non-rectangular geometries like trapezoidal enclosures have significant impact on internal flow and temperature profiles in mixed convection. A limited numbers of previous studies have considered mixed convective flow in trapezoidal enclosures. The most popular lid-driven trapezoidal enclosure has a short stationary base wall and inclined side walls whereas the long top wall is moving in its own plane. Hossain et al. [3], Chowdhury et al. [4], Hasan et al. [5] and Mamun et al. [6] considered this type of enclosure in their studies under different thermal boundary conditions.

Recently, nanofluid plays an important role for the enhancement of heat transfer due to higher thermal conductivity of metallic nanoparticles like copper, aluminum, silver, etc., added to the mixture of base fluid such as water, ethylene glycol or propylene glycol. A large number of articles reported mixed convection heat transfer enhancement with nanofluid in regular geometries (e.g., square, rectangular or triangular enclosures). So far, no author has paid attention to the problem of mixed convection in a trapezoidal enclosure filled with nanofluid. Cheng [7] discussed the characteristics of mixed convection in a 2D lid-driven square enclosure with the variation of Richardson and Prandtl numbers. However, a similar investigation for the case of trapezoidal enclosure with the presence of nanofluid is still lacking.

The objective of the present study is to investigate the characteristics of mixed convection heat transfer in a lid-driven trapezoidal enclosure filled with water-$Al_2O_3$ nanofluid at $Ri = 1$ by evaluating the magnitude of average Nusselt number. The values of both $Gr$ and $Re$ are increased gradually from the steady, laminar flow regimes to a situation where transition takes place. The presence of both plain fluid and nanofluid is also observed on the quantitative prediction of the transition phase.

Problem Formulation

A trapezoidal enclosure of aspect ratio of $L/H = 0.732$ is considered in the present study which is shown in figure 1 along with boundary conditions. The adiabatic side walls of the enclosure is inclined at an angle, $\gamma = 15^\circ$, with the vertical $y$-axis since Chowdhury et al. [4] recommended this configuration better than the lid-driven square enclosure for mixed convection heat transfer. For this configuration, the domain area of the trapezoid becomes unity relative to the area of a square of length $H$. The short base wall is stationary and heated at constant temperature, $T_b$, while the long top wall is moving in its own plane at a constant speed $u_a$ and is kept at constant surrounding temperature, $T_s(< T_b)$.

The working fluids (water and water-$Al_2O_3$ nanofluid) are assumed to be incompressible Newtonian fluid and their proper-
ties are constant except density variation of the nanofluid with
temperature according to the Boussinesq approximation. By ne-
glecting viscous dissipation and radiation effect with no internal
heat generation, the non-dimensional forms of continuity, mo-
mentum and energy equations are expressed as,

\begin{equation}
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{1}
\end{equation}

\begin{equation}
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + \frac{\mu_{nf}}{\rho_{nf} \nu_{nf} \text{Re}} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right), \tag{2}
\end{equation}

\begin{equation}
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + \frac{\mu_{nf}}{\rho_{nf} \nu_{nf} \text{Re}} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{(\rho \beta)_{nf}}{\rho_{nf} \nu_{nf} T}, \tag{3}
\end{equation}

\begin{equation}
U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} = \frac{\alpha_{nf}}{\sqrt{\text{Re} \text{Pr}}} \left( \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \right), \tag{4}
\end{equation}

where, the dimensionless parameters in the above equations are
defined as,

\begin{equation}
X = \frac{X}{H}, Y = \frac{Y}{H}, U = \frac{u}{u_0}, V = \frac{v}{u_0}, P = \frac{p}{\rho_{nf} \nu_{nf}^2}, \Theta = \frac{T - T_e}{T_h - T_e}, \tag{5}
\end{equation}

and the non-dimensional governing parameters can be ex-
pressed as,

\begin{equation}
Re = \frac{u_0 H}{v_f}, Gr = \frac{g \beta_f (T_h - T_e) H^3}{v_f^2}, Pr = \frac{v_f}{\alpha_f}, Ri = \frac{Gr}{Re^2}. \tag{6}
\end{equation}

The thermo-physical properties of the nanofluid such as ef-
effective density \((\rho_{nf})\), effective viscosity \((\mu_{nf})\), thermal expansion
coefficient \((\beta_{nf})\), effective thermal diffusivity \((\alpha_{nf})\), effective
thermal conductivity \((k_{nf})\) and the heat capacitance of the
nanofluid can be obtained from the following relations:

\begin{equation}
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s, \tag{7}
\end{equation}

<table>
<thead>
<tr>
<th>Property</th>
<th>Water</th>
<th>Al₃O₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_p) (J/kgK)</td>
<td>4179</td>
<td>685</td>
</tr>
<tr>
<td>(\rho) (kg/m³)</td>
<td>997.1</td>
<td>3970</td>
</tr>
<tr>
<td>(k) (W/mK)</td>
<td>0.613</td>
<td>25</td>
</tr>
<tr>
<td>(\beta \times 10^5) (1/K)</td>
<td>21</td>
<td>0.85</td>
</tr>
<tr>
<td>(\mu) (Pa.s)</td>
<td>0.001003</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1. Thermo-physical properties of water and Al₃O₅ nanoparticles [8].

