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Cancellation effects introduced by the "enthalpy flux–momentum flux" coupling term in the acoustic spectrum of heated flows at supersonic speeds

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Abstract

Accurate prediction of the jet noise in heated flows is of considerable interest. In this paper, we develop a self-consistent model for the jet noise in heated flows using the generalized acoustic analogy derived by Goldstein [1]. As opposed to previous models, we do not neglect the "enthalpy flux – momentum flux" covariance (or the coupling term). Theory shows, the coupling term can give a cancellation to the acoustic spectrum when the acoustic Mach number is supersonic and the observation point is close to the jet axis. The coupling term, therefore, provides a new and simple means of jet noise reduction.

Introduction

Exhaust jet flows from aircraft engines operate at higher temperatures than the free stream. A fundamental understanding of heated jets, within the context of a unified mathematical theory, is, therefore, of great importance especially if it is capable of explaining observed trends in the acoustic data across the Mach number range of interest. The aim of this paper is to clarify the role played by a phenomenon that is usually neglected in jet noise modeling: namely the influence "enthalpy flux – momentum flux" coupling on the aero-acoustics of heated flows. We show that the effect of this coupling term on the acoustic spectrum becomes increasingly important for near transonic and supersonic heated flows. This term may help explain why the experiments (Tanna [2] and Harper-Bourne [3]) and computational studies (Bodony & Lele [4]) exhibit a *noise quieting* at supersonic conditions.

Most of these computational studies on heated jets were based on Lighthill's [5] formulation because of its simplicity. But there are technical difficulties in using it as a starting point for mathematical modeling in heated flows, because the entropy can increase even in the absence of temperature fluctuations (Lilley [6]). For example, Freund's [7] DNS computation shows that the entropy term can be important even for isothermal jets. A mathematical model based on Lighthill's formalism would therefore have to retain the entropy term even for cold flows. There, therefore, appears to be three requirements that should be imposed on any mathematical model in heated flows. Namely:

(i) Heat-associated terms must be negligible when the jet flow is cold, or isothermal.

(ii) The turbulence model must be realistic as possible.

(iii) There should be a manageable number of source terms in the final formula for the acoustic spectrum.

The present paper is concerned with using the axi-symmetry model of the Reynolds stress auto-covariance originally introduced and verified in Afsar *et al* [8] to obtain a self consistent model for the enthalpy coupling term in the formula for the acoustic spectrum. The coupling term, which has not been considered in previous studies, is shown to play an important role at high acoustic Mach numbers using data from recent measurements of the of the correlations function amplitudes taken at the NASA Glenn Research Center. We show how this term can become negative at supersonic speeds and conclude that it may, therefore, provide a new basis for noise reduction methods, particularly for the peak noise at small angles to the jet axis.

Basic equation

The acoustic spectrum,

$$I_\omega(\underline{x}) = \int_{-\infty}^{+\infty} e^{i\omega\tau_0}\overline{p^2}(\underline{x},\tau_0)\,d\tau_0 \tag{1}$$

at the observation point $\underline{x}$, i.e. the Fourier transform of the far-field pressure auto-covariance

$$\overline{p^2}(\underline{x},\tau_0) = \frac{1}{2T}\int_{-T}^{+T} p(\underline{x},t)p(\underline{x},t+\tau_0)\,dt \tag{2}$$

can be expressed in terms of $I_\omega(\underline{x}\mid\underline{y})$, the acoustic spectrum at $\underline{x}$, due to a unit volume of turbulence at $\underline{y}$, by the equation:

$$I_\omega(\underline{x}) = \int_{V_\infty(\underline{y})} I_\omega(\underline{x}\mid\underline{y})\,d\underline{y}, \tag{3}$$

where $V_\infty(\underline{y})$ denotes integration over all space with respect to $\underline{y}$. Goldstein & Leib [8] show that this latter quantity is given by

$$I_\omega(\underline{x}\mid\underline{y}) = (2\pi)^2\Gamma_{\nu j}(\underline{x}\mid\underline{y};\omega)\int_{V_\infty(\underline{\eta})}\Gamma^*_{\mu l}(\underline{x}\mid\underline{y}+\underline{\eta};\omega)\mathcal{H}_{\nu j,\mu l}(\underline{y},\underline{\eta},\omega)\,d\underline{\eta} \tag{4}$$

where the asterisks denote complex conjugate, Greek suffixes range from 1 to 4, and Latin suffixes from 1 to 3. The second rank tensor $\Gamma_{\nu j}(\underline{x}\mid\underline{y};\omega)$ is the Fourier transform of a "propagator" $\gamma_{\nu j}(\underline{x},t\mid\underline{y},\tau)$ that depends on the adjoint vector Green's function $g^a_{\mu\nu}(\underline{y},\tau\mid\underline{x},t)$, determined by equations (4.8) and (4.11) of Goldstein & Leib [9], which shows that the Green's function and, therefore, $\gamma_{\nu j}(\underline{x},t\mid\underline{y},\tau)$ can be calculated once the mean flow is known.

