

Viscous Drag Force and Heat Transfer from an Oscillating Micro-Wire

P.L. Woodfield¹ and A. D. Seagar¹

¹School of Engineering
 Griffith University, Queensland 4222, Australia

Abstract

An experimental apparatus is being developed in order to investigate heat and momentum transfer from oscillating surfaces submerged in various fluids. A fine wire is situated in a magnetic field and immersed in a fluid of interest. The wire is forced to oscillate using an alternating electrical current flowing perpendicular to the magnetic field direction. The motion of the wire is deduced by measuring the magnitude and phase shift of the induced voltage across the wire with a lock-in amplifier. By changing the frequency of the driving current in the vicinity of the natural frequency of the wire, a resonance curve is extracted. The shape of the resonance curve is strongly affected by the drag force on the wire. Simultaneous heat transfer measurement is achieved by superimposing a DC component and a high frequency component to the current through the wire.

Introduction

The theory of the vibrating wire in a magnetic field originally linked by Tough et. al [6] to Stokes' theory for oscillating cylinders [5] has become the basis for one of the state-of-the-art techniques in fluid viscosity measurement (e.g. [1,3,8]). Vibrating wire viscometers work on the principle that the motion of the micro-wire is damped by the viscosity of the surrounding fluid. The transient hot wire technique (e.g. [7]) is now widely considered as the state-of-the-art in fluid thermal conductivity measurement. It works on the principle that the transient temperature history of an electrically heated wire is strongly influenced by heat conduction into the surrounding fluid. The two techniques use very different principles of operation but similar instrumentation. Moreover, both rely on electrically manipulating micro-wires to take the measurement of the desired transport property. One obvious difference in probe design is that the thermal conductivity measurement is done in the absence of a magnetic field. The present work represents the beginning of a fundamental study on simultaneous heat and momentum transfer from an oscillating micro-wire with possible applications such as developing a device that can measure temperature, viscosity and thermal conductivity of a fluid simultaneously. Moreover, because electrical currents through the wire may be manipulated easily, the possibility of studying the fundamental fluid behaviour well beyond the Stokes regime is easily accessible.

Theory

Figure 1 shows the basic electromechanical circuit for analysis. The wire is fixed at both ends and y is the instantaneous deflection due to vibration at a distance x from one end of the vibrating wire. The dashed line is the undeflected position of the wire and the magnitude of y in the figure is exaggerated for clarity. A magnetic field of strength B is perpendicular to the plane of vibration. R_{sens} is the electrical resistance of the wire (shown separate from the wire in figure 1 for clarity) and R_{circ} is the electrical resistance in the circuit excluding the resistance of the sensor. R_{tot} is the total electrical resistance of the circuit. The wire is forced to vibrate as a result of the oscillating voltage source V_s .

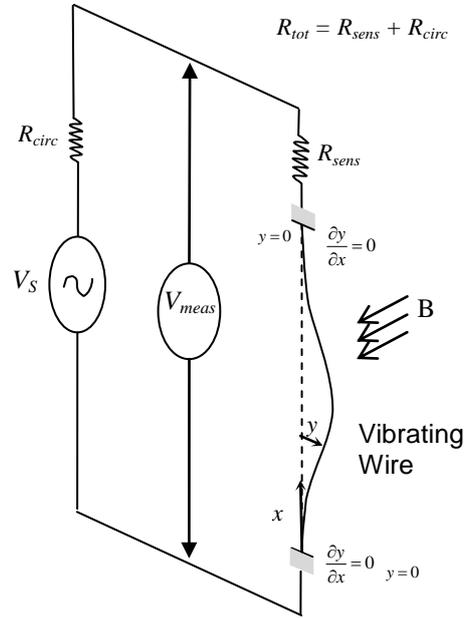


Figure 1. Circuit diagram and boundary conditions for the vibrating wire

The voltage signal to be analysed is V_{meas} . In general, R_{sens} is a function of temperature and V_s may be a combination of a D.C. voltage offset and oscillating voltages at different frequencies. Analysing the circuit shown in figure 1 and assuming that the deflection of the wire is small and perpendicular to the direction of the magnetic field, applying Faraday's Law gives the measured voltage V_{meas} as:

