

Medium- and Fluid-Compressibility Effects in Flows through Hot Sedimentary Aquifers

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Abstract

Hot sedimentary aquifer systems are currently attracting considerable commercial interest in Australia as clean and renewable energy sources. The feasibility of such technology depends not only on the flow properties of the aquifer, but also on the properties of the aqueous fluids. The steady-state flow field between parallel injection and production wells is expressed in terms of a potential constructed from the best available equations of state for water and the porous medium. These calculations serve to elucidate the qualitative and quantitative importance of aquifer- and fluid-compressibility effects in such systems.

Introduction

Extraction of geothermal energy from deep, hot sedimentary aquifers is currently of considerable interest in Australia [3]. Simulations of fluid flows through porous media can be expected to play a crucial role in the design of such energy-harvesting schemes. The predicted flow behavior will depend in an obvious way on the porosity and permeability of the reservoir. But the properties of the fluid are possibly even more significant, considering that the isothermal compressibility of water can assume large values in the pressure and temperature ranges approaching the critical point. It is well known that accurate prediction of volumetric properties in the critical region is an exacting test of any equation of state; as we will see, the flow of such a fluid through a porous medium poses a potentially challenging problem in fluid mechanics, especially when combined with the highly non-uniform flow distribution between parallel cylinders, which arises in this particular application.

Mathematical models of flows through porous media are usually based on Darcy's Law,

$$\mathbf{v} = -\frac{K}{\eta} \nabla p, \quad (1)$$

where η is the viscosity and K is the permeability. The velocity \mathbf{v} is related to the pore fluid density ρ by the continuity equation,

$$\frac{\partial(\rho\phi)}{\partial t} = -\nabla \cdot (\rho\mathbf{v}), \quad (2)$$

where ϕ is the porosity of the medium. Use of Darcy's Law for \mathbf{v} results in the 'diffusivity equation'

$$\frac{\partial(\rho\phi)}{\partial t} = \nabla \cdot \left(\rho \frac{K}{\eta} \nabla p \right). \quad (3)$$

Solution of this nonlinear equation requires specification of the equation of state relating the pressure and density, and further assumptions concerning the dependence of η , K , and ϕ on p . Functional relationships of the form $\phi = \phi_0 \exp[\beta_v(p - p_0)]$, $\rho = \rho_0 \exp[\beta_l(p - p_0)]$, and $K = K_0 \exp[\beta_K(p - p_0)]$ are often assumed, where β_v , β_l , and β_K (Pa^{-1}) are constant and subscript 0 denotes values at a reference pressure p_0 . If η is inde-

pendent of p , use of

$$\nabla \left(\frac{\rho K}{\eta} \right) = \left(\frac{\rho K}{\eta} \right) (\beta_K + \beta_l) \nabla p, \quad \frac{\partial}{\partial t} (\rho\phi) = (\beta_v + \beta_l) \rho\phi \frac{\partial p}{\partial t} \quad (4)$$

allows equations 1 and 2 to be combined into

$$\frac{\partial p}{\partial t} = D_0 e^{(\beta_K - \beta_v)(p - p_0)} [\nabla^2 p + (\beta_l + \beta_K) \nabla p \cdot \nabla p], \quad (5)$$

where $D_0 = K_0/\eta\phi_0(\beta_l + \beta_v)$ is the hydraulic diffusivity at p_0 . A recent paper by Marshall [9] showed that the constant-compressibility equation of state (with ρ_0 and β_l equal to their saturated-liquid values) is surprisingly accurate (errors are less than 1% for temperatures to 600 K and pressures to about 20 MPa), and that analytical solutions of equation 5 are possible if β_v and β_K are constant, and $\beta_v \approx \beta_K$.

This paper deals with the more general situation where the fluid compressibility β_l and the flow properties of the medium are dependent on p . Assuming that $\phi = \phi_0 \exp[\beta_v(p - p_0)]$, and using the second of equations 4 to express the time derivative of ρ in terms of the time derivative of p , the form of equation 3 that serves as the basis of the present work is found to be

$$[\beta_v + \beta_l] \phi_0 e^{\beta_v(p - p_0)} \rho \frac{\partial p}{\partial t} = \nabla \cdot \left[\frac{K(p)\rho(p)}{\eta(p)} \nabla p \right], \quad (6)$$

