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A New Velocity Scale for Turbulent Boundary Layers with Adverse Pressure Gradients

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Abstract

Turbulent boundary layers undergoing adverse pressure gradient are frequently encountered in aeronautics, turbomachinery, ship hulls and other industrial applications. The turbulence behaviour of such flows departs significantly from the canonical turbulent flows and the understanding of such flows is important for the calibration of turbulence models and flow control schemes. Finding suitable velocity scales for such boundary layer flows is still an active field of research. In the present paper, a new velocity scale $U_e\beta^{0.1}$ is proposed for turbulent boundary layers undergoing streamwise adverse pressure gradient, where U_e is the freestream velocity and β is non-dimensional pressure gradient parameter. Nine data-sets of turbulent boundary layers undergoing adverse pressure gradient are chosen from literature to investigate the validity and universality of the new scaling. It is found that the new velocity scale $U_e\beta^{0.1}$ collapses the velocity defect profiles in the outer region ($y/\delta > 0.2$) of the studied turbulent boundary layers with adverse pressure gradient over flat or mildly curved surfaces.

Introduction

A fluid flow is subjected to adverse pressure gradient (APG) whenever a solid boundary moves away from the mean flow direction in external flows like the flow over the trailing edge of an airfoil, turbine blades, helicopter rotors, in diverging channels like diffusers, over aft hulls of ships and airships, etc. APG, if strong enough, can cause flow separation which leads to a large loss of total pressure alongwith loss of lift and control. Therefore turbulent boundary layer subjected to APG is an important phenomenon in engineering and is of great technological interest. In several numerical and experimental studies related to non-equilibrium APG turbulent boundary layers, it was observed that the mean velocity and Reynolds stresses do not scale with the friction velocity $u_\tau = \sqrt{\tau_w/\rho}$ where τ_w is the wall shear stress and ρ is the fluid density. Furthermore, for flows with separation, the use of u_τ is not suitable. Several alternate scalings have been proposed for APG boundary layers. Stratford [13] proposed a "pressure" velocity scale $u_p = (\frac{\nu}{\rho}\frac{dP}{dx})^{1/3}$. Mellor and Gibson [8] proposed another pressure-based velocity scale $u_p = u_\tau\beta^{1/2}$ for flows near separation. Zagarola and Smits [14] proposed $U_{zs} = U_e\delta^*/\delta$, which proved to be a valid scale for the outer layer of turbulent boundary layers. Their scale not only removed Reynolds number effects but also the upstream condition effects were noted to disappear. Elsberry et al [6] performed experiments on a boundary layer on the verge of separation and showed that the traditional outer layer scaling U_e did not collapse the stress profiles, although it did collapse the mean velocity profile. They then proposed U_eU_{emax} as an outer layer scaling for Reynolds shear stress and U_{emax}^2 for the Reynolds normal stresses, where U_{emax} is the maximum freestream velocity. Aubertine and Eaton [2] carried out Laser Doppler Anemometry (LDA) studies in a turbulent boundary layer with a mild APG and proposed $u_{\tau,ref}^2(1-0.5\Pi_{ref}^2)$ for for streamwise normal stress in the outer layer. Π_{ref} and $u_{\tau,ref}$ are the wake parameter and friction velocity evaluated at a reference location upstream. The scale was tested on the data of Samuel and Joubert [10] but failed to achieve a good collapse. Maciel et al [7] performed PIV experiments on turbulent boundary layer subjected to a strong APG and showed that the Reynolds normal and shear stresses did not scale with local freestream velocity U_e^2, friction velocity u_τ, mixed scaling U_eu_τ and even those proposed by Mellor and Gibson [8]. In the present work, a new velocity scale $U_e\beta^{0.1}$ for the attached turbulent boundary layers undergoing APG is proposed. Present study is limited to application of the proposed scaling to defect velocity profiles and the streamwise rms velocity fluctuations.