$$V_{meas} = \frac{R_{sens}}{R_{tot}} V_s + \left(1 - \frac{R_{sens}}{R_{tot}}\right) \int_0^L B \frac{\partial y}{\partial t} dx \quad (1)$$

where L is the length of the wire. Thus if we know details of the magnetic field B and the mode shape of the oscillating wire, the measured voltage V_{meas} can be used to determine the velocity of any point on the wire.

Analysis for the Stokes Regime

For very small oscillations, Stokes [5] showed that the drag force per unit length on an oscillating cylinder is given by

$$F_{DStokes} = -\pi \rho_{fluid} a^2 \omega k'(m_{st}) \frac{\partial y}{\partial t} - c' \frac{\partial^2 y}{\partial t^2} \quad (2)$$

ρ_{fluid} is the density of the fluid, a is the radius of the wire, ω the angular frequency, $\partial y / \partial t$ the velocity, and the function k' is given by [3] the imaginary part of

$$k + ik' = 1 - \frac{\sqrt{2}(1-i) H_1^{(1)}(z)}{m_{st} H_0^{(1)}(z)} \quad (3)$$

$H_0^{(1)}(z)$ and $H_1^{(1)}(z)$ are Hankel functions where z is a complex number given by:

$$z = \sqrt{2}(1+i)m_{st} \quad (4)$$

The acceleration term in equation (2) may be understood as the effect of the inertia of fluid that oscillates with the wire where the coefficient c' is given by

$$c' = \pi \rho_{fluid} a^2 k(m_{st}) \quad (5)$$

And m_{st} is given by

$$m_{st} = \frac{a}{s} \sqrt{\omega \rho_{fluid} / \mu} \quad (6)$$

In equation (6), μ is the viscosity of the fluid. In this special case where equation (2) applies, the drag force is proportional to the velocity and the equation of motion for the wire can be written in the form:

$$(\pi a^2 \rho_w + c') \frac{\partial^2 y}{\partial t^2} = -\frac{\pi a^4}{4} E \frac{\partial^4 y}{\partial x^4} + T \frac{\partial^2 y}{\partial x^2} - D \frac{\partial y}{\partial t} + BI \quad (7)$$

In equation (7), the term involving Young's Modulus E represents the force per unit length from bending a slender cylindrical beam, the term involving the second space derivative is the restoring force per unit length due to the tension T in the wire and the last term on the right hand side is the Lorentz force assuming that the wire (with electrical current of magnitude I), the magnetic field (magnitude B) and the displacement are mutually perpendicular. For the Stokes regime, D is the coefficient of $\partial y / \partial t$ in equation (2). Equation (7) may be solved analytically (e.g. [3]) and combined with the circuit model to provide equations suitable for extracting the unknown viscosity (e.g. [3,8]) via a least-squares fitting procedure with experimentally determined resonance curves.

Experiment – Fluid Viscosity Measurement

Figure 2 shows the basic configuration for measurement of the resonance curves from which the fluid viscosity (or drag force on the wire) can be determined. The oscillating electrical current to drive the experiment is produced by a function generator with a maximum output of ± 10 V. Typically the magnitude of the driving voltage for viscosity measurement is of the order of millivolts or smaller. The function generator is best modelled as an ideal voltage source with a 50Ω resistor in series [7]. The function generator has a separate output channel which can act as the reference signal input for the lock-in amplifier.

