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The Dynamics of a Rising Pivoted Cylinder

B. Stappenbelt, J. Neville, B. Murdoch and A. Johnstone
School of Mechanical, Materials and Mechatronic Engineering,
University of Wollongong, Wollongong, NSW 2522, Australia

Abstract

The existence of a critical mass ratio for cylinders undergoing vortex-induced vibration (VIV) in a translational system has been well established. Below this critical point, the reduced velocity at VIV lock-out tends to infinity. It has been surmised that a corresponding mass moment of inertia ratio must exist for a pivoted cylinder arrangement. To the authors' knowledge there has been no investigation published substantiating this premise. The aim of the present investigation then was to examine the critical point for cylinders in a rotational system. The approach adopted involved measuring the VIV amplitude response of a positively buoyant, and hence rising, pivoted cylinder at very high reduced velocity. High reduced velocity was attained by establishing a very low system natural frequency through the omission of external restoring forces. The key finding of this study is the presence of a critical point with a value similar to that of the critical mass ratio in translational systems. This critical point does not however appear to be governed by the mass moment of inertia ratio but rather by the force moment ratio.

Introduction

Fluid flow past a circular cylindrical object generates vorticity due to the shear present in the boundary layer. This vorticity in the flow field coalesces into regions of concentrated vorticity, known as vortices, on either side of the cylinder. Flow above a threshold Reynolds number allows perturbations in the flow upstream to cause one of the vortices to grow larger. This vortex, with higher flow velocities and accompanying lower pressures, draws the smaller vortex from the opposing side across the wake centreline. The opposite vorticity from this smaller vortex severs the vorticity supply of the larger vortex, allowing it to convect downstream [15]. This process is repeated in the reverse sense, leading to alternating vortex shedding from the cylinder.

When the cylinder is elastically restrained and natural frequencies are introduced, a fluid-elastic instability known as vortex-induced vibration (VIV) results. The time-varying non-uniform pressure distribution around the cylinder resulting from the vortex shedding causes structural vibrations both inline and transverse to the flow. Near the natural frequency of the structure, the vortex-shedding frequency synchronises with the natural frequency and the vibration frequency. One of the primary mechanisms responsible for this synchronisation is the change in hydrodynamic mass, as demonstrated in the experiments of Vikestad [16]. The range of reduced velocity over which this synchronisation occurs is known as the lock-in range. Mostly, the ensuing vibrations are undesirable, resulting in increased fatigue loading and component design complexity to accommodate these motions. The transverse vibrations also result in higher dynamic relative to static drag coefficients.

With decreasing mass ratio, an increase in the amplitude response is generally evident [13]. Also, the smaller the mass ratio, the larger the relative influence of the hydrodynamic mass on the vibration response of the structure.

Various definitions for the mass ratio are widely employed. In this work, the mass ratio is defined as the ratio of the oscillating structural mass, m, to the displaced fluid mass, m_d, as

$$m^* = \frac{m}{m_d}. \tag{1}$$

The structural mass, m, includes any enclosed fluid, but excludes the hydrodynamic mass. Note that the mass ratio is equivalent to the magnitude of the ratio of the weight, W, and buoyancy, B, forces since

$$m^* = \frac{W}{B} = \frac{mg}{m_d g}. \tag{2}$$

The mass ratio parameter influences both the amplitude and frequency response of the cylinder. With higher mass ratios (e.g. a cylinder vibrating in air, with a mass ratio O(100)), changes in added mass are relatively insignificant due to the low density of the fluid. The natural frequency then remains relatively unchanged throughout the lock-in range. When the fluid medium under consideration is much denser (e.g. a cylinder vibrating in water), distinct changes in the natural frequency are observed. The increasing natural frequency observed with increasing reduced velocity is directly attributable to the decreasing added mass throughout the lock-in range [14, 16]. An overview of the characteristics of low mass damping VIV is given in the review paper by Gabbai and Benaroya [2].