Scaling of Turbulent Boundary Layers with APG

The parameters that characterize the turbulent wall-bounded flow subject to adverse pressure gradient are the wall-normal coordinate y, density ρ, kinematic viscosity ν, center-line velocity or freestream U_e, characteristic length such as pipe radius R or boundary layer thickness δ and pressure gradient dP/dx. In the outer region, we can reduce the functional form of the velocity defect profile for flow over smooth and flat surfaces as

$$(U_e - u) = g\left(y, \delta, U_e, \tau_w, \frac{dP}{dx}\right) \tag{1}$$

Boundary conditions are $U = V = 0$ at $y = 0$ and $U = U_e$ at $y = \delta$. Application of the Buckingham's Π-theorem to equation (2) leads to the following non-dimensional functional relationship,

$$(U_e - u)/U_e = G\left(\frac{y}{\delta}, \frac{\delta}{\tau_w}\frac{dP}{dx}\right) \tag{2}$$

As shown by the equation (2), a velocity scale for the outer region of an APG boundary layer should take into account external velocity U_e and streamwise pressure gradient parameter $\delta/\tau_w dP/dx$. A possible velocity scale was formulated by taking into account U_e and $\beta = \delta^*/\tau_w dP/dx$, which is $U_e\beta^\alpha$. Values of α from 1 to 1/10 were tested on several data sets and it was found that 1/10 gives optimum results.

Eight data-sets (numerical and experimental) with different experimental techniques, Reynolds numbers and pressure gradients were chosen to test the validity and universality of the proposed scaling. All cases pertain to incompressible turbulent boundary layers over solid and smooth surfaces. Table 1 lists all the data-sets, their important global parameters, like the nature of the data-set, Reynolds number based on momentum thickness Re_θ, inlet velocity U_{in} and the maximum of the non-dimensional pressure gradient parameter β. In the table 1, HWA stands for Hot-Wire Anemometry, DNS for Direct Numerical Simulation, PIV for Particle Image Velocimetry and LDA for Laser-Doppler Anemometry. For all data-sets, only the streamwise locations undergoing APG have been included. Data-sets were chosen such as to have combination of traditional equilibrium and non-equilibrium boundary layers. The idea of equilibrium turbulent boundary layers was introduced by Clauser [5]. According to Clauser [5], turbulent boundary layers where the

Case	*Nature*	Re_θ	$U_{ref}(m/s)$	β_{max}
Newman(1951)	HWA	5509-26129	36.5	182
Clauser(1954)	HWA	5637-17404	13.00	2.34
Bradshaw(1965)	HWA	14492-36669	33.5	5.40
Bradshaw(1966)	HWA	10061-22578	33.5	0.915
Samuel(1974)	HWA	—	26.2	8.02
Spalart(1993)	DNS	600-1600	6.5	2.0
Skåre(1994)	HWA	25400-53970	-	21
Aubertine(2005)	LDA	3350-6320	20.5	2.31
Maciel(2006)	PIV	3360-14300	8.9	∞

Table 1: Global characteristics of the selected data-sets.

plots of velocity defect normalised with the local wall friction velocity $(U_e - u)/u_\tau$ versus y/δ at $Re \to \infty$ are invariant (self-similar) at succeeding intervals of the streamwise coordinate are called equilibrium boundary layers. These boundary layer were then associated with constant streamwise values of the parameter $\beta = \delta^*/\tau_w dP_e/dx$.

Figures 1, 2, 3, 4,5, 6, 7, 8 and 9 show the velocity defect profiles for Bradshaw-1 [3], Bradshaw-2[4], Clauser [5], Skåre and Krogstad [11], Spalart and Watmuff [12], Aubertine and Eaton [1], Newman [9], Samuel and Joubert [10] and Maciel et al [7] respectively with the proposed scaling. The data-sets of Newman [9], Clauser [5], Bradshaw-1 [3], Bradshaw-2 [4], Samuel and Joubert [10], Skare and Krogstad [11] and Spalart and Watmuff [12] show a good collapse of the velocity defect profiles. These flows are different from each other with respect to pressure gradients, geometries and equilibrium. Newman [9] is a non-equilibrium strong APG airfoil flow where the parameter β increases rapidly from 3.23 to 182.77. Bradshaw-2 [3] is an equilibrium flow in mild APG developing on the flat bottom of the wind tunnel with β remaining almost constant at 0.9. Bradshaw-1 [4] is a relaxing flow where an equilibrium boundary layer in a mild APG passes into a region of zero pressure gradient with β remaining almost constant with a range of 5.1-5.4. Thus it is a case of moving-equilibrium flow. Samuel and Joubert [10] presents a moderate APG non-equilibrium flow. The data-set of [11] pertains to a strong APG where the parameter β remains nearly constant at 20 in the equilibrium zone. The collapse of velocity profiles for these different flows shows the robustness of the proposed scale for the defect velocity profiles. Self-similarity shown by both equilibrium and non-equilibrium flow with the present scale corroborates the proposition of Maciel et al [7], "a flow can be in equilibrium even if a particular similarity theory would define it as non-equilibrium flow. Conversely, if a flow is found to be in equilibrium according to a given similarity analysis then it is in equilibrium regardless of the similarity assumptions". Maciel et al [7] (figure 9) does not show collapse for all four stations before separation. It is evident because the very strong APG and subsequent flow separation takes the parameter β to infinity. Still the last two stations show excellent collapse. Figures 10, 11 and 12 show the rms velocity fluctuations for Samuel and Joubert[10], Aubertine and Eaton [1] and Spalart and Watmuff [12] respectively. The profiles do show an average collapse for $y/\delta > 0.2$, however,the collapse is not as good as for the mean velocity and needs further testing.

