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MHD Falkner-Skan Boundary Layer Flow past a Moving Wedge with Suction (Injection)

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Abstract

The behaviour of laminar boundary layer flow field over a solid surface moving with constant speed plays a significant role in several applications of science and technology. This paper examines the steady, laminar incompressible boundary-layer flow of a viscous electrically conducting fluid past a moving wedge with suction (injection) in the presence of an applied magnetic field. The set of partial differential equations governing Falkner-Skan wedge flow is first transformed into ordinary differential equation using similarity transformations which is later solved numerically, using an implicit finite - difference scheme known as the Keller-box method. Numerical results are presented graphically to illustrate the influence of magnetic parameter and suction/injection on local skin friction coefficient and velocity field. Further, it is demonstrated that magnetic field and suction plays a noteworthy role in controlling the laminar boundary layer separation from the moving wedge surface.

Introduction

The problem of steady, two-dimensional flow of a viscous incompressible fluid past a static wedge shaped bodies constitutes one of the classical results of the Prandtl's boundary layer theory. With a similarity transformation the governing boundary layer equation is reduced to an ordinary differential equation, which is well known as the Falkner-Skan equation [9]. The variety of applications and the importance of the Falkner-Skan equation for the understanding of the physical features of laminar boundary layer flow have motivated many researchers [2,3,4,6,7,8,10,11,12,13,14], employing various analytical and numerical methods acquiescent for different flow situations. Nevertheless, studies reported above related to the Falkner-Skan boundary layer flow over a fixed wedge placed in a moving fluid. Recently, Anuar Ishak et. al [1] have considered the Falkner-Skan problem for the flow past a moving wedge with the application of suction or injection.

In recent years a great deal of interest has been generated in the study of magneto-hydrodynamic (MHD) boundary layer research due to its extensive practical applications in technological processes; such as MHD power generator designs, design for cooling of nuclear reactors, construction of heat exchangers, installation of nuclear accelerators, blood flow measurement techniques and on the performance of many other systems using electrically conducting fluids. Further, it has been long recognized that surface mass transfer (suction or injection) energetically influences the development of a boundary layer along a surface and, in particular, can prevent or at least delay separation of the viscous region [15].

In view of the above mentioned applications, the present study investigates the Falkner-Skan boundary layer flow past a moving wedge with an applied magnetic field and suction (injection). Using the similarity transformations, the governing equations have been transformed into a third order ordinary differential equation, which is nonlinear in nature and cannot be solved analytically; consequently, Keller box method has been used for solving it.

Problem Formulation

The physical configuration of the present investigation [See figure 1] consists of a cartesian coordinate system where x is measured along the surface of the wedge and y is normal to it.

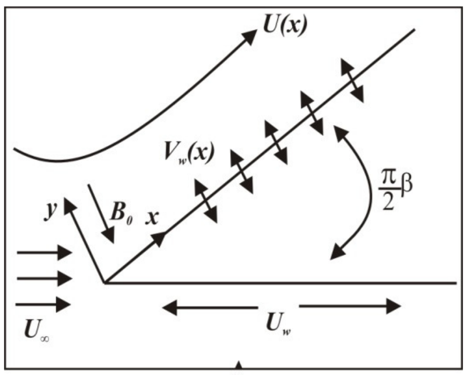

Figure 1. Physical model and coordinate system

Let us consider the steady, two-dimensional laminar incompressible flow of a viscous electrically conducting fluid due to a moving wedge with a constant velocity U_w in the direction opposite to the mainstream. The moving wedge is considered permeable with a lateral mass flux of velocity $V_w(x)$ and the outer flow velocity is $U(x)$. A uniform magnetic field of strength B_0 is applied in the direction normal to the wedge surface. It is also assumed that the magnetic Reynolds number is small and the electric field due to polarization of charges is negligible. Under the boundary layer approximations, the governing equations for the continuity and momentum transfer are, respectively given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\partial U}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^{\,2}}{\rho}(u-U) \quad (2)$$

where u and v are the velocity components in the $x-$ and $y-$ directions of the fluid flow, respectively; ν is the kinematic viscosity of the fluid, σ and ρ are, respectively, electrical conductivity and density of the fluid. The physical boundary conditions for the problem are given by

$$u(x,\,0)=U_w=-U_w(x/L)^m,$$
$$v(x,\,0)=V_w(x) \quad for\ x>0,$$
$$u\to U(x)=U_\infty(x/L)^m$$
$$as\ \ y\to\infty \quad for\ \ x>0 \quad (3)$$

where U_W and U_∞ are constants characterizing the moving wedge velocity and mainstream velocity, respectively. Further, L is a characteristic length and m is the Falkner-Skan power-law parameter and x is measured from the tip of the wedge. The subscripts w and ∞ denotes conditions at wall and infinity, respectively.