where β_l , ρ , and η are understood to be functions of pore-fluid pressure p . The permeability depends on pressure indirectly through ϕ , as $K(p) \equiv K(\phi(p))$. Assuming that the values of the pressure-dependent quantities ρ , β_l , K , and η are expressed relative to the respective reference values ρ_0 , β_{l0} , K_0 , and η_0 corresponding to the same pressure p_0 , this equation can be simplified somewhat by defining

$$P = p - p_0, \quad \rho^* = \frac{\rho}{\rho_0}, \quad K^* = \frac{K}{K_0}, \quad \eta^* = \frac{\eta}{\eta_0}, \quad (7)$$

and introducing

$$X_i = \frac{x_i}{L}, \quad \tau = \frac{D_0 t}{L^2}, \quad D_0 = \frac{K_0}{(\beta_v + \beta_{l0})\eta_0\phi_0}, \quad (8)$$

where L is a reference length and D_0 is a reference hydraulic diffusivity. Thus, the dimensionless form of equation 6 is

$$e(P) \frac{\partial P}{\partial \tau} = \nabla \cdot [g(P) \nabla P] \quad (9)$$

where

$$e(P) \equiv \frac{\beta_v + \beta_l(P)}{\beta_v + \beta_{l0}} e^{\beta_v P} \rho^*(P), \quad g(P) \equiv \frac{K^*(P)\rho^*(P)}{\eta^*(P)} \quad (10)$$

are the dimensionless specific storage and hydraulic conductivity, respectively (such that $e(0) = g(0) = 1$), and the gradients are taken with respect to the scaled coordinates.

Medium and Fluid Properties

The large body of published experimental data for water and steam was critically evaluated by Wagner and Pruß [10], who developed a reference-quality equation of state parameterized to represent the best available measurements of multiple properties to within experimental error. This equation relates the measurable properties of water to the first and second partial derivatives of the potential $\Phi = a_r/RT$ (where a_r is the non-ideal Helmholtz energy per unit mass) with respect to the reduced density $\rho_r = \rho/\rho_c$ and reciprocal reduced temperature $\tau = T_c/T$. (The critical parameters are $\rho_c = 322 \text{ kg}\cdot\text{m}^{-3}$ and $T_c = 647.096 \text{ K}$.) The isothermal compressibility is

$$\beta_r \rho_c R T = \frac{1}{\rho_r} \left[1 + 2\rho_r \frac{\partial \Phi}{\partial \rho_r} + \rho_r^2 \frac{\partial^2 \Phi}{\partial \rho_r^2} \right]^{-1}, \quad (11)$$

where $R = 0.46151805 \text{ kJ}\cdot\text{K}^{-1}\cdot\text{kg}^{-1}$ is the specific gas constant for water. Iterative calculation of the density at specified temperature and pressure is always required. The value and the pressure dependence of the compressibility depend strongly on temperature. The Wagner - Pruß [10] equation of state for water gives a compressibility of about $5 \times 10^{-10} \text{ Pa}^{-1}$ (almost constant for pressures up to about 100 MPa) at ambient temperature, and about $4 \times 10^{-9} \text{ Pa}^{-1}$ at 600 K and 20 MPa, decreasing to about $1.5 \times 10^{-9} \text{ Pa}^{-1}$ at 100 MPa.

The best available representation of the pressure dependence of η is the correlation recommended by IAPWS [6] which expresses η as a function of the reduced temperature and density. Use of this in conjunction with the Wagner-Pruß equation of state shows that the pressure dependence of the viscosity is slight. For example, at 30°C, the viscosity on the liquidus curve at $(p_s, \rho_s) = (0.004 \text{ MPa}, 995.602 \text{ kg}\cdot\text{m}^{-3})$ is 0.7972 mPa s, and is 0.7973 mPa s at $(p, \rho) = (20.004 \text{ MPa}, 1004.338 \text{ kg}\cdot\text{m}^{-3})$. At 300°C, the viscosities at $(p_s, \rho_s) = (8.588 \text{ MPa}, 712.329 \text{ kg}\cdot\text{m}^{-3})$ and $(p, \rho) = (28.588 \text{ MPa}, 748.563 \text{ kg}\cdot\text{m}^{-3})$ are 0.0859 and 0.0928 mPa s, respectively.