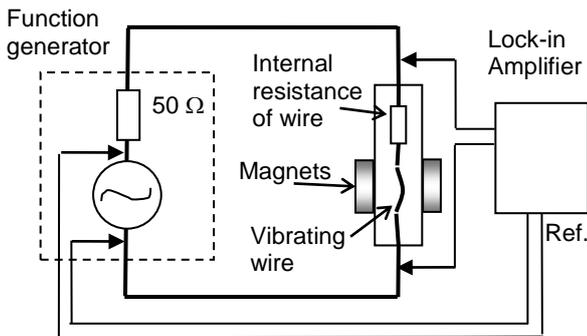


Figure 2. Basic circuit for measuring effect of viscous drag on vibrating wire

The probe is placed in the chamber of a dry-block temperature control unit along with a traceable platinum resistance

thermometer calibrated according to the ITS-90 international temperature scale.

Typical experimental results of the AC voltage output measured by the lock-in amplifier are shown in figure 3. In this case measurements were done in air using a $50 \mu\text{m}$ diameter tungsten wire. The part of the signal in phase with the driving voltage from the function generator shows a smooth symmetrical peak with a resonant frequency of 294.2 Hz. The quadrature component indicates that a frequencies below resonance the phase shift of the measured signal with respect to the driving voltage is positive while at frequencies higher than resonance it is negative. The model fit is excellent for both the in-phase and quadrature components.

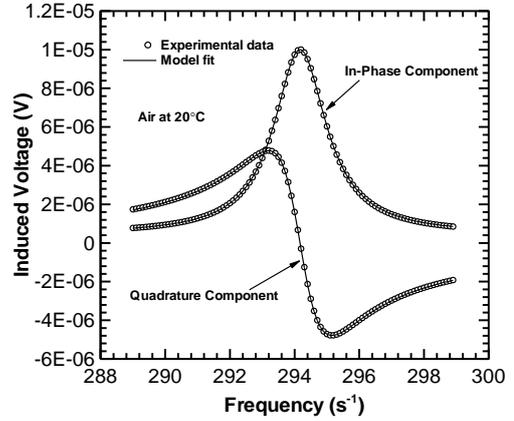


Figure 3. Sample resonant frequency response for $50 \mu\text{m}$ diameter tungsten wire. The fluid is air at 20°C . Motion of the wire is in the Stokes regime.

Table 1 shows a comparison of viscosities measured with the vibrating wire and reference data from the literature [8,9]. The measured viscosity is a model parameter determined by least-squares fitting of the model to the measured resonance curve. As can be seen, the viscosity measurements for water are in good agreement with the reference data while that for air is about 40% higher than expected. This can be explained by the fact that other losses such as the internal friction of the wire are not included in the model. The effect of such losses is small compared with the effect of the viscosity of water but significant when compared with that of the viscosity of air.

| Fluid | Temp. | Measured Viscosity (Pa.s) | Ref [2,4] (Pa.s) |
|-------|----------------------|---------------------------|-----------------------|
| Air | 20.5°C | 2.6×10^{-5} | 1.83×10^{-5} |
| Water | 21.8°C | 9.6×10^{-4} | 9.59×10^{-4} |
| Water | 22.1°C | 9.5×10^{-4} | 9.52×10^{-4} |

Table 1 Viscosity measurements with $50 \mu\text{m}$ diameter tungsten wire

Experiment – Viscosity, Temperature and Heat Transfer

Figure 4 shows the experimental setup for simultaneous measurement of the wire temperature, the viscosity and heat transfer from the wire. The function generator used for the experiment has the ability to add a direct current (DC) offset voltage (V_{DC}) and combine the main oscillating test signal (V_I) with another oscillating signal at a higher frequency (V_H). The concept is to use the added high frequency voltage to heat the

wire while the lock-in amplifier measures the resonance curves at low frequencies. The voltage drop (V_I) over a $10\ \Omega$ fixed resistor was used to measure the electrical current flowing through the circuit and the DC component of the voltage drop across the probe (V_P) was measured with a digital multimeter in order to determine the electrical resistance of the wire (R_{sens}). Thus the power to the wire and the temperature of the wire can be determined.

