18th Australasian Fluid Mechanics Conference
Launceston, Australia
3-7 December 2012

A direct measure of the frequency response of hot-wire anemometers.

N. Hutchins[1], J. P. Monty[1], R. Baidya[1], M. Hultmark[2] & A. J. Smits[2]

[1]Department of Mechanical Engineering
University of Melbourne, Parkville, VIC 3010 Australiaa

[2]Department of Mechanical and Aerospace Engineering
Princeton University, Princeton, NJ 08544, USA

Abstract

Based on the need to investigate the accuracy of hot-wire anemometers in high Reynolds number turbulent boundary layers, we pursue a direct method for testing the dynamic response of a HWA system to very high frequency velocity fluctuations (up to 50 kHz). To achieve this we will use a fully-developed turbulent pipe-flow to provide the input velocity perturbations. Having established the accuracy of this method of testing, we will also seek to test the extent to which the amplitude and sign of velocity fluctuations close to the wall in a turbulent boundary layer influences the anemometer response.

Introduction

There are two principle techniques to determine the temporal response of a HWA system, both involving the introduction of some known perturbation into the system and observing the response of the anemometer's output signal. *Direct* methods refer to cases where this perturbation is introduced as a velocity fluctuation into the flow in which the hot-wire probe is measuring. Such methods are attractive in the sense that this is the true response that we wish to know. However, the need to accurately impose high frequency velocity perturbations on to a flow poses numerous experimental challenges. The majority of users rely on *electronic* testing to estimate the system frequency response. In this case a square-wave or sin-wave perturbation is injected into the bridge and the response of the anemometer's output signal is examined. Though there are numerous theoretical studies suggesting that the square-wave test can provide a reliable indication of the true response of HWA systems to a velocity impulse (for example Freymuth [1]), there are also a few notable exceptions in the literature highlighting cases where the square-wave response seems less reliable. For example, both Hutchins *et al.*[2] and Valente & Vassilicos [3] seem to suggest that the true temporal response of the AA Lab Systems AN series anemometer is substantial slower than that indicated from electronic testing (even when the response appears to conform to optimally-damped second order behaviour, [4])

Rationale

The concept of Reynolds number similarity suggests that two fully developed turbulent pipe-flows with differing characteristic velocity, lengthscale and kinematic viscosity, yet the same Reynolds number, must be identical in all appropriately scaled statistics. For these experiments, we exploit this assumption to vary the frequency content of the turbulent spectrum in order to investigate the frequency response of hot-wire anemometers.

The most convenient Reynolds number for turbulent pipe flow is the friction Reynolds number defined as,

$$Re_\tau = \frac{U_\tau R}{\nu}, \qquad (1)$$

where U_τ is the friction velocity, ν is kinematic viscosity and R is the pipe radius. For a given facility, the radius R is fixed, and Re_τ is typically altered by changing the centerline velocity (which will alter U_τ). For these experiments, we make use of the Princeton Superpipe [5], a pressurized turbulent pipe flow facility in which we also have the option of using pressurization to alter ν (by a factor of up to 100). We exploit this unique capability to test multiple identical turbulent boundary layers, all at matched Reynolds numbers, but with different centerline velocities and thus vastly differing frequency content.

The rationale here is very simple. Close to the wall in a turbulent boundary layer, the smallest scales of turbulence have a size that can be expressed in terms of the viscous length-scale ν/U_τ and a convection velocity that approximately scales on U_τ. Therefore the small-scale turbulent energy will have a frequency observed by a stationary observer that will scale as,