The drained pore compressibility β_v is commonly inferred from measurements of the mechanical properties of saturated porous media, by application of poroelastic theory. Perhaps as a result of the difficulties inherent in such measurements [2], reliable numerical values are considerably less easy to come by than accurate volumetric and transport property data for fluids, and almost no publicly-available data are available for the temperature dependence of β_v . For present purposes, we regard β_v as a fixed parameter between the limits 10 and 100 times 10^{-6} psi^{-1} (or 1.45×10^{-9} and $1.45 \times 10^{-8} \text{ Pa}^{-1}$)¹ recently observed by Jalal [7] for Hungarian sandstone cores.

The assumption of an exponential dependence of permeability on pressure is potentially inconsistent with numerous flow models in which the permeability is expressed in terms of the porosity. For granular media described by the Carman-Kožený (CK) equation, according to which

$$K \propto \frac{\phi^2}{(1-\phi)^3} \implies \ln K = \text{const.} + 2 \ln \phi - 3 \ln(1-\phi). \quad (12)$$

Since the proportionality constant is independent of pressure,

$$\frac{K(p)}{K(p_0)} = \left(\frac{\phi}{\phi_0} \right)^2 \left(\frac{1-\phi_0}{1-\phi} \right)^3. \quad (13)$$

The deviation from log-linear pressure dependence can be seen

¹Conversion factors: 1 psi = 6.894757 kPa, 10⁶ psi = 6.894757 GPa, (10⁶ psi)⁻¹ = 1.450377 × 10⁻¹⁰ Pa⁻¹.

more clearly by substituting $\phi/\phi_0 = \exp[\beta_v(p-p_0)]$ and making use of identities such as $1-\phi = 1-\exp[\ln \phi]$, whereupon

$$\frac{K(p)}{K(p_0)} = e^{\frac{1}{2}\beta_v(p-p_0)} \left[\frac{\sinh \frac{1}{2} \ln \phi_0}{\sinh \frac{1}{2} (\ln \phi_0 + \beta_v(p-p_0))} \right]^3. \quad (14)$$

As an example, consider $\phi_0 = 0.3$, $\beta_v = 1.45 \times 10^{-8} \text{ Pa}^{-1}$, and $p-p_0 = 10 \text{ MPa}$, for which the expression on the right hand side of equation 13 is about 1.645, indicating a 64.5% enhancement of permeability due to pore dilation. It is worth noting that the use of power laws of the form $K \propto \phi^c$ (as in the studies of porous rocks by Gouze and Coudrain-Ribstein [5] and Wark and Watson [11]) leads to rather simpler expressions for pressure-dependence of permeability. Since the theoretical justification for such a dependence is not obvious, the Carman-Kožený equation is adopted in the present study. In this regard, it is also worth remarking that the experimental measurements of [7] indicate a dependence of β_v on porosity, which will result in a still more complicated dependence of K on p .

Flow of Highly-Compressible Fluids

The theoretical treatments referred to in [9]) do not work when the compressibility is pressure-dependent. Al-Hussainy et al. [1] showed that flows of fluids with equations of state $p = \rho R T z$ (where z is the compressibility factor) can be described in terms of the pseudopressure ψ , with dimensions of pressure per unit time, defined by

$$\psi = 2 \int_{p_0}^p \frac{p}{\eta z} dp. \quad (15)$$

If the pressure dependence of K is insignificant compared to that of ρ , equation 3 reduces to

$$\frac{\partial \psi}{\partial t} = D(\psi) \nabla^2 \psi, \quad (16)$$

where $D(\psi)$ is the hydraulic diffusivity. Equation 15 is a special case of the Kirchhoff transformation (compare [8, page 13]), which identifies $\rho \mathbf{v}$ as the gradient of a scalar potential. The scalar potential defined more generally by which identifies $\rho \mathbf{v}$ as the gradient of a scalar potential. In the present context, the scalar potential defined more generally by

$$\Psi = \int_{p_0}^p g(p) dp \quad \text{or} \quad \Psi = \int_0^P g(P) dP \quad (17)$$

proves more useful than equation 15, since it has the correct dimensions of pressure, and incorporates all the limiting cases of interest. If K , η , and β_l are constant, this definition can be shown to be equivalent to the Cole-Hopf transformation with a different choice of integration constants from that used in [9]. The derivatives of ψ are related to those of P by

$$\nabla \psi = \frac{\rho^* K^*}{\eta^*} \nabla P, \quad \frac{\partial \psi}{\partial \tau} = \frac{\rho^* K^*}{\eta^*} \frac{\partial P}{\partial \tau}. \quad (18)$$