Similarity Analysis

We apply the following similarity transformations:

$$\psi=x^{(1+m)/2}\sqrt{\frac{2\nu U_\infty}{(1+m)L^m}}f(\eta);$$
$$u=\frac{\partial\psi}{\partial y}; \qquad v=-\frac{\partial\psi}{\partial x}; \quad (4)$$
$$\eta=\sqrt{\frac{1+m}{2}\frac{U_\infty}{\nu L^m}}\left(\frac{y}{x^{(1-m)/2}}\right)$$

to equations (1) and (2), we find that continuity equation.(1) is identically satisfied and momentum equation (2) is transformed to:

$$F''+fF'+\left(\frac{2m}{1+m}\right)\left(1-F^2\right)+M\left(1-F\right)=0 \quad (5)$$

where

$$v=-\sqrt{\frac{2}{1+m}\frac{\nu u_\infty}{L^m}}\,x^{(m-1)/2}\left\{\left(\frac{1+m}{2}\right)f+\eta\left(\frac{m-1}{2}\right)f'\right\};$$

$$\frac{u}{U}=f'=\frac{\partial f}{\partial\eta}=F; \quad M=\frac{2L\sigma B_0^{\,2}}{\rho u_\infty(m+1)} \quad (6)$$

Here, ψ and f are dimensional and dimensionless stream functions, respectively; $F(=f')$ is dimensionless velocity; M is the dimensionless magnetic parameter. Also, η is the similarity variable and prime $(')$ denotes derivative with respect to η.

From (3) and (6), we have

$$V_w(x)=-\sqrt{\frac{(1+m)\nu U_\infty}{2L^m}}\,x^{(m-1)/2}f(0) \quad (7)$$

And, in order that similarity solutions of equations (1) and (2) to exist, we take

$$V_w(x)=-\sqrt{\frac{(1+m)\nu U_\infty}{2L^m}}\,x^{(m-1)/2}f_w \quad (8)$$

where $f(0)=f_W$ is a constant. Notice that $V_W>0$ (i.e., $f_W<0$) is for mass injection and $V_W<0$ (i.e., $f_W>0$) is for mass suction, while $V_W=0$ (i.e., $f_W=0$) is for impermeable surface.

The transformed boundary conditions are:

$$f(0)=f_w,\ \ F(0)=-\lambda,\ \ F(\infty)=1 \quad (9)$$

The parameter λ is defined as $\lambda=U_w/U_\infty$ i.e., λ is the velocity ratio of the surface to the mainstream. Further, $\lambda>0$ and $\lambda<0$ correspond to moving wedge in opposite and same directions to the mainstream, respectively, while $\lambda=0$ corresponds to a fixed wedge.

We note that in equation (5), the parameter m is connected with the apex angle $\beta(\pi/2)$ by the relation $m=\beta/(2-\beta)$ or $\beta=2m/(m+1)$. It is worth mentioning that β is a measure of the pressure gradient dp/dx. If β is positive, the pressure gradient is negative or favourable, and negative β denotes an unfavourable positive pressure gradient, while $\beta=0$ denotes the flow past a flat plate. Further, in the present study, the numerical computations have been carried out for entire range of realistic flow i.e., for the range $0\le m\le 0.5$ (corresponds to wedge angle ranging from 0^o to 60^o), as the Falkner-Skan one-parameter family of solutions of the boundary layer equations has proved to be very useful in the interpretation of fluid flows at large Reynolds number[16].

The quantity of physical significance namely the local skin friction coefficient C_f , defined as

$$C_f (\mathrm{Re}_L)^{1/2} = \frac{\tau_w}{\rho U^2/2} = 2\sqrt{\frac{1+m}{2}}\,(F')_{\eta=0} \quad (10)$$

where $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is shear stress along the surface of the wedge, where μ is dynamic viscosity and $\mathrm{Re}_L = \frac{Ux}{\nu}$ is the local Reynolds number.

Method of Solution

The nonlinear partial differential equation (5) subject to boundary conditions (9) is solved numerically using an implicit finite-difference scheme known as Keller-box method, as described in Cebeci and Bradshaw[5]. This method is unconditionally stable and has a second order convergence. The method has the following four main steps:

(i) Reduce (5) to a system of first order equations;

(ii) Write the difference equations using central differences;

(iii) Linearize the resulting algebraic equations by Newton's method:

(iv) Write them in matrix- vector form and use the block-tridiagonal-elimination technique to solve the linear system.

For the sake of brevity, the details of the complete solution procedure are not presented here.

Since the physical domain in this problem is unbounded, whereas the computational domain has to be finite, we apply the far field boundary conditions for the pseudo-similarity variable η at a finite value denoted by $\eta_{\max}$. We ran our bulk of computations with $\eta_{\max}$ = 6, which is sufficient to achieve asymptotically the far field boundary conditions, for all values of the pertinent parameters considered. To ensure the convergence of the numerical solutions to the exact solution, step size $\Delta\eta$ has been optimized. For achieving this, the computed values of skin friction parameter (F'_w), with a step size $\Delta\eta$ are compared with those obtained using reduced step sizes viz., $(\Delta\eta/2), (\Delta\eta/4)$ and so on and the results presented here independent of $\eta_{\max}$ = 6. Further, a uniform step size $\Delta\eta$ = 0.01 is found to be satisfactory and the requisite solutions are obtained with an error tolerance of 10^{-6}.