$$f \propto \frac{U_\tau^2}{\nu} \qquad (2)$$

Comparing equations for the Reynolds number (equation 1) and the characteristic frequency (equation 2) we note that Reynolds number scales on U_τ/ν, while the maximum frequency in the flow scales on U_τ^2/ν. If a set of experiments are conducted for different centerline velocities (and hence U_τ), but with ν adjusted (via pressurization) such that Re_τ remains constant, we will be able to probe turbulent pipe flows with different frequency content but identical viscous scaled statistics. Consider a hot-wire probe, inserted into a fully developed turbulent pipe flow at a fixed distance z from the wall for this set of experiments. The single-normal hot-wire probe has a sensor length l and diameter d. In terms of key experimental parameters, the viscous scaled wall-normal position z^+ $(= zU_\tau/\nu)$ for the fixed probe will not change between experiments with matched Re_τ. In addition the viscous scaled sensor length l^+, will also remain constant and thus there will be no variation in the spatial resolution of the probe. Thus all key experimental conditions will remain unchanged other than the frequency content of the turbulence. Based on the assumption of Reynolds number similarity, any differences between the spectral content of the two measured signals, can therefore only be attributed to the frequency response of the anemometry system.

Validation of technique

The results discussed here will be for an in-house MUCTA anemometer. A slightly modified Dantec 55P15 boundary-type probe was used, with prong spacing 1.5 mm. Wollaston wires are soldered to the prong tips and etched to produce a 2.5μm diameter platinum sensing element of length 0.5 mm. The anemometer is operated in constant temperature mode with an overheat ratio of 1.8. Table 1 shows the set of matched Re_τ experiments conducted for the MUCTA anemometer. The Superpipe was pressurized from approximately 20 atm up to approximately 300 atm over a sequence of 5 experiments. Based on the pressurization and the temperature in the facility, we were able to calculate the value of kinematic viscosity ν. For each pressurization, the centerline velocity (U_∞) was carefully adjusted

e	pressure (kPa)	ν $(\times 10^6)$ (m^2s^{-1})	U_∞ (ms^{-1})	U_τ (ms^{-1})	Re_τ	z (mm)	z^+	l (mm)	l^+	probe	$f^+ = 0.01$ (kHz)
1	248.8	6.235	27.03	1.00	10385	0.5	80.3	0.5	80.3	55P15	1.61
2	328.8	4.731	20.52	0.76	10391	0.5	80.3	0.5	80.3	55P15	1.22
3	458.9	3.392	14.72	0.55	10395	0.5	80.4	0.5	80.4	55P15	0.88
4	758.2	2.054	8.92	0.33	10401	0.5	80.4	0.5	80.4	55P15	0.53
5	2156.1	0.723	3.13	0.12	10383	0.5	80.3	0.5	80.3	55P15	0.19

Table 1: Experimental parameters for the 5 matched cases of the MUCTA Anemometer

such that the ratio U_∞/ν was held constant across the 5 experiments. We thus knew that the ratio S (where $S = U_\infty/U_\tau$) would also remain constant, and hence the Reynolds number remained matched. To determine the initial value for S, the static pressure drop was measured in the unpressurised case to determine skin friction, yielding a value of $S = 27$ at $Re_\tau \approx 10\,000$. The highlighted columns in Table 1 demonstrate that by varying the pressure and the centerline velocity, the Reynolds number of the boundary layer (and hence the z^+ and l^+ of the sensor) remained fixed to within $\pm 0.3\%$ across the range of experiments. The final column shows the frequency corresponding to a non-dimensional frequency of $f^+ = 0.01$ which is the approximate characteristic timescale for the near-wall cycle. Clearly the dimensional frequency associated with this scale of turbulence reduces by almost an order of magnitude, from 1.61 to 0.19 kHz for experiments 1 to 5 respectively.

Figure 1(a) shows power spectral density of voltage fluctuations Φ_{EE}, in pre-multiplied format as a function of frequency for the experiments described in Table 1. Since these spectra are based on uncalibrated voltage signals we avoid some of the experimental uncertainty associated with calibration drift. The pre-multiplied spectra are normalised such that the area under the low frequency portion of the curve ($1000 < t^+ < 4000$) is forced to 1 for all experiments. The notation of vertical bars is used to denote normalised spectra (i.e. $|f\Phi_{EE}|$). Figure 1(a) clearly demonstrates that this set of matched experiments has a widely varying frequency content. The darkest line, representing experiment 1 with the highest U_∞ (and lowest pressure) clearly has turbulent fluctuations at much higher frequencies than the other experiments (shown by the lighter shaded lines). Figure 1(b) shows the same spectra normalised by the viscous time-scale (same data, with f^+ on the x axis). Reynolds number similarity suggests that all spectra normalised in this way for a constant Reynolds number should be identical, and indeed the collapse is quite convincing (albeit with the usual convergence issues at the low frequency end). However, a closer inspection of figure 1(b) reveals some systematic variation in the higher frequency end of the spectra.