Since the hydrodynamic mass variation is largely responsible for synchronisation of the shedding and vibration frequencies, typically much wider lock-in regions are experienced at low mass ratio[13, 16]. The limit of this trend is found at the critical mass ratio of around 0.54 [5], below which there exists no de-coherence region and VIV occurs at all velocities above the initial lock-in. In fact, at mass ratios below the critical point the lower response branch can never be reached.

The initial discovery of the critical mass ratio resulted from the examination of elastically constrained cylinder experimental data [3]. Subsequent transverse amplitude tests on translational cylindrical systems where restoring forces have been removed (i.e. with the reduced velocity, $U_r \rightarrow \infty$) have been conducted with results as illustrated in figure 1 [4]. At $U_r \rightarrow \infty$ resonance is seen below the critical mass ratio and forced vibrations above. The development of understanding of the critical mass ratio for a single degree of freedom cylindrical system is chronicled well in the review by Williamson and Govardhan [18] and in the publications by Govardhan and Williamson [4, 5] in which a critical mass ratio 0.542 ± 0.01 is claimed.

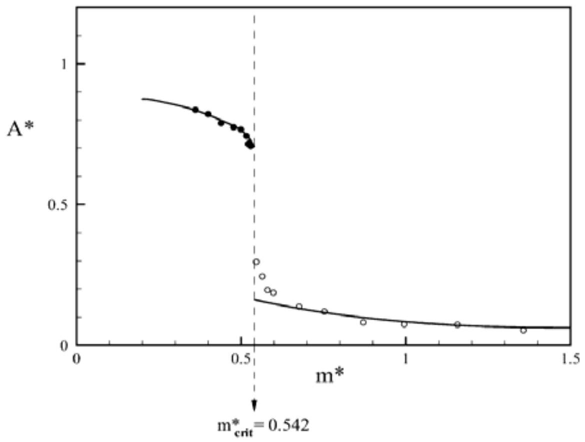

Figure 1. Experimentally determined value of the critical mass ratio [4].

The low Reynolds number study by Ryan, Thompson and Hourigan [11] revealed a Reynolds number dependency of the critical mass ratio. This was supported by the study by Morse and Williamson [9] which showed an increase in critical mass ratio from 0.36 to 0.54 over the Reynolds number range of 4000 to 30000.

The investigation by Horowitz and Williamson [6] where the VIV motions of a rising and falling cylinder were examined yielded a critical mass ratio of 0.54. This arrangement, despite allowing the cylinder multiple degrees of freedom, produced results in close agreement with previous experiments. A system free to vibrate inline and transverse to the flow has the potential at low mass ratio to display a super-upper response branch [10, 12] rather than the upper response branch observed in transverse only experiments. By extrapolation of their data, the two degree of freedom system experiments by Jauvtis and Williamson [7] revealed a critical mass ratio value of 0.522.

Very few prior investigations have examined the critical point for a pivoted cylinder. Those that have are generally of limited use in elucidating the critical point for a rotational system. The studies by Leong and Wei [8] and Voorhees, Dong, Atsavapranee, Benaroya and Wei [17] for example attempt to apply the concept of the mass ratio to a rotational system. Insufficient information is provided in these papers to ascertain the mass moment of inertia of the cases covered. The former study presents only partial response curves for limited mass ratios and the latter provides experimental results above the critical point.

In the pivoted cylinder study by Flemming and Williamson [1] the mass moment of inertia ratio is introduced as the governing parameter. The mass moment of inertia ratio, I^*, defined as the ratio of the mass moment of inertia of the structural components, I, to that of the displaced fluid, I_d, is

$$I^* = \frac{I}{I_d} = \frac{\oint r^2 dm}{\oint r_d^2 dm_d}, \quad (3)$$

where r and r_d respectively refer to the distance of the structural and displaced fluid mass elements (i.e. dm and dm_d) from the point of rotation.