<table>
<thead>
<tr>
<th>Boundary wall</th>
<th>Velocity</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top wall</td>
<td>(U = 1, V = 0)</td>
<td>(\Theta = 0)</td>
</tr>
<tr>
<td>Bottom wall</td>
<td>(U = V = 0)</td>
<td>(\Theta = 1)</td>
</tr>
<tr>
<td>Side walls</td>
<td>(U = V = 0)</td>
<td>(\partial \Theta / \partial n = 0)</td>
</tr>
</tbody>
</table>

Table 2. Non-dimensional boundary conditions of the present problem.

\begin{align*}
\mu_{nf} &= \frac{\mu_f}{(1 - \phi)^{\frac{2}{3}}}, \tag{8} \\
(\rho \beta)_{nf} &= (1 - \phi) (\rho \beta)_f + \phi (\rho \beta)_s, \tag{9} \\
\alpha_{nf} &= \frac{k_{nf}}{(\rho C_p)_f}, \tag{10} \\
k_{nf} &= \frac{k_f + 2k_f - 2 \phi (k_f - k_s)}{k_f + 2k_f + \phi (k_f - k_s)}, \tag{11} \\
(\rho C_p)_{nf} &= (1 - \phi) (\rho C_p)_f + \phi (\rho C_p)_s. \tag{12}
\end{align*}

Here, \(\phi\) is the solid-volume fraction of the nanoparticles and the
subscripts ‘\(s\)’, ‘\(f\)’ and ‘\(nf\)’ represent the properties of nanopar-
cicles, base fluid and nanofluid, respectively. The thermo-
physical properties of the base fluid (water) and the nanopar-
cicles (Al₃O₅) are presented in table 1.

The non-dimensional boundary conditions for the present
problem are listed in table 2. The characteristics of mixed convection
heat transfer can be assessed by evaluating the average Nusselt
number of the heated bottom wall and is expressed by the fol-
lowing relation,

\begin{equation}
Nu = \frac{h L}{k_f} = \frac{k_{nf}}{k_f} \frac{H}{L} \int_0^L \left( \frac{\partial \Theta}{\partial Y} \right) \mathrm{d}X. \tag{13}
\end{equation}

Simulation Procedure

The numerical procedure used to solve the governing equations
(1–4) for the present problem follows the Galerkin finite element
method which is well documented by Zienkiewicz and
Taylor [9]. A non-uniform eight-noded quadrilateral mesh
element is implemented in the solution domain especially adopt-
ing finer elements near the solid boundary to capture the rapid
changes in the dependent variables. All eight nodes are asso-
ciated with velocities as well as temperature, only the corner
nodes are associated with pressure. The relative tolerance for
the error criteria is considered to be 10⁻⁶.

Grid Sensitivity Check

Test for the accuracy of grid sensitivity is examined for the ar-
rangements of seven different non-uniform mesh with the fol-
lowing number of elements within the trapezoidal enclosure:
50 × 50, 60 × 60, 65 × 65, 70 × 70, 75 × 75, 80 × 80 and 85 × 85.

The results in terms of average Nusselt number of the heated
wall for constant \(Ri, Re\) and \(Gr\) condition are shown in table 3.

From these comparisons, it is clear that 70 × 70 non-uniform
mesh elements are sufficient to produce the optimum result.
However, 75 × 75 mesh is selected for the present simulation
in order to attain more accuracy.
Table 3. Grid sensitivity check using the variation of Nu with mesh elements for $\phi = 0.05$, $Re = 1$, $Re = 100$ and $Gr = 10^4$.

<table>
<thead>
<tr>
<th>Element numbers</th>
<th>Node numbers</th>
<th>Nu</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50 \times 50$</td>
<td>43405</td>
<td>7.000359</td>
</tr>
<tr>
<td>$60 \times 60$</td>
<td>62285</td>
<td>7.001171</td>
</tr>
<tr>
<td>$65 \times 65$</td>
<td>73000</td>
<td>7.001416</td>
</tr>
<tr>
<td>$70 \times 70$</td>
<td>84565</td>
<td>7.001734</td>
</tr>
<tr>
<td>$75 \times 75$</td>
<td>96980</td>
<td>7.001917</td>
</tr>
<tr>
<td>$80 \times 80$</td>
<td>110245</td>
<td>7.002160</td>
</tr>
<tr>
<td>$85 \times 85$</td>
<td>124360</td>
<td>7.002191</td>
</tr>
</tbody>
</table>

Table 4. Comparison of average Nusselt number from the present code with the results of Abu-Nada and Chamkha [8] for $Gr = 100$ and $\phi = 0.05$ (water-$Al_2O_3$ nanofluid).