The lowest speed experiment (exp. 5, the lightest line, hereafter referred to as $e5$) will have the lowest frequency content, and thus will be the most immune to temporal resolution issues. This curve can be considered as the baseline (the most 'correct' measurement), against which the other experiments are compared. Any differences between the baseline case ($e5$) and the other experiments must be due to temporal resolution issues. For each experiment number e, and at each value of f^+ we can define a difference function χ_e, as the fractional variation of the pre-multiplied spectra for a given experiment e (where $e = 1, 2, 3$ or 4) about the baseline case ($e5$),

$$\chi_e(f^+) = \frac{|f\phi_{EE}(f^+)|_e - |f\phi_{EE}(f^+)|_5}{|f\phi_{EE}(f^+)|_5} \tag{3}$$

So for a given experiment e, the function χ_e is a measure of the missing measured energy at each non-dimensional frequency f^+. A non-zero result for χ_e must be indicative that the anemometer is struggling to measure energy at the higher frequency f_e where,

$$f_e = \frac{f^+\nu_e}{U_{\tau_e}^2} \tag{4}$$

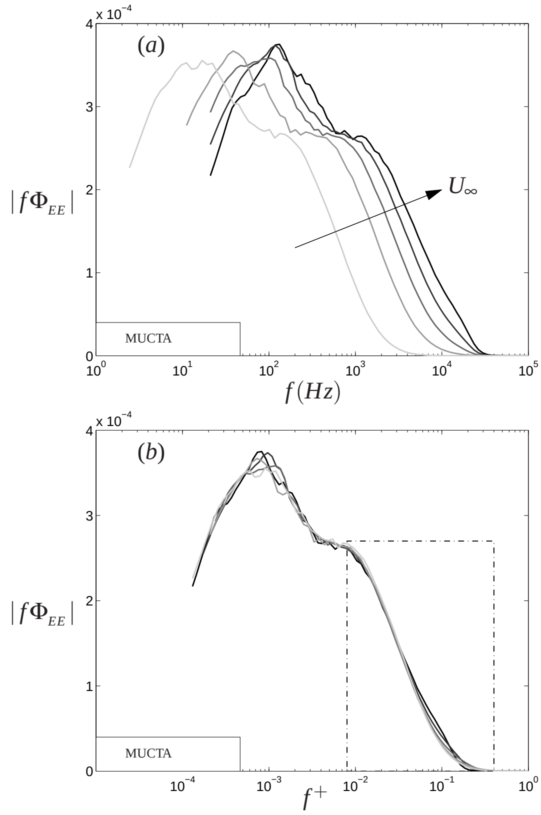

Figure 1: Pre-multiplied energy spectra from the MUCTA experiment described in table 1 shown for (a) dimensional frequency in Hz; (b) non-dimensional frequency f^+. Line shading indicates centerline velocity from light (lowest U_∞, $e = 5$) to dark (highest U_∞, $e = 1$).

The plot of χ_e versus f_e gives a *direct* measure of the attenuation due to the anemometer as a function of frequency. This result is shown in figure 2(a) for experiments 1 to 4. It is clear from figure 2(a) that the attenuation (and amplification) due to the temporal response of the anemometer system is relatively consistent across all experiments. Figure 2(a) shows that for frequencies in excess of $f_e \approx 5000$ Hz, and peaking at $f_e \approx 18000$ Hz, there is an increasing over-representation of energy. At this peak frequency, the anemometer records energy that is approximately 60 - 70 % greater than the baseline. These results