Adopting the mass moment of inertia ratio is the logical choice as it is the rotational analogy of the mass ratio in a translational system. In the investigation by Flemming and Williamson [1], three mass moment of inertia ratio cases are presented, ranging from I^*=7.69 to I^*=1.03. These experiments are performed well above the critical point value (I^*_{cr}≈0.5) surmised in the same paper and are therefore of limited value in illuminating the critical point for a pivoted cylinder.

The aim of the present study was therefore to examine the dynamics of a pivoted cylinder to ascertain the existence and characteristics of a critical VIV point for rotational systems.

Methodology

The present investigation consists an experiment utilising a pivoted cylinder rotating as a result of the moment due to the buoyancy and weight forces. Inline and transverse motions were not restrained. Transverse vibrations were measured by the placement of accelerometers at the end of the cylinder. No rotation of the cylinder about its longitudinal axis was permitted.

For a translating system, the mass ratio is equal to the magnitude of the force ratio (i.e. W/B). Since the rotational equivalents of these (i.e. the mass moment of inertia and force moment ratios) are not equivalent, the present study considered both parameters.

The force moment ratio, M^*, is defined as the ratio of the moment about the point of rotation due to the weight force acting on the structural mass to that acting on the displaced fluid mass as

$$M^* = \frac{M}{M_d}. \quad (4)$$

Note that equation 4 is equivalent to the magnitude of the ratio of moments due to the structural weight (i.e. $W.r_W.sin\theta$ where r_W is the distance of the centre of gravity (cog) to the centre of rotation) and buoyancy (i.e. $B.r_B.sin\theta$ where r_B is the distance of the centre of buoyancy (cob) to the centre of rotation) forces in the plane of transverse oscillations,

$$\text{i.e.} \quad M^* = \left|\frac{Wr_W sin\theta sin\alpha}{Br_B sin\theta sin\alpha}\right| = \left|\frac{Wr_W}{Br_B}\right|. \quad (5)$$

Figures 2 and 3 illustrate the experimental configuration and act as a parameter definition sketch. The angular displacement relative to the initial position of the cylinder in the plane of transverse oscillation is designated θ and α is the angular position relative to the horizontal.

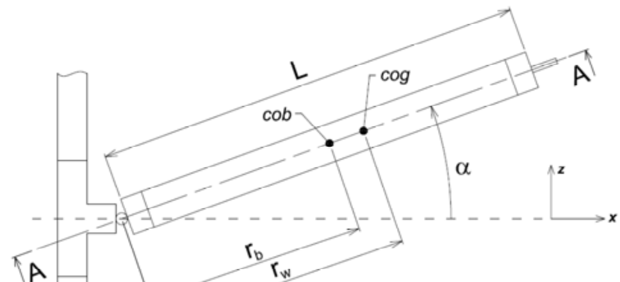

Figure 2. Experimental apparatus elevation. Buoyancy driven motion of the cylinder is rotation in the positive α direction.

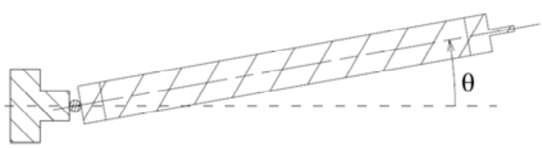

Figure 3. Experimental apparatus in the plane of transverse oscillations (section A-A from figure 2).

Table 1 details the parameter values for the experiment. The force moment and mass moment of inertia ratios were experimentally controlled by the addition of lump masses at the end of the cylinder.

Parameter	Value	Units
Fluid	Water	
Mass moment of inertia ratio range, I^*	0.360-1.110	
Moment ratio range, M^*	0.350-0.846	
Cylinder length, L	1507	mm
Cylinder diameter, D	43	mm
Angular range of motion about the horizontal, α	-30 to +30	deg
Structural damping, ζ	0.006	
Cylinder end mean Reynolds number range	2.0×10^4 to 2.7×10^4	

Table 1. Experimental parameter values.

