Abstract

A numerical study based on three-dimensional direct numerical simulations is performed to investigate horizontal thermal convection in a long channel at a large Rayleigh number, Ra. Differential thermal forcing is applied at the bottom boundary over two equal regions. The steady-state circulation is achieved after the net heat flux from the boundary becomes zero. A stable thermocline forms above the cooled base and is advected over the heated part of the base, confining small-scale three-dimensional convection to the heated base and end wall region. At the end-wall a narrow turbulent plume rises through the full depth of the channel. The less energetic return flow is downward in the interior, upon which eddy motions are imposed. This work, for the first time, focuses on the three-dimensional instabilities and structures of the flow. The conversions of mechanical energy are examined in different regions of the flow (boundary layer, plume and interior) and help to understand overall circulation dynamics.

Introduction

Horizontal convection (HC) is driven by a horizontal difference in temperature or heat flux at a single horizontal boundary of a fluid. In a thermally equilibrated state net heat flux over the boundary is zero and circulation cell involves a horizontal boundary flow, turbulent plume motion at the end wall and weak interior return flow, covers the entire the flow domain [5]. HC is of interest given [3]’s proposed model of the meridional overturning circulation (also known as global thermohaline circulation) based on a convective flow which is driven by a horizontal surface temperature gradient.

Experimental observations [10] show a single convecting cell of marked asymmetric structure. Based on a buoyancy-viscous balance gave scaling the boundary layer thickness and the Nusselt number. On the other hand recent 3D numerical studies by [6] provide a different view about the dynamical nature of HC, in temperature or heat flux at a single horizontal boundary of two equal regions. The steady-state circulation is achieved after the net heat flux from the boundary becomes zero. A stable thermocline forms above the cooled base and is advected over the heated part of the base, confining small-scale three-dimensional convection to the heated base and end wall region. At the end-wall a narrow turbulent plume rises through the full depth of the channel. The less energetic return flow is downward in the interior, upon which eddy motions are imposed. This work, for the first time, focuses on the three-dimensional instabilities and structures of the flow. The conversions of mechanical energy are examined in different regions of the flow (boundary layer, plume and interior) and help to understand overall circulation dynamics.

Nevertheless, contracting opinions exist among theoretical oceanographers [8, 16, 9, 3, 6] regarding the capability of HC, or more precisely surface buoyancy forcing, to contribute to the circulation, some appealing to wind and tides as the only sources of mechanical energy to maintain the observed stratification in the abyssal ocean and drive the circulation. [9] have derived expression for volume integrated dissipation rate and an upper bound on this at infinite Ra which is later supported by [6] using two-dimensional numerical simulation. [9] has also restricted the HC from being a turbulent flow on the basis vanishing dissipation vanishing diffusivity (hence at infinite Rayleigh number). On the other hand recent 3D numerical studies by [11] provide a different view about the dynamical nature of HC, based on the geometrical statistics that indicate a turbulent flow.

Figure 1: Schematic of the domain used for the simulation: the hot plate of temperature $T_h = 40\degree C$ on the left-half of the base ($0 \leq x < L/2$) and the cold plate temperature of $T_c = 10\degree C$ on the right half of the base ($L/2 < x \leq L$). Superposed is a snapshot from the numerical of the horizontal velocity field, $u$(m/s) in x-z plane at thermally equilibrated state.

Recently, [4], [14] provide the framework in which energy conversions in the global ocean can be understood. It is still unclear what drives the circulation observed previously in the numerous laboratory experiments [10, 7, 1] and the fully resolved 3D DNS simulations [11] given the strong constraint on total dissipation and on the net mount of conversion from potential to kinetic energy. We therefore, use DNS to examine energetics of convection at large Rayleigh number.

Formulation of the problem

The simulation corresponds closely to conditions used in the laboratory experiments by [7]. The channel domain has length $L = 1.25 \text{ m}$, height $H = 0.2 \text{ m}$ and width $W = 0.05 \text{ m}$ (in which direction the domain is assumed to be periodic). Constant and uniform temperatures $T_h = 40\degree C$ and $T_c = 10\degree C$ are each applied over half of the base as shown in figure 1. All other boundaries are assumed to be adiabatic. The working fluid is water with kinematic viscosity $v = 10^{-6} \text{ m}^2/\text{s}$ and thermal diffusivity, $\kappa_T = 2 \times 10^{-7} \text{ m}^2/\text{s}$.