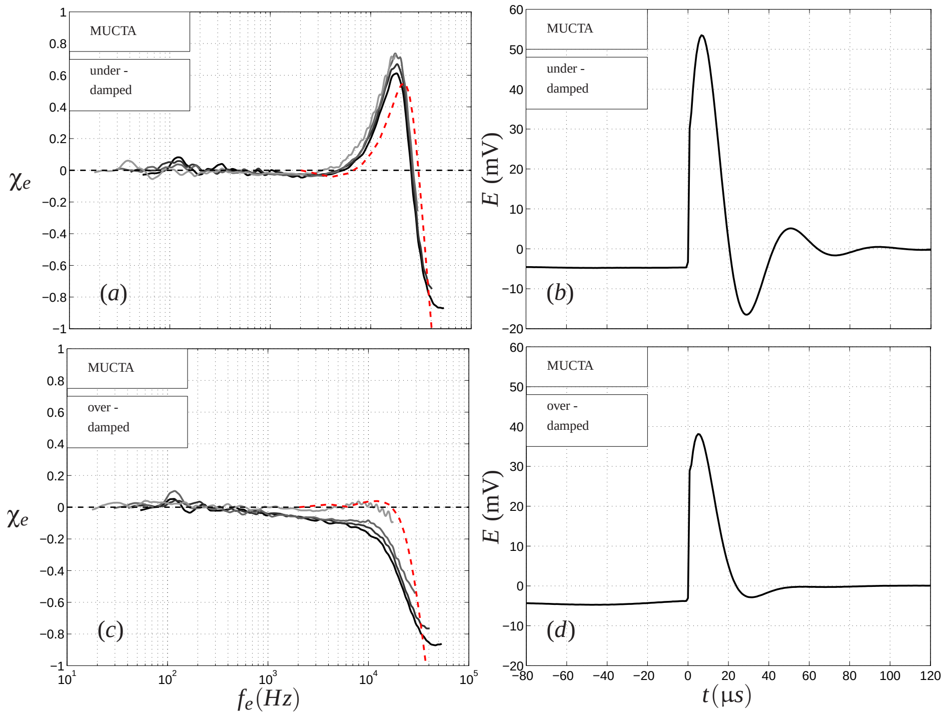

Figure 2: The difference function (a, c) and the corresponding square wave output (b, d) for (top plots) the under-damped MUCTA and (bottom plots) the over-damped MUCTA. The red dashed lines in plots (a) and (c) show the normalised Fourier transform of the square wave output. Line shading indicates centerline velocity from light (lowest U_∞) to dark (highest U_∞).

would seem to indicate that quite serious systematic errors may be present in hot-wire measured data due to temporal resolution issues. Figure 2(b) shows the measured response of the MUCTA anemometer to an *electronic* square wave test. The red dashed curve on figures 2(a) shows the normalised Fourier transform of the response of the bridge to a step input in voltage (the FFT of the square wave response shown in plot b). This 'Bode' plot closely approximates the experimentally observed behaviour given by χ, suggesting that in this instance the *electronic* test gives a good indication of the *direct* response of the anemometry system. To reassure that the effect demonstrated by figure 2(a) is really a product of the anemometer frequency response and not merely an artifact of experimental technique, a further test of the MUCTA anemometer was conducted with increased damping. Figure 2(d) shows the measured square-wave response from *electronic* testing of the over-damped system. Figure 2(c) shows the experimentally determined transfer function χ obtained from the comparison of viscous scaled energy spectra at matched Reynolds numbers. The different damping setting has clearly altered the measured response. The red dashed curve on figure 2(c) shows the corresponding Bode plot (the Fourier transformed square wave response from plot d). Again, there is reasonable agreement between the response determined from the *direct* and *electronic* methods.