The lock-in amplifier is very effective in isolating the component of the measured signal at the reference frequency. However, in order to produce a significant heating effect for the wire, the amplitude of the heating component of the signal V_H was typically set several orders of magnitude larger than the amplitude of the measuring signal V_L . This exceeded the allowable 'dynamic range' for the lock-in amplifier so a simple low-pass RC filter was inserted between the lock-in amp and the probe as shown. The filter overcame the problem, but introduced a small, but correctable distortion to the measured signal.

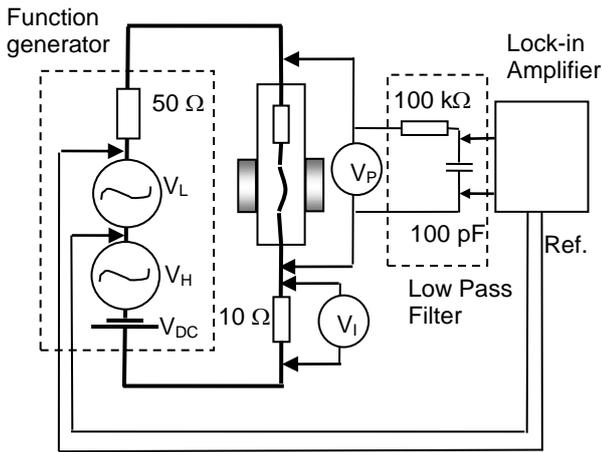


Figure 4. Circuit for studying both viscous drag and heat transfer from vibrating wire

Temperature Calibration

Since platinum is a standard material for temperature measurement, a $10\ \mu\text{m}$ diameter platinum wire, 29 mm in length was selected as the wire for the test probe. The small diameter also made it easier to heat the wire using the available apparatus. The temperature – resistance characteristics of the wire were calibrated against the platinum resistance thermometer over the range of temperatures shown in figure 5.

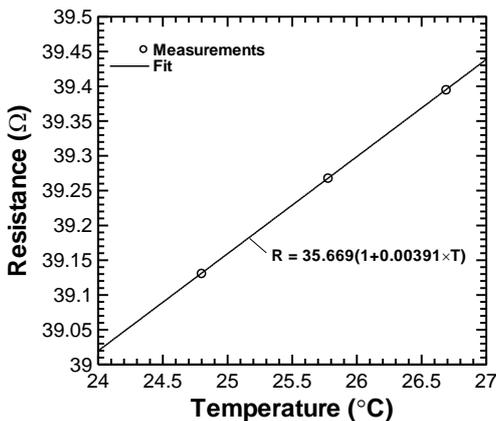


Figure 5. Temperature resistance characteristics for the $10\ \mu\text{m}$ diameter platinum probe

Measurement Issues

Using the finer wire tended to highlight a few measurement issues that are not as obvious in the data shown in figure 3. Figure 6 shows a sample of measured data from the lock-in amplifier for the in-phase component of the voltage across the $10\ \mu\text{m}$ probe. At 50 Hz a disturbance in the measured signal appears. A similar but smaller disturbance typically appeared at all multiples of the power line frequency. The simplest remedy for this problem is to remove from the analysis any data that falls within $\pm 1\ \text{s}^{-1}$ of exact multiples of the line frequency. A somewhat more serious issue is the appearance of a smaller secondary peak in the resonance curve as shown at around $80\ \text{s}^{-1}$ in figure 6. The cause of this peak (as was first pointed out by Bruschi and Santini [1]) is that for each mode of vibration of a straight wire there are usually two possible planes of oscillation. Bruschi and Santini overcame this problem by changing the orientation of the magnetic field to so that the field only excited one of the modes. Another strategy is to use a curved or a bent wire [3,8].