Conclusions

A new outer velocity scale $U_e\beta^{0.1}$ for turbulent boundary layers undergoing adverse pressure gradient over flat or mildly curved surfaces is derived using Buckingham Π-theorem, where U_e is the freestream velocity and β is the non-dimensional pressure gradient parameter. The validity and universality of the proposed velocity scale is tested by applying it to nine data-sets (both numerical and experimental) with different experimental techniques, Reynolds number ranges, flow geometries and pressure gradients. It is found that for the data-sets studied, proposed velocity scale collapses the velocity defect profiles in the outer region of turbulent boundary layers ($y/\delta > 0.2$) undergoing streamwise adverse pressure gradient without flow separation.

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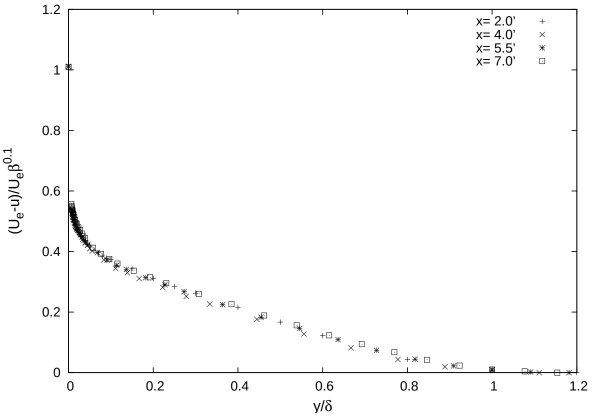

Figure 1: Velocity defect profiles scaled by $U_e\beta^{0.1}$ for Bradshaw-1 [3].

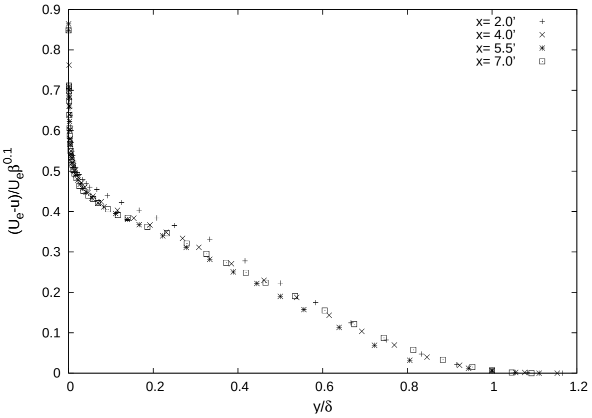

Figure 2: Velocity defect profiles scaled by $U_e\beta^{0.1}$ for Bradshaw-2 [4].

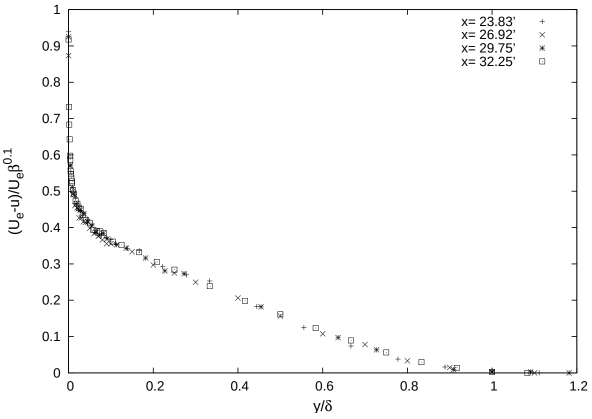

Figure 3: Velocity defect profiles scaled by $U_e\beta^{0.1}$ for Clauser [5].