The effect of turbulence on the frequency response

Having established the veracity of our novel direct technique, and having verified that the Bode plot (formed from the square-wave response) gives a reasonable measure of the true response of the system, we now turn our attention to analysing the influence of turbulent fluctuations on this response. For this set of experiments, the same standard probe (Dantec 55P15) is placed at $z^+ = 15$ in a turbulent boundary layer at $Re_\tau \approx 8000$. This is close to the peak in the variance of streamwise velocity fluctu-

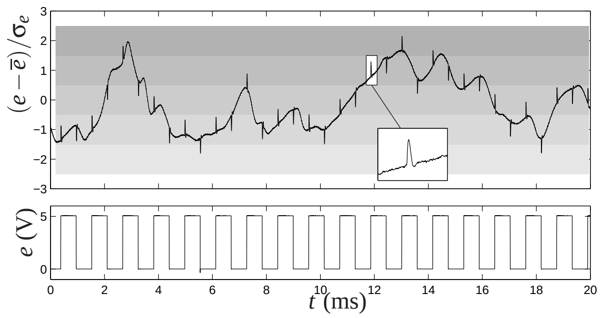

Figure 3: (top) fluctuating voltage signal (with superimposed square wave response) from MUCTA anemometer with probe at $z^+ = 15$ in a turbulent boundary layer at $Re_\tau \approx 8000$. (bottom) corresponding square wave input to anemometer.

ations. Measurements were made in the high Reynolds number boundary layer wind-tunnel (HRNBLWT) at the University of Melbourne, at a location 18 m downstream of the tripped inlet to the working section and at a freestream velocity of 15 ms^{-1}. The probe is operated in constant temperature mode using the MUCTA anemometer. The square wave is input to the bridge while the hot-wire sensor experiences the turbulent flow. The square wave input and the fluctuating voltage signal from the turbulent flow are sampled at 1 MHz. Figure 3 shows the fluctuating voltage output from the MUCTA anemometer and corresponding square wave input. Where ever there is a rising edge on the square wave input (bottom plot), the corresponding square wave response can be seen superimposed onto the fluctuating signal (as highlighted by the inset). This average response can be extracted from the signal shown in figure 3(top) by conditioning on the rising edge of the square wave shown

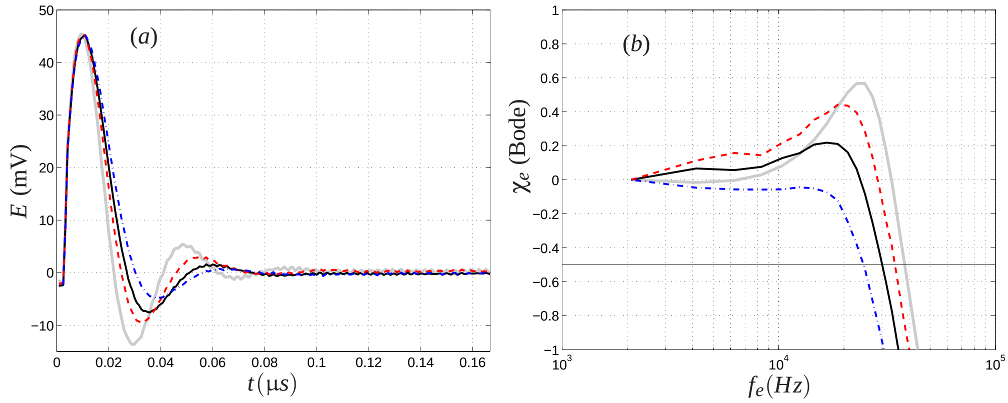

Figure 4: (a) square wave response from electronic testing and (*b*) corresponding Bode plot for (—) unconditional in freestream; (—) unconditional at $z^+ = 15$; (- -) positive velocity fluctuation at $z^+ = 15$; (· - ·) negative velocity fluctuation at $z^+ = 15$.

in figure 3 (bottom). The resulting 'unconditional' square wave response (the response for all states of the turbulence signal) is shown by the solid black line in figure 4(*a*) and the corresponding Bode plot is shown in plot (*b*). This response is at $z^+ = 15$, where the local velocity is much lower than the freestream velocity (about one third). The 'unconditional' square wave response in the freestream (at $U_\infty = 15$ ms^{-1}) is shown in figure 4 by the faint gray lines. Clearly there is a big discrepancy between the response obtained in the freestream and the one that is actually attained deep in the turbulent boundary layer (this is to be expected; it is well known that the frequency response is a function of the mean flow over the probe). Proper experimental technique for turbulent boundary layer measurements should involve checking (and preferably recording) the square wave response at a velocity that matches the mean at the lowest measurement stations in the boundary layer.