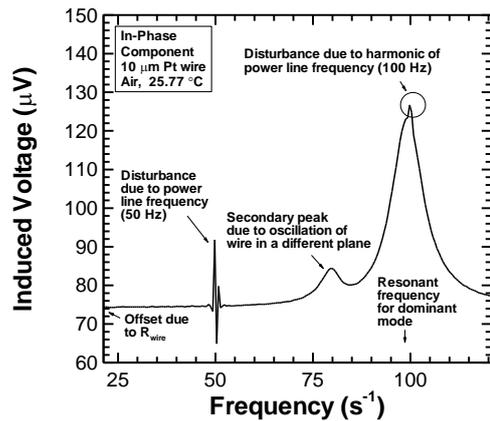


Figure 6. Typical in-phase component of the signal using the apparatus shown in figure 4

A further obvious difference between the in-phase component in figure 3 and that shown in figure 6 is the large offset voltage far from the resonance peak. In figure 6 the offset is about $74\ \mu\text{V}$ while in figure 3 it is less than $1\ \mu\text{V}$. The reason for this difference is the difference in the electrical resistance of the wire which was around $39\ \Omega$ for the $10\ \mu\text{m}$ platinum wire but only $1.2\ \Omega$ for the $50\ \mu\text{m}$ tungsten wire.

Effect of DC offset

For the purpose of heat transfer measurement, a seemingly more obvious way to heat the wire would be to increase the DC component of the output from the function generator. However, changing the DC offset even a few millivolts had the unexpected effect of changing the resonant frequency. Figure 7 shows the effect of changing V_{DC} in Figure 4 from 0 to 60 mV. The position of the peak moves dramatically with negligible heating effect on the wire. The magnitude of the peak diminishes somewhat as V_{DC} is increased. Moreover, the position and magnitude of the secondary peak also change as V_{DC} is changed. The sensitivity of the resonant frequency to the direct current flowing through the wire can be explained simply by noting that the direct current changes the tension in the wire as a result of the Lorentz force from the magnetic field. The change in the position and magnitude of the secondary peak may be because initially the wire had negligible tension and then adding the direct current caused the wire to bow slightly in a plane perpendicular to the magnetic field.

The behaviour shown in figure 7 has the merit that we can very easily change the location of the resonant peak without

modifying the probe. This shows a potential of the instrument for investigating non-Newtonian fluids where the viscous drag effect may change with both magnitude and frequency. Moreover, the ability to select a resonant frequency such that the secondary peak is reduced or removed is another desirable feature for a viscometer.

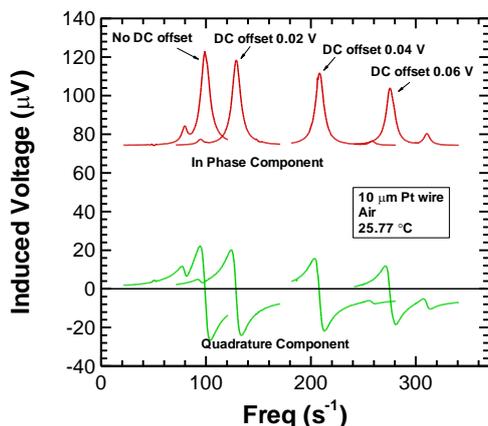


Figure 7. Effect of changing the direct current offset voltage setting V_{DC} shown inside the function generator in figure 4

Effect of Amplitude of Driving Voltage

To remain in the Stokes regime, the amplitude of the oscillation of the wire needs to be very small. Figure 8 shows the effect of increasing the magnitude of V_L in figure 4. At 0.2 mV root-mean square (rms) the resonant curve for the in-phase component is symmetrical and consistent with the typical shape for the Stokes region shown in figure 3. As V_L is increased, the shape of the resonant curve becomes more and more distorted as the drag force ceases to be linearly related to the wire velocity.