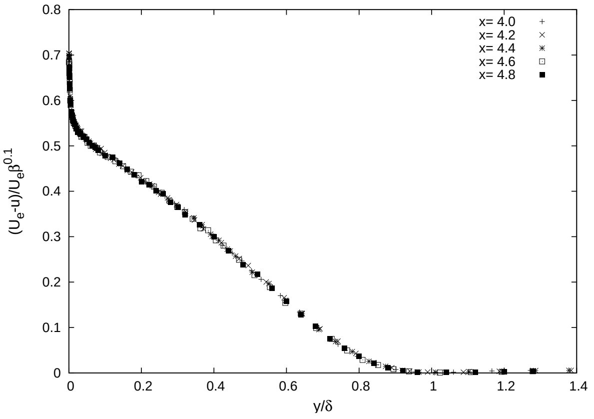

Figure 4: Velocity defect profiles scaled by $U_e\beta^{0.1}$ for Skåre and krogstad [11].

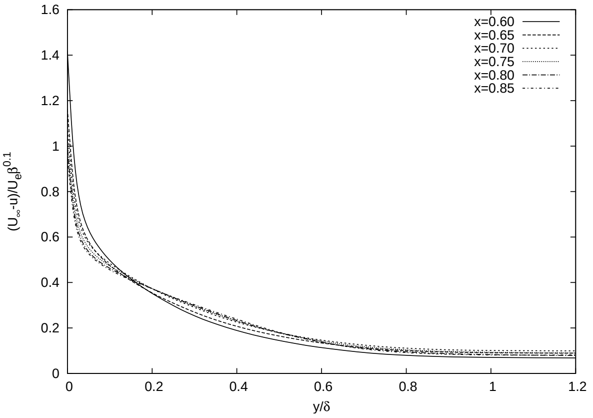

Figure 5: Velocity defect profiles scaled by $U_e\beta^{0.1}$ for Spalart and Watmuff [12].

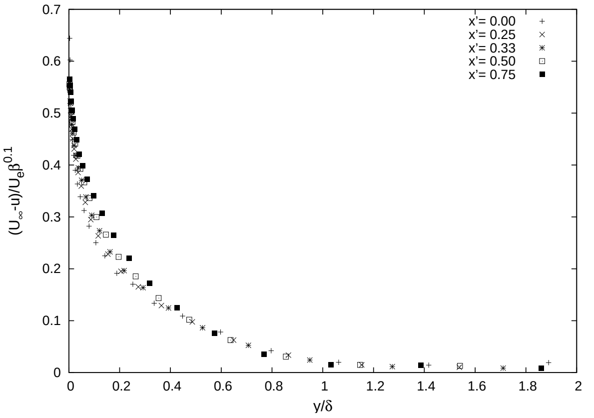

Figure 6: Velocity defect profiles scaled by $U_e\beta^{0.1}$ for Aubertine and Eaton [1].

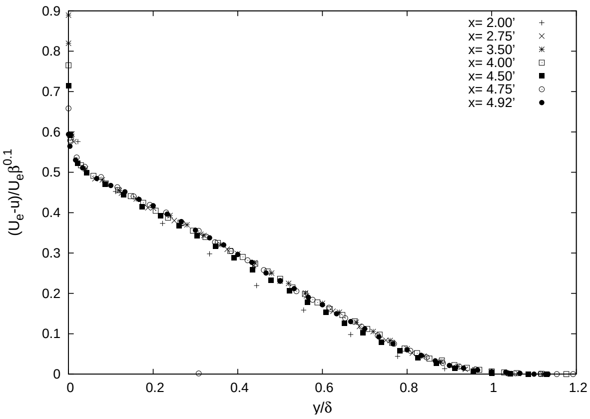

Figure 7: Velocity defect profiles scaled by $U_e\beta^{0.1}$ for Newman [9].

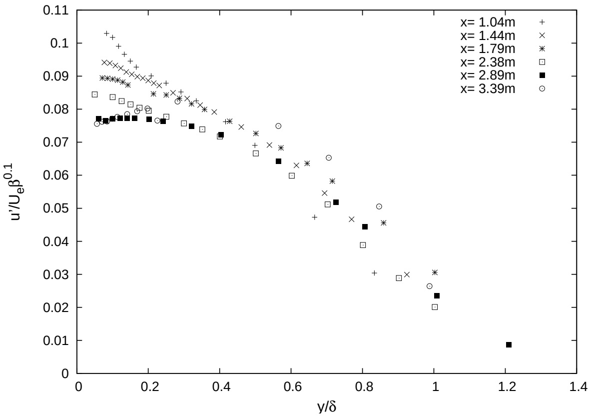

Figure 10: Streamwise velocity fluctuations scaled by $U_e\beta^{0.1}$ for Samuel and Joubert [10].