The question we wished to answer was whether the response of the anemometer was dependant on the local state or phase of the turbulent cycle. To answer this, the data is used to determine conditionally averaged square-wave responses for different amplitude fluctuations about the mean. This is achieved by conditionally averaging the square wave response on the rising edge of the input *and* situations where the instantaneous turbulence signal is within certain limits. The red dashed curve in figure 4(*a*) shows the square wave response where $-2.5\sigma_e < e < -1.5\sigma_e$ (corresponding to a high magnitude positive velocity fluctuation). The blue dot-dashed curve in figure 4(*a*) shows the square wave response where $1.5\sigma_e < e < 2.5\sigma_e$ (corresponding to a high magnitude negative velocity fluctuation). Clearly the sign (and magnitude) of the turbulent signal has a large bearing on the overall response of the signal, with the anemometer (not unexpectedly) exhibiting a slower response during negative velocity fluctuations, and a faster response during high speed fluctuations. Thus the response in a turbulent signal is itself time varying. The red and blue curves in figure 4(*b*) show the Bode plots associated with these conditions. Clearly the over-amplification and attenuation characteristics (and hence the -3dB cut-off, shown by the black horizontal solid line) are quite different depending on the sign of the turbulent fluctuations. This suggests that the output of an anemometry system, operating in turbulent flows beyond its flat response limit, cannot properly be corrected with a simple transfer function based on the Bode plot. If there is turbulent energy at high enough frequency, a situation could potentially arise where negative velocity fluctuations are temporally filtered to a greater extent than positive velocity fluctuations, which could effect higher order statistics (such as skewness).

Conclusions

A new *direct* method for testing the frequency response of hot-wire anemometers to velocity fluctuations has been proposed. For the in-house anemometer tested, the direct measured response is very similar to that predicted from electronic testing. For accurate turbulence measurements we note that the -3dB test as suggested by Freymuth [1] is not a suitable measure of anemometer performance (since the -3dB cut-off frequency is for 50% attenuation of energy). The Bode plot calculated from the square response gives a much better indication of the performance of the system, showing that all anemometers will experience quite serious errors for frequencies in excess of approximately $f \approx 8$ kHz. The electronic square wave response has been used to analyse the response of the anemometer system in an actual turbulent flow. The first observation is rather obvious; for accurate measurements close to the wall, the anemometer response must be tuned at a velocity that approximates the local mean in this location. Otherwise an optimally-damped second order response in the freestream, will lead to over-damped behaviour close to the wall. The second observation is that the true response in a turbulent signal is itself time varying (the response is faster during positive velocity fluctuations and slower during negative).

Acknowledgements

The authors gratefully acknowledge support from the Australian Research Council.

References

[1] Freymuth, P., Frequency response and electronic testing for constant-temperature hot-wire anemometers., *J. Phys. E: Sci. Instrum.*, **10(7)**, 1977, 705–710.

[2] Hutchins, N., Nickels, T. B., Marusic, I. and Chong, M. S., Hot-wire spatial resolution issues in wall-bounded turbulence., *J. Fluid Mech.*, **635**, 2009, 103–136.

[3] Valente, P. C. and Vassilicos, J. C., Comment on "dissipation and decay of fractal-generated turbulence", *Phys. Fluids*, **23**, 2011, 119101.

[4] Watmuff, J. H., An investigation of the constant-temperature hot-wire anemometer, *Exp. Therm. and Fluid Sci.*, **11**, 1995, 117 – 134.

[5] Zagarola, M. V. and Smits, A. J., Mean-flow scaling of turbulent pipe flow, *J. Fluid Mech.*, **373**, 1998, 33–79.