Governing equations

Directional numerical simulation (DNS) is used to solve dimensionless the continuity, Navier-Stokes and temperature equations for linear Boussinesq fluid:

$$\nabla \cdot \mathbf{u} = 0, \quad \frac{Du}{Dt} = -\nabla p^* + Pr
\nabla^2 \mathbf{u} + RaPr T^* \mathbf{k}, \quad \frac{DT}{Dt} = \nabla^2 T
$$

where the dimensionless quantities $\mathbf{u} = (u, v, w)$ is the velocity field and $T$ is temperature, $p^*$ is deviation from the background hydrostatic pressure, $T^*$ denotes the deviation from the background state and $t$ is time. The governing equations have three nondimensional parameters: Rayleigh number $Ra$, Prandtl number $Pr$ and aspect ratio, $Ar$, where

$$Ra \equiv \frac{\alpha g\Delta T_{\text{b}} H^3}{\nu \kappa T}, \quad Pr \equiv \frac{v}{\kappa T}, \quad Ar = H/L. \quad (2)$$

Here dimensional quantities in the problem are horizontal temperature difference at the bottom boundary $\Delta T_{\text{b}}$, length of the channel, $L$, $H$ and the fluid properties: molecular viscosity, $\nu$, molecular thermal diffusivity, $\kappa_T$ and the temperature gradient at the bottom boundary $\Delta T_{\text{b}}$. The governing equations are solved in a fully three-dimensional domain using the open-source library OpenFOAM. Figure 1: Schematic of the domain used for the simulation: the hot plate of temperature $T_h = 40\degree C$ on the left-half of the base ($0 \leq x < L/2$) and the cold plate temperature of $T_c = 10\degree C$ on the right half of the base ($L/2 < x \leq L$). Superposed is a snapshot from the numerical of the horizontal velocity field, $u$(m/s) in x-z plane at thermally equilibrated state.
thermal diffusivity, $\kappa_T$, thermal expansion coefficient, $\alpha_T$, and reference density, $\rho_0$.

The variables are nondimensionalized as follows:

$$
\begin{align*}
t &= \frac{t_d}{L^2/\kappa_T}, \quad &\mathbf{x} &= \left(\frac{x_d,y_d,z_d}{L}\right), \quad &p^* &= \frac{p_d}{\rho_0 \kappa_T / L^2}; \\
u &= \left(u,v,w\right) = \left(\frac{u_d,v_d,w_d}{\kappa_T / L}\right), \quad &T &= \frac{T_d}{\Delta T_d}
\end{align*}
$$

(3)

**Numerical method**

The simulations use a mixed spectral/finite difference algorithm. Periodicity is imposed in the spanwise, $y-$ direction and derivatives are evaluated with a pseudo-spectral method. The grid is staggered in the vertical and streamwise directions and the corresponding derivatives are computed with second-order finite differences. A low-storage third-order Runge-Kutta-Wray method is used for time stepping, except for the viscous terms which are treated implicitly with the alternating direction implicit (ADI) method. The code has been parallelized using the message passing interface (MPI). Variable time stepping with a fixed Courant-Friedrichs-Lewy (CFL) number of 1.0 is used. Time steps are the order of $10^{-2}$.

**Initialization & Domain resolution**

The simulation is initialized with an isothermal tank of water at temperature, $T_{in} = 35^\circ C$ and no motion the tank interior. Small amount of white noise is applied at initial time after damping outside the bottom boundary. The Prandtl number is chosen to be $Pr = 5$. Based values values of the dimensional parameters Rayleigh number of the flow is $Ra = 5.86 \times 10^{11}$. The computational grid has $513 \times 128 \times 257$ points in the $x$, $y$ and $z$ directions, respectively. We have checked the grid resolution by comparing with the Batchelor scale $\eta_0 \sim Pr^{-1/2} (v^3/\epsilon)^{1/4}$. We have followed the criteria $\eta / \eta_0 \leq \pi$ as proposed by [12]. Here, $\eta$ is the resolution any given direction. Resolution in the streamwise direction is also confirmed by examining scalar dissipation spectra, $2k^2 \tilde{E}_0$ (shown in figure 2) as a function of $\eta_0 \xi$ at three different locations in the domain: inside the bottom boundary layer, the bottom and upper part of the end wall plume. The present simulations are well resolved $k_{\eta, max} \eta_0 \sim 2$.