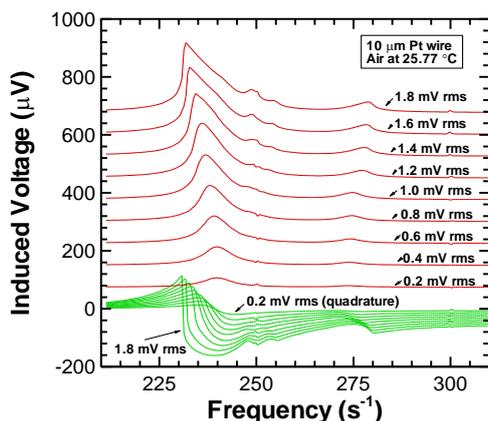


Figure 8. Effect of changing the amplitude of the driving oscillating current

Simultaneous Temperature, Viscosity and Heat Transfer from the Oscillating Wire

Using the apparatus as set up in figure 4 and the calibration curve shown in figure 5 it is possible to simultaneously measure the wire temperature, the heat transfer from the wire and the viscosity of the fluid. Figure 8 shows an example of measurements taken with the heated wire where $V_H = 2.5$ Vpp, $V_L = 0.0002$ V rms and $V_{DC} = 0.05$ V. At steady-state the wire temperature has increased from 25.75 °C to 31.05 °C. The measured power to the wire is 3.132 mW and (unlike the case for

the thicker wire shown in figure 3) the measured viscosity is within a few percent of the reference data [2].

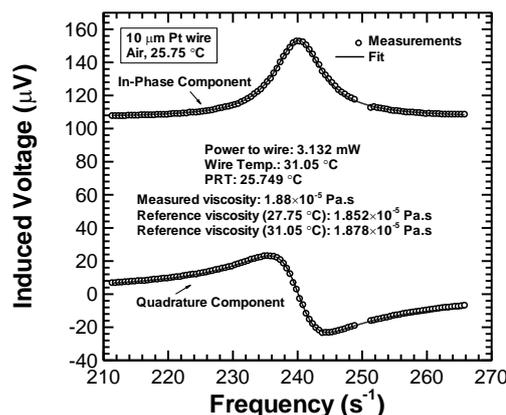


Figure 8. Resonance curves for heated 10 μm platinum wire

Conclusions

The main findings of this study so far are:

1. It is possible to measure the temperature of the wire and the viscosity of the fluid at the same time. This has value for applications such as measurement of viscosity in fluids undergoing chemical reactions where the temperature needs to be monitored.
2. Adding a DC offset to the oscillating driving current has the effect of changing the resonant frequency.

Acknowledgments

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References

- [1] Bruschi, L. & Santini, M., Vibrating Wire Viscometer, *Rev. Sci. Instrum.*, **46**, 1975, 1560-1568.
- [2] Kadoya, K., Matsunaga, N. & Nagashima, A., Viscosity and Thermal Conductivity of Dry Air in the Gaseous Phase, *J. Phys. Chem. Ref. Data* **14**, 1985, 947-970.
- [3] Morishita, M., Kuroda, T., Sawada, A. & Satoh, T., Mean Free Path Effects in Superfluid ^4He , *J. Low Temp. Phys.*, **76**, 1989, 387-415.
- [4] NIST Standard Ref. Database 69, June 2006: NIST Chemistry WebBook, <http://webbook.nist.gov/chemistry/>
- [5] Stokes, G. G., *Mathematical and Physical Papers Vol. III*, Cambridge University Press, London, 1922, 38-55.
- [6] Tough, J.T., McCormick, W.D. & Dash, J.G., Vibrating Wire Viscometer, *Rev. Scientific Instruments*, **35**, 1964, 1345-1348.
- [7] Woodfield, P.L., Moroe, S., Fukai, J., Fujii, M., Shinzato, K., Kohno, M., & Takata, Y., Techniques for Accurate Resistance Measurement in the Transient Short-Hot-Wire Method Applied to High Thermal-Diffusivity Gas, *Int. J. Thermophys.*, **30**, 2009, 1748-1772.
- [8] Woodfield, P.L., Seagar, A., Hall, W., Magnetic Damping Effects in Forced-Oscillation Vibrating-Wire Viscometers, *Int. J. Thermophys.*, **33**, 2012, 259-278.