Result and Discussion

To validate the numerical method used, the skin friction parameter (F'_w) and velocity field [$F(\eta)$] results are compared with those of Anur Ishak et.al [1] [See figure 2], for a static wedge $(\lambda = 0)$, for non- magnetic case $(M = 0)$.

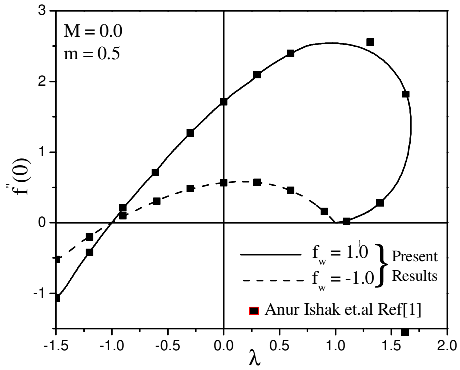

Figure 2. Comparison of skin friction parameter when $M = 0.0$ with Anur Ishak et.al [1].

We observed that the results obtained for the present study are found to in very good agreement with those of [1], correct to four decimal places of accuracy. Therefore, the developed code has been used with confidence to analyse the problem under consideration in the presence of magnetic field parameter $(M \neq 0)$.

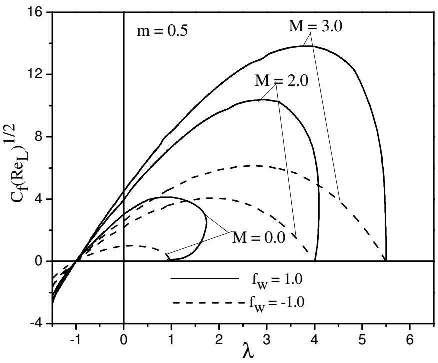

Figure 3. Effect of magnetic field (*M*) on skin friction

Figure 3 portrays the effect of magnetic parameter ($M \neq 0$) on the skin friction coefficient [$C_f(\mathrm{Re}_L)^{1/2}$] and on the velocity profiles (F) when $m = 0.5$ (corresponding to the larger included wedge angle 60° , considered in this study) for both suction ($f_w > 0$) and injection ($f_w < 0$). These results are computed when the wedge is moving in the direction opposite to that of the mainstream ($\lambda > 0$). It is observed that as magnetic field increases, skin friction increases irrespective of whether there is injection or suction. This is attributed to the fact that the transverse magnetic field has a tendency to create a drag force (known as the Lorentz force which opposes the transport phenomena), which leads to the deceleration of the flow, enhancing the surface shear stress at the wall. Skin friction values are higher during suction as compared during the process of injection. The dual solutions noticed in the values skin friction coefficient (i.e., $C_f(\mathrm{Re}_L)^{1/2}$) for M=0, just before the separation, are imperceptible [figure 3], under the influence of both suction/injection and applied magnetic field. Thus, the magnetic field parameter $(M \neq 0)$ acts as a remarkable parameter to control the surface shear stress.

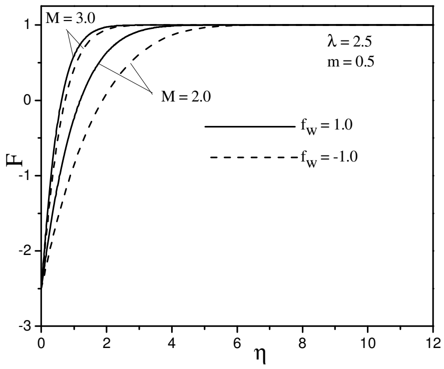

Figure 4. Velocity profile for both suction and injection when $m = 0.5(60^0)$

The velocity profiles (F), shown in figure 4 for $m = 0.5$ and $\lambda = 2.5$, reveal that the momentum boundary layer thickness decreases with the increase of magnetic field parameter $(M \neq 0)$ for both suction($f_w = 1.0$) and injection ($f_w = -1.0$). Further, an increased value of the parameter $\lambda\,(\lambda = 2.5)$ leads to the speeding up of the fluid flow inside the laminar boundary layer. Also, it is evident that the velocity boundary layer becomes thinner for suction and thicker for blowing which in turn, confirming the fact that suction controls the laminar boundary layer separation and helps to bring the stability in the flow.

It is found that when the wedge and the fluid move in the same direction $(\lambda < 0)$, the skin friction solution is unique for all pertinent physical parameters considered in this study. Indeed, the said results are not presented in detail here, for the sake of brevity.

Concluding remarks

In this paper, the effects of applied magnetic field and suction/injection on the steady flow of a viscous, incompressible electrically conducting fluid over a moving wedge have been investigated.

From the present study, following conclusions are drawn:

(i) Applied magnetic field increases the skin friction throughout process of suction as well as injection.

(ii) Increase in the value moving wedge parameter speeds up the fluid flow and, separation does not occur when the wedge and the fluid moving in the same direction.

(iii) Flow velocity increases during suction, in the presence of magnetic field, as compared to injection.

(iv) Suction plays a key role in the control of laminar boundary layer separation and brings stability in the flow.

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