**Results**

**Flow structures**

Thermally equilibrated state was reached after a time, $t = 8 \; hr$ in the simulation and flow field is similar to that observed in previous experimental studies [10, 7]. A large scale overturning circulation fills the channel and consists of a stably-stratified boundary layer above the base which is feeding into a strong, unsteady and narrow plume against the endwall at the heated end. Fluid leaving the plume in an outflow against the upper boundary enters the interior, which is characterized by broad and gradual downwelling.

Details examination of the flow structures inside the boundary layer is illustrated in figure 3 at steady state ($t \sim 10 \; hr$). The stable thermocline is maintained above the cooled base and is advected across the heated half of the base, capping the small scale convection. But the convection driven by bottom heating erodes the stable temperature gradient from the beneath. Coherent rolls which are shown by isosurface of counter-rotating streamwise vortices, $\Omega = \left| \mathbf{v} \times \mathbf{u} \right|$ are aligned with the boundary layer flow near the middle of the bottom surface. These coherent rolls survive for a small distance of $\sim 70-100$ mm from the cold base. Similar streamwise convection rolls were observed in the previous laboratory experiments [7] with an imposed heat influx and a constant bottom base temperature. These vortices have a strong impact on the temperature field by developing spanwise corrugations of isotherms as shown by temperature isosurface of magnitude, $T = 30^\circ C$ in figure 3. (Recently, [2] have observed a similar type of three dimensional convective roll associated with spanwise density corrugation during the convective instability of internal waves at a critical slope.) These counter rotating vortices interact and merge with each other approximately $350-400$ mm away from the end wall over the heated base, resulting in fully three dimensional complex structures.
Following [15] and [4], we define for a linear Boussinesq fluid the mean kinetic energy, $E_k = \rho_0/2 \int \bar{u}^2 dV$, the turbulent kinetic energy, $E_t = \rho_0/2 \int u' \bar{w} dV$, the potential energy, $E_p = g \int z \bar{\rho} dV$, the background potential energy, $E_b = g \int z' \rho dV$ and the available potential energy, $E_a = E_p - E_b$, where $z' = z^\prime(\rho)$ is the height at which a parcel of density $\rho$ would reside if the entire density field is allowed to relax adiabatically to equilibrium (the background state). The rates at which mechanical energy is transferred between these form is shown schematically in figure 4 (see [4] for more details.)

The turbulent mixing efficiency for HC is defined as $\eta = (\Phi_h - \Phi_i)/\Phi_d + \Phi_e) = 1 - \Phi_i/\Phi_d$ and for the present simulation of $\eta \approx 0.927$. Note that previous definition of the overall mixing efficiency as $\eta = \Phi_d/(\Phi_h + \Phi_d)$ by [11] is misleading, because it will reflect the highest efficiency of $\eta \approx 1$ even for a stationary stratified fluid, where $\Phi_i \approx 0$.

### Table 1: Table for various energy conversion terms ($\times 10^{-7}$)

<table>
<thead>
<tr>
<th>$\Phi_T$</th>
<th>$\Phi_e$</th>
<th>$\Phi'_c$</th>
<th>$\Phi_c'$</th>
<th>$\Phi_z$</th>
<th>$\Phi_f$</th>
<th>$\Phi_{b1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>-2.10</td>
<td>-1.42</td>
<td>1.98</td>
<td>3.52</td>
<td>45.5</td>
<td>-45.5</td>
</tr>
</tbody>
</table>

Only non-zero forcing term in figure 4 is $\Phi_{b1} = g\kappa_f \int z^\prime (\bar{\rho} \bar{u}/\bar{x}_i) n dS$ (i.e. $\Phi_{b1} = \Phi_i = \Phi_{b3} = 0$). We proceed to evaluate the remaining terms from the the simulation: $\Phi_T = -\rho_0 \int (\bar{u}_i/\bar{x}_i) \bar{w} dV$, $\Phi_e = g \int \bar{\rho} \bar{u} \bar{w} dV$, $\Phi_c = \rho_0 V \int (\bar{u}^2 \bar{x}_i) \bar{\rho} dV$, $\Phi_c = -g \rho_f \int (z^2 \bar{\rho} /\bar{x}_i) dV$ and $\Phi_i = -g \kappa_f A (\bar{p}_{top} - \bar{p}_{bottom})$, which take the mean values given in table 1.