The corresponding statement of Darcy's Law is therefore

$$\rho \mathbf{v} = -\frac{\rho K}{\eta} \nabla P = -\frac{\rho_0 K_0}{\eta_0} \nabla \psi. \quad (19)$$

If the functions $e(p)$ and $g(p)$ in equation 6 are $h(\psi)$ and $f(\psi)$, respectively, when rewritten in terms of ψ , then since

$$\nabla \psi = g(p) \nabla p, \quad \frac{\partial \psi}{\partial t} = g(p) \frac{\partial p}{\partial t} \quad \frac{\partial p}{\partial t} = \frac{1}{f(\psi)} \frac{\partial \psi}{\partial t} \quad (20)$$

the equation for ψ becomes

$$\frac{1}{D^*(\psi)} \frac{\partial \psi}{\partial t} = \nabla^2 \psi, \quad D^* = \frac{f(\psi)}{h(\psi)}. \quad (21)$$

Flow Between Parallel Wells

Bipolar Coordinates

The two-dimensional flow field produced by completely-penetrating wells is most conveniently described by adopting a bipolar coordinate system, with foci at points $(-a, 0)$ and $(a, 0)$, on the line joining the axes. A variety of coordinate systems can be devised from this arrangement [4], by imposition of different conditions on the distances d_1 and d_2 between the two foci and a given field point. Specification of $d_1/d_2 = e^U$ and the angle V between these lines generates families of non-concentric circles. In these terms, $U = 0$ for all points on the y axis, $V = \pi$ for all points on the x -axis, and $V \rightarrow 0$ as $y \rightarrow \infty$ for a given value of x : $(U, V) = (0, 0)$ is evidently the image of $(x, y) = (0, \infty)$. This coordinate system therefore maps the infinite plane onto an infinite strip of width π . The equations relating x, y to U, V are

$$x = a \frac{\sinh U}{\cosh U - \cos V} \quad y = a \frac{\sin V}{\cosh U - \cos V}. \quad (22)$$

Curves of constant U are circles

$$(x - a \coth U)^2 + y^2 = \frac{a^2}{\sinh^2 U} \quad (23)$$

while curves of constant V are

$$x^2 + (y - a \cot V)^2 = \frac{a^2}{\sin^2 V}. \quad (24)$$

It is possible to select a value of a so that U is constant on each surface. Thus, if the radius of each well is r_0 , the equation of the cylinder on the negative x axis is

$$\left(x + \frac{d}{2}\right)^2 + y^2 = r_0^2, \quad (25)$$

and U will be constant on this circle if

$$r_0^2 = \frac{a^2}{\sinh^2 U}, \quad \frac{d}{2} = a \coth U, \quad (26)$$

or

$$U = \cosh^{-1} \left(\frac{d}{2r_0} \right), \quad a = r_0 \sinh \left[\cosh^{-1} \left(\frac{d}{2r_0} \right) \right] \quad (27)$$

With $d = 1$ m and $r_0 = 0.1$ m, these equations give $U = 2.29243167$ and $a = 0.489897949$ or 49.0 cm, which is slightly less than $d/2$. The infinite porous medium bounded by these two cylinders therefore corresponds to the rectangular strip of width π , bounded at the ends by the vertical lines $U = \pm 2.29243167$. As d/r_0 becomes large, $a \rightarrow d/2$.

Steady Flow Distribution

By standard methods of analysis it can be shown that the scale factors for the computation of the various vector derivatives are

$$h_V = h_U = \frac{a}{\cosh U - \cos V}. \quad (28)$$

Thus the mass flux according to Darcy's Law is

$$\mathbf{J} \equiv \rho \mathbf{v} = - \frac{\cosh U - \cos V}{a} \frac{\rho_0 K_0}{\eta_0} \left[\frac{\partial \psi}{\partial U} \mathbf{e}_U + \frac{\partial \psi}{\partial V} \mathbf{e}_V \right]$$

where \mathbf{e}_U and \mathbf{e}_V are orthogonal unit vectors normal to the curves of constant U and constant V , respectively. The appropriate form of the continuity equation for steady flow is

$$-\nabla \cdot (\rho \mathbf{v}) = \frac{(\cosh U - \cos V)^2}{a^2} \frac{\rho_0 K_0}{\eta_0} \left[\frac{\partial^2 \psi}{\partial U^2} + \frac{\partial^2 \psi}{\partial V^2} \right] = 0. \quad (29)$$