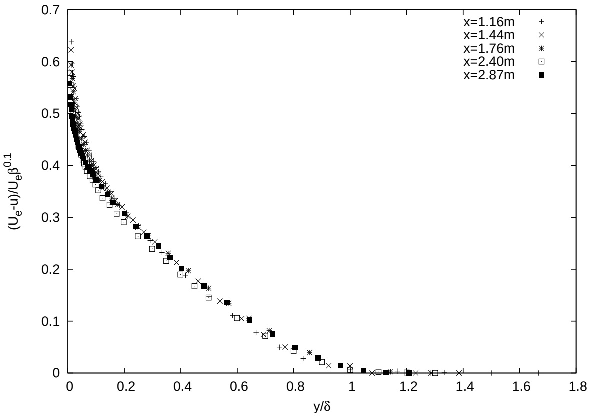

Figure 8: Velocity defect profiles scaled by $U_e\beta^{0.1}$ for Samuel and Joubert [10].

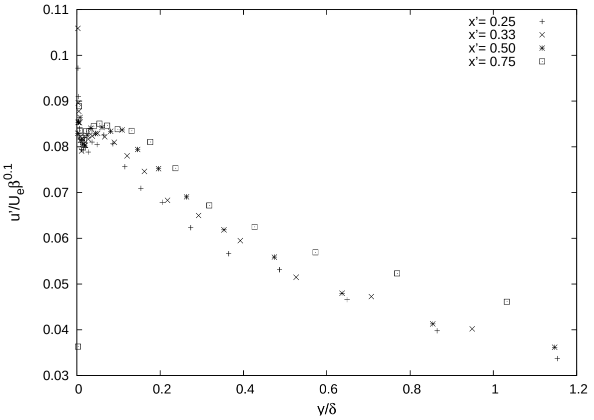

Figure 11: Streamwise velocity fluctuations scaled by $U_e\beta^{0.1}$ for Aubertine and Eaton [1].

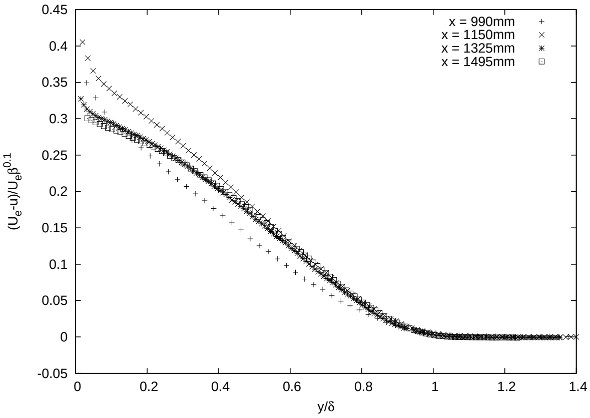

Figure 9: Velocity defect profiles scaled by $U_e\beta^{0.1}$ for Maciel et al [7].

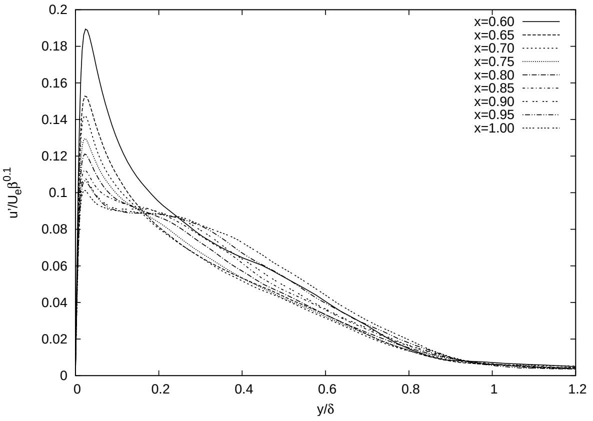

Figure 12: Streamwise velocity fluctuations scaled by $U_e\beta^{0.1}$ for Spalart and Watmuff [12].