The approach adopted in the present study to determine the location of the critical point was to examine the nature of the vortex-induced vibrations at very high reduced velocity (i.e. as $U_r \rightarrow \infty$). The amplitude of the response indicated either resonant (i.e. below the critical point) or forced vibration (i.e. above the critical point).

The reduced velocity is given by

$$U_r = \frac{U}{f_n D} \, . \tag{6}$$

This is equivalent to equation 7 if θ_D is equal to the arc subtended by a chord and radius equal to the diameter and length of the cylinder respectively.

$$U_r = \frac{\dot{\alpha}}{f_n \theta_D} \tag{7}$$

The natural frequency, f_n, is proportional to the square root of the angular restoring force coefficient, k_θ. Considering the sum of the moments about the cylinder pivot point it may be shown that

$$k_\theta = (W r_W - B r_B) sin\alpha = B r_B (M^* - 1) sin\alpha \, . \tag{8}$$

To test for resonance as $U_r \rightarrow \infty$, the restoring force coefficient, k_θ, must tend to zero. From equation 8 it may be seen that this condition is approached as $M^* \rightarrow 1$ or $\alpha \rightarrow 0$. Maximum cylinder rising velocity is also attained around the latter point, creating an angular region about the horizontal plane with very high reduced velocity.

Results and Discussion

The amplitude response of the cylinders over the mass moment of inertia and force moment ratios covered in the experiment are presented in Figures 5 and 6. Each data point in these plots represents the averaged maximum cylinder-end amplitude response of between five and ten trials recorded. Two time series examples are also provided in figure 4.

It is clear from these figures that a transition from low amplitude forced vibration response to high amplitude resonant response does occur across the parameter range covered. This indicates that a critical point does exist within this range. Figure 4a is an example of the nature of the resonant vibrations below the critical point and figure 4b an example of the forced vibration response above the critical point.

Since the force moment and mass moment of inertia ratios both vary throughout the experiment, several trials were conducted where one of these parameters was held constant. This was achieved through the addition of lump masses at both the end of the cylinder and mounted along the cylinder in the wake region. This data is represented by the symbol □ in figures 5 and 6. When the force moment ratio is constant, there is no change in the amplitude response of the cylinder with variation of the mass moment of inertia ratio. When the mass moment of inertia ratio is constant however, the data aligns consistently with the amplitude response data collected with both parameters varying (designated by the symbol • in figures 5 and 6). The data collected with one parameter held constant clearly indicates that it is the force moment ratio that is the governing parameter in determining the critical point, not the mass moment of inertia ratio.

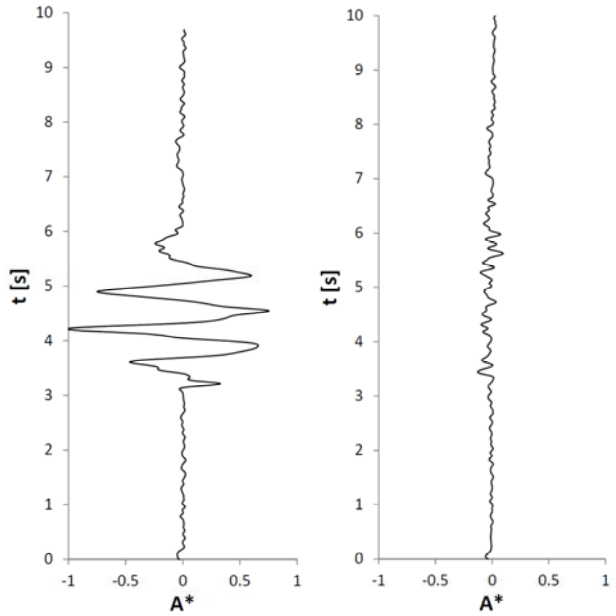

Figure 4. Cylinder displacement time series at a) M*=0.350 and b) M*=0.819.