The steady flow potential therefore satisfies Laplace's equation

$$\frac{\partial^2 \psi}{\partial V^2} + \frac{\partial^2 \psi}{\partial U^2} = 0 \quad (30)$$

in a rectangle of width π and length defined by the values of U corresponding to the diameters of the cylinders. On the horizontal boundaries $V = 0$ and $V = \pi$, homogeneous Neumann conditions are physically appropriate, while Neumann or Dirichlet conditions can be applied at either or both of the vertical boundaries. The potential within the $U - V$ rectangle is therefore to be sought as a Fourier cosine series

$$\psi = A + BU + \sum_{n=1}^{\infty} C_n h_n(U) \cos nV, \quad (31)$$

where the h_n are hyperbolic functions whose form depends on the type of boundary conditions to be satisfied at the vertical boundaries. If the cylinders are equipotential surfaces, the values of ψ on the corresponding vertical boundaries of the rectangle are independent of V , and all the Fourier coefficients vanish. If $\psi(U_1) = \psi_1$ and $\psi(U_2) = \psi_2$, A and B are

$$B = \frac{\psi_1 - \psi_2}{U_1 - U_2} \quad A = \psi_1 - \frac{\psi_1 - \psi_2}{U_1 - U_2} U_1. \quad (32)$$

The required potential is therefore

$$\psi(U, V) = \psi_1 + \frac{\psi_2 - \psi_1}{U_2 - U_1} (U - U_1). \quad (33)$$

The V component of the gradient evidently vanishes, and the U component is proportional to the scale factor. The flux $\mathbf{J} = (J_U, J_V)$ is therefore one-dimensional with $J_V = 0$ and

$$J_U = - \frac{\psi_2 - \psi_1}{U_2 - U_1} \frac{\rho_0 K_0}{\eta_0} \frac{\cosh U - \cos V}{a}. \quad (34)$$

Clearly, $\psi_1 - \psi_2 > 0$ for flow from U_1 to U_2 . It is useful to rewrite this expression in terms of the one-dimensional flux

$$J_0 = \frac{\rho_0 K_0}{\eta_0} \frac{\psi_1 - \psi_2}{d} \quad (35)$$

arising from a potential difference $\psi_1 - \psi_2$ between two planar surfaces separated by distance d . In these terms,

$$\frac{J_U}{J_0} = - \frac{\cosh U - \cos V}{U_1 - U_2} \frac{d}{a} = \frac{\cosh U - \cos V}{U_2 - U_1} \frac{d}{a}; \quad (36)$$

with $d = 1$ m and $r_0 = 0.1$ m, $U_2 - U_1 = 4.58486334$, and d/a is 2.04124145.

Numerical Calculations

Calculation of the pressure dependence of ψ is necessary in order to determine the pressure difference needed to sustain a given flow rate. Assuming a reference permeability of about 1 darcy = 1×10^{-12} m² and aquifer temperature 350°C, $p_0 = p_s = 16.529$ MPa, $\rho_0 = 574.471$ kg·m⁻³, $\eta_0 = 0.658$ mPa s, and $K_0 \rho_0 / \eta_0 = 8.73 \times 10^{-7}$ s. For higher pressures, the flow potential ψ can be determined by numerical integration of g with respect to the excess pressure $p - p_0$, from 0 to 10 MPa. Results are shown in figure 1, for $\beta_v = 1.45 \times 10^{-8}$ Pa⁻¹ and 1.45×10^{-9} Pa⁻¹. For the lower value, the curve is almost the straight line that would be expected if the integrand were constant. This result can be explained by observing that the pressure effects on the density and viscosity are comparable, and approximately cancel each other out. (This cancellation is also to be expected at the lower temperatures (up to 200°C) characteristic of most

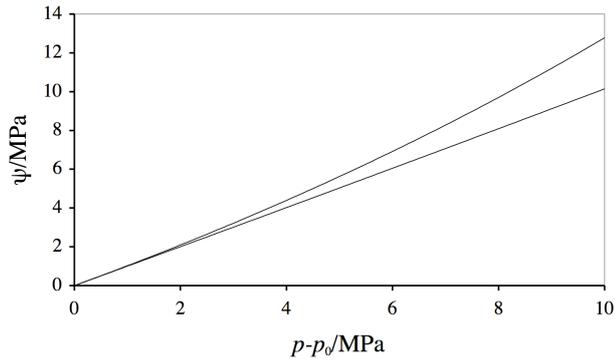


Figure 1: Flow potential ψ at 350°C, calculated for $p_0 = p_s$. Upper curve: $\beta_v = 1.45 \times 10^{-8} \text{ Pa}^{-1}$; lower curve: $\beta_v = 1.45 \times 10^{-9} \text{ Pa}^{-1}$.