<table>
<thead>
<tr>
<th>$Re$</th>
<th>Present Code</th>
<th>Abu-Nada and Chamkha [8]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>2.781107</td>
<td>2.866557</td>
</tr>
<tr>
<td>0.5</td>
<td>2.328368</td>
<td>2.365185</td>
</tr>
<tr>
<td>2</td>
<td>1.743216</td>
<td>1.741458</td>
</tr>
<tr>
<td>5</td>
<td>1.459776</td>
<td>1.453406</td>
</tr>
</tbody>
</table>

Figure 2. Combined effect of Reynolds and Grashof numbers on average Nusselt number of the heated wall for constant Richardson number, $Ri = 1$. Solid black line represents plain fluid whereas red line represents nanofluid with $\phi = 0.05$. Onset of laminar-chaos transition in mixed convection regime is clearly marked by two black and red points, A and B.

Table 5. List of critical parameters for transition from laminar to chaos in the present problem.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\phi = 0$</th>
<th>$\phi = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re(A)$</td>
<td>116</td>
<td>134</td>
</tr>
<tr>
<td>$Gr(A)$</td>
<td>13456</td>
<td>17956</td>
</tr>
<tr>
<td>$Re(B)$</td>
<td>146</td>
<td>174</td>
</tr>
<tr>
<td>$Gr(B)$</td>
<td>21316</td>
<td>30276</td>
</tr>
<tr>
<td>$\Delta Nu(A-B)$</td>
<td>1.159668</td>
<td>1.1525785</td>
</tr>
</tbody>
</table>

Results and Discussions

The present investigation is carried out on pure mixed convection region at $Ri = 1$ for both plain fluid (water) and nanofluid (water-$Al_2O_3$) containing 5% solid-volume fraction of nanoparticles. Reynolds number is varied from 0.1 to $10^3$ at $Ri = 1$ and thus Grashof number simultaneously changes from 0.01 to $10^6$ with a view to understand the combined effect of $Re$ and $Gr$ on the nature of mixed convection inside a lid-driven trapezoidal enclosure.

Table 5 lists the critical parameters for transition from laminar to chaos in the present problem.

The variation of streamline and isotherm contours with both Reynolds and Grashof numbers where $Ri = 1$ for plain fluid is depicted in figure 3. It is easily understood from these figures that at point A, buoyancy flow converges with core flow to form a single clockwise rotating cell of semicircular structure and thus indicates dominating flow field. A minor counter-clockwise rotating vortex is observed at the bottom right corner. Further increase of Reynolds and Grashof numbers from point A to point B reveals that the counter-clockwise cell at the bottom right corner starts to expand in size and thus with increasing strength opposes the circulating flow. Like streamlines, the isotherm contours show significant change with increasing $Re$ and $Gr$.

For $Re = 116$, the isotherm contours are clustered near the heated bottom wall, which indicate the existence of steep...
temperature gradients and thin thermal boundary layers in the vertical direction. But when $Re = 146$, the temperature gradients close to the bottom wall and at the place of secondary developing vortex of the bottom right corner are less steep as compared to the case of $Re = 116$. The mixing of the cold and the hot fluids also becomes weak which further initiates the reduction of the conduction and convection modes of heat transfer inside the trapezoidal enclosure.

Figure 4 is dedicated for the purpose of comparing the presence of nanofluid on flow and temperature profiles in relation to the plain fluid. Apparently, it looks like the profiles of flow fields are unchanged, but the careful observation reveals that the size of the cell increases with increasing $\phi$, which corresponds to a faster flow field due to increased mechanical effect of the moving lid at higher $Re$ (points A and B). Moreover, isothermal contours are comparatively more distorted for nanofluid and thus indicating better mixed convection heat transfer. Since $Nu$ is always higher for nanofluid in compared with plain fluid, the resulting variation of critical parameters (values of $Re$ and $Gr$ at points A and B) is influenced by the improved thermo-physical properties of the nanofluid.

Conclusions

The present study focuses on the phenomenon how pure mixed convection heat transfer at $Ri = 1$ inside short base lid-driven trapezoidal enclosure changes with the variation of both $Gr$ and $Re$ and thus illustrates the change from laminar to chaos. Reynolds number is varied over a suitable range so that the transition from laminar to chaos region can be observed. Quantitative predictive criteria for the beginning and the end of transition are presented. It is found that both the flow and the temperature profiles are influenced by the combined effect of Reynolds and Grashof numbers to a great extent. The profiles also vary with the presence of nanofluid. The limit of critical parameters relating to the onset of the transition significantly depends on the presence of nanofluid as compared with plain fluid.

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References