The mean dissipation, $\bar{\varepsilon}$, and the mean buoyancy flux, $\bar{\Phi}_z$, are the dominant terms in the $KE$ budget. Mean buoyancy flux oscillates more, compared to $\bar{\varepsilon}$, around its mean values over the steady-state. For the present case the surface buoyancy flux $\Phi_{b3}$ is the dominant source of available potential energy, and is balanced by the rate dissipation due to irreversible mixing i.e. $\Phi_{b2} = \Phi_d$. This exact balance has also been predicted by [14] and [4].

The total dissipation ($\bar{\varepsilon} + \bar{\Phi}_z$) is equal to $\Phi_i$, which is equal to total $APE - KE$ conversion (total buoyancy flux, $\bar{\Phi}_z + \Phi_i$). As $\Phi_i$ is dependent on the difference of the averaged value of buoyancy $\Delta \rho$ between the top and the bottom surface multiplied by thermal diffusivity $\kappa_f$, it must equal the total dissipation, which must equal the total conversion from $APE - KE$. We argue that [9]'s conjecture of non turbulence nature of horizontal convection at the limit of $\kappa_f \rightarrow 0$ is misleading. At infinite Rayleigh number, $Ra \rightarrow \infty$ based on $\kappa_f \rightarrow 0$, $\nu \rightarrow 0$, the imposed surface buoyancy flux also vanishes ($\Phi_{b2} \rightarrow 0$) and we end up with zero dissipation from the kinetic energy reservoir and zero energy influx for the buoyancy field, ($\Phi_{b3} \rightarrow 0$). (Based on the same argument the limit of $\nu \rightarrow 0$, dissipation from the wind field also becomes zero, Ref. Eqn. (1.6) by [3]). In contrast, at finite value of surface buoyancy flux, available potential energy is created and returned to the background potential energy via irreversible mixing. The rate generation of $APE$ and irreversible mixing ($\Phi_d$) are one order of magnitude larger than box integrated dissipation (and $\Phi_i$). This demonstrates that consideration of the $KE$ budget alone overlooks the dominant physics of the flow.

We have also examined APE to KE conversion in four of the flow regions: (1) lower boundary layer, (2) plume region, (3) upper boundary region and (4) interior. In particular, comparable and oppositely-signed rates of APE to KE conversion take place in the plume and interior regions and each is more than three orders of magnitude greater than the other terms in the kinetic energy budget. This reveals that integrating over the whole volume obscures physically important APE to KE conversion.

**Flow energetics**

Following [15] and [4], we define for a linear Boussinesq fluid the mean kinetic energy, $E_k = \rho_0/2 \int \bar{u}^2 dV$, the turbulent kinetic energy, $E_t = \rho_0/2 \int u' \bar{w} dV$, the potential energy, $E_p = g \int z \bar{\rho} dV$, the background potential energy, $E_b = g \int z' \rho dV$ and the available potential energy, $E_a = E_p - E_b$, where $z' = z^\prime(\rho)$ is the height at which a parcel of density $\rho$ would reside if the entire density field is allowed to relax adiabatically to equilibrium (the background state). $\bar{\varepsilon}$ denotes an average of the quantity calculated by averaging over the spanwise direction. The rates at which mechanical energy is transferred between these form is shown schematically in figure 4 (see [4] for more details.)

For thermally-quilibrated horizontal convection, the
convection in a long channel driven by differential heating over the bottom surface. A strong circulation exists over the entire water column, similar to the earlier experimental observations. Convective mixed layer is formed over the heated base by flow parallel role instability and breakdown to small-scale 3D plumes. At the end wall a large turbulent plume, fed by the bottom convective mixed layer, penetrates through the full depth of channel. Importantly, the simulation shows how horizontal convection can be vigorous while satisfying the constraint of \[9\]. The results shows large conversions of available potential energy to kinetic energy by buoyancy flux in the end wall plume and the reverse in the interior of the circulation. A large value of turbulent mixing efficiency, \(\eta \sim 0.927\), claims that horizontal convection is highly efficient in the sense that a strong overturning circulation and irreversible mixing occurs with minimal viscous dissipation.

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References