HSA systems, but the magnitude of the pore compressibility effect is impossible to predict in the absence of experimental data on the temperature dependence of β_v .) Figure 1 can be used to estimate the penetration depth needed to sustain a flow rate of 100 kg/s. Assuming upstream and downstream pressures $p_s + 10 \text{ MPa}$ (the hydrostatic pressure of a 2.6 km column of water) and p_s , respectively, the corresponding difference in flow potential is about 12.8 MPa. From equation 35 with $d=1 \text{ m}$, the one-dimensional mass flux is therefore $(8.73 \times 10^{-7} \text{ s}) \times (1.28 \times 10^7 \text{ Pa}) = 11.3 \text{ kg m}^{-2} \text{ s}^{-1}$. The required penetration area is 8.85 m^2 , a depth of about 14 m if $r_0 = 0.10 \text{ m}$. Rather small flow rates such as these suggest that in practice, multiple injection wells might be needed, or that the permeability would need to be increased by hydrofracturing. In any event, the energy output cannot be reliably assessed without solution of the combined fluid-flow and heat transfer problem.

The flow distribution evolves over a time scale determined by the reference length and the reference hydraulic diffusivity, $D_0 = K_0 / (\eta_0 \phi_0 (\beta_v + \beta_{l0}))$ (where $\beta_{l0} = 1.69 \times 10^{-11} \text{ Pa}^{-1}$ at 350°C), which has the value $3.495 \text{ m}^2 \text{ s}^{-1}$ if $\beta_v = 1.45 \times 10^{-8} \text{ Pa}^{-1}$, and $34.95 \text{ m}^2 \text{ s}^{-1}$ with $1.45 \times 10^{-9} \text{ Pa}^{-1}$. If the reference length is $d = 1 \text{ m}$, the corresponding values of the time scale d^2/D_0 are 0.2861 s and 0.02861 s. The hydraulic diffusivity depends on the pressure, or equivalently, on the flow potential, as shown in figure 2. The contrast between these two curves results primarily from the enhancement of the permeability given by equation 14. The diffusivity values that result when the quantity shown in Figure 2 is multiplied by D_0 indicate an effectively instantaneous response to changes in the upstream pressure. In fact, considering that the speed of sound through water is nearly 1.5 km/s, the time scale for the response of the flow through the medium would be shorter than the time needed for the propagation of a pressure change down an injection well with a depth of 2-3 km. These observations apply *a fortiori* for aquifers at lower temperatures, for which the liquid compressibility and pressure effects on the viscosity are even smaller.

Conclusions

Flow of water through compressible HSAs can be described in terms of a flow potential that can be calculated directly from the pressure dependence of the fluid and medium properties, with no further analytical approximations. Use of the best available equations of state for water and published data for the pore compressibility of sandstones leads to the tentative conclusion that aquifer compressibility effects will predominate under the relevant conditions. Calculations indicate that measurements of pore compressibility and its temperature dependence will be important in feasibility studies of the exploitation of HSAs.

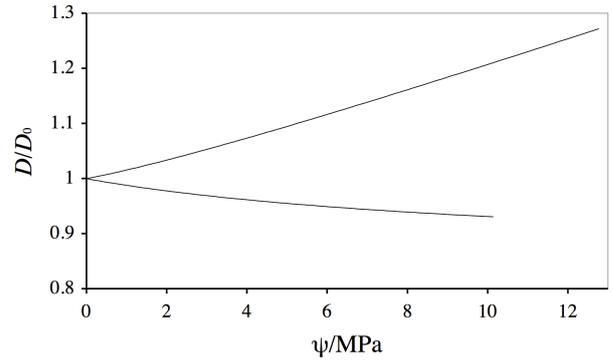


Figure 2: Dimensionless hydraulic diffusivity as a function of flow potential ψ at 350°C, calculated for $p_0 = p_s$. Upper curve: $\beta_v = 1.45 \times 10^{-8} \text{ Pa}^{-1}$; lower curve: $\beta_v = 1.45 \times 10^{-9} \text{ Pa}^{-1}$.

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