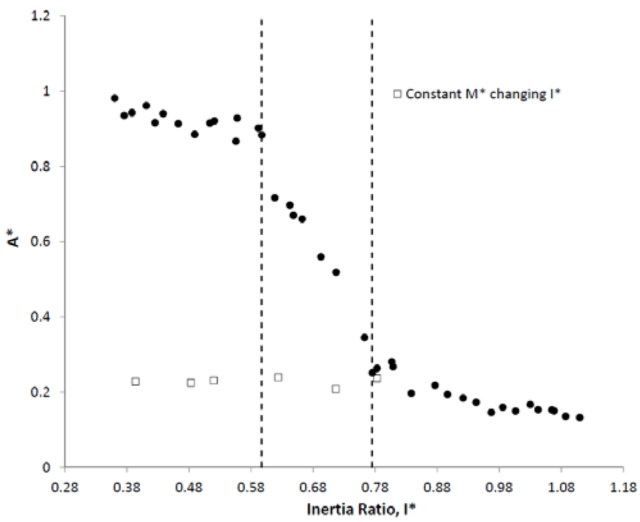

Figure 5. Response amplitude as a function of mass moment of inertia ratio (□ constant M*=0.677; • varying M^* and I^*).

The transition from resonant to forced vibration appears to occur between approximately M^*=0.51 and 0.63. As previously stated, if the rotational system behaves in a manner analogous to the translational system, this would suggest a critical point somewhere in this region.

Relative to previous critical mass ratio experiments (e.g. figure 1), the transition point is not as clearly defined in the present investigation. There are several factors contributing to this, including a changing restoring force in the plane of oscillation (i.e. section A-A in figure 3) with angular position of the cylinder relative to the horizontal. The cylinder rotational velocity also changes with varying force moment ratios and angular position.

Despite an angular range near the horizontal where the restoring force is sufficiently small that the natural frequency tends to zero (i.e. $U_r \rightarrow \infty$), maximum vibration amplitude may not be reached due to the short time span it takes the cylinder to traverse this range. As the critical point is approached, the system may take longer to attain maximum amplitude and, as is the general case for VIV near the lock-out point, may switch intermittently between resonant and forced vibration. This notion is consistent with the observation that the greatest variability in maximum amplitude between trials was found in this transition region.

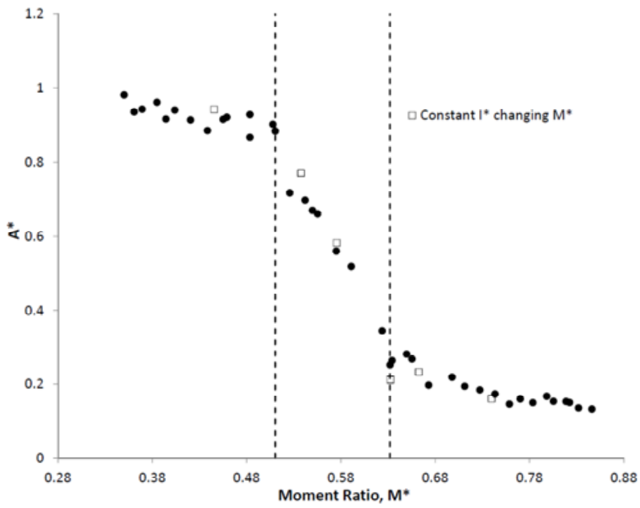

Figure 6. Response amplitude as a function of force moment ratio (□ constant I*=0.436; • varying M^* and I^*).

Conclusions

A critical point for vortex-induced vibration in a rotational system does appear to exist. The evidence collected through the experiment presented shows a clear transition from forced to resonant vibration at very high reduced velocities.

The governing parameter for the critical point for a pivoted cylinder appears to be the force moment ratio, not the mass moment of inertia ratio. When considering the corresponding translational system, it is perhaps more correct in characterising the system dynamics to use the force ratio, rather than the mass ratio in light of this result.

The transition from resonant to forced vibration appears to occur between approximately M^*=0.51 and 0.63, suggesting a critical point somewhere in this region. Remarkably, the previously reported critical mass ratio values (i.e. approximately 0.54) fall within this critical force moment ratio range.

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