The rank four tensor, $\mathcal{H}_{\nu j,\mu l}(\underline{y},\underline{\eta},\omega)$ is related to the generalized fluctuating stress tensor

$$R_{\nu j,\mu l}(\underline{y},\underline{\eta},\tau_0) =$$

$$\frac{1}{2T}\int_{-T}^{+T}\left[\rho v'_\nu v'_j - \overline{\rho v'_\nu v'_j}\right](\underline{y},\tau)\left[\rho v'_\mu v'_l - \overline{\rho v'_\mu v'_l}\right](\underline{y}+\underline{\eta},\tau+\tau_0)\,d\tau \tag{5}$$

by the simple linear transformation,

$$\mathcal{H}_{\nu j,\mu l} = \epsilon_{\nu j,\sigma m} H_{\sigma m,\lambda n}\epsilon_{\mu l,\lambda n}, \tag{6}$$

where

$$H_{\nu j,\mu l}(\underline{y},\underline{\eta},\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-i\omega\tau_0} R_{\nu j,\mu l}(\underline{y},\underline{\eta},\tau_0)\,d\tau_0 \tag{7}$$

$\tilde{v}_k$ denotes the Favre averaged mean flow velocity, the overbar denotes the time average with time period T, and,

$$\epsilon_{\nu j,\sigma m} = \left(\delta_{\nu\sigma}\delta_{jm} - \frac{\gamma-1}{2}\delta_{\nu j}\delta_{\sigma m}\right) \tag{8}$$

$v'_\mu(\underline{y},\tau) \equiv v_\mu(\underline{y},\tau) - \tilde{v}_\mu(\underline{y})$ denotes a four dimensional velocity fluctuation, with the fourth component defined by

$$v'_4(\underline{y},\tau) = (\gamma-1)\left[h' + \frac{1}{2}v'^2\right](\underline{y},\tau) \tag{9}$$

where h' denotes the enthalpy fluctuation and $v'_4/(\gamma-1)$ denotes the moving frame stagnation enthalpy fluctuation, which is expected to be small relative to $U_{\rm jet}v'_i$ for unheated, or cold jets (where $U_{\rm jet}$ is the nozzle exit velocity).

Equation (5) shows that the generalized Reynolds stress auto-covariance tensor $R_{\nu j,\mu l}(\underline{y},\underline{\eta},\tau_0)$ possesses pair symmetry in the in the suffixes (ν, j) when $\nu = (1,2,3)$ and in the suffixes (μ, l) when $\mu = (1,2,3)$. It, therefore, follows from equations (6) - (8) that the transformed tensor $\mathcal{H}_{\nu j,\mu l}$ also has this property. Then since this symmetry does not exist when the corresponding Greek suffix is equal to 4, it makes sense to re-write equation (4) as

$$I_\omega(\underline{x}\mid\underline{y}) =$$

$$(2\pi)^2\Gamma_{ij}(\underline{x}\mid\underline{y};\omega)\int_{V_\infty(\underline{\eta})}\Gamma^*_{kl}(\underline{x}\mid\underline{y}+\underline{\eta};\omega)\mathcal{H}_{ij,kl}(\underline{y},\underline{\eta},\omega)\,d\underline{\eta}$$

$$+\ (2\pi)^2\Gamma_{ij}(\underline{x}\mid\underline{y};\omega)\int_{V_\infty(\underline{\eta})}\Gamma^*_{4l}(\underline{x}\mid\underline{y}+\underline{\eta};\omega)\mathcal{H}_{ij,4l}(\underline{y},\underline{\eta},\omega)\,d\underline{\eta}$$

$$+\ (2\pi)^2\Gamma_{4j}(\underline{x}\mid\underline{y};\omega)\int_{V_\infty(\underline{\eta})}\Gamma^*_{kl}(\underline{x}\mid\underline{y}+\underline{\eta};\omega)\mathcal{H}_{4j,kl}(\underline{y},\underline{\eta},\omega)\,d\underline{\eta}$$

$$+\ (2\pi)^2\Gamma_{4j}(\underline{x}\mid\underline{y};\omega)\int_{V_\infty(\underline{\eta})}\Gamma^*_{4l}(\underline{x}\mid\underline{y}+\underline{\eta};\omega)\mathcal{H}_{4j,4l}(\underline{y},\underline{\eta},\omega)\,d\underline{\eta} \tag{10}$$

But since, $R_{\nu j,\mu l}(\underline{y},\underline{\eta},\tau_0) = R_{\mu l,\nu j}(\underline{y}+\underline{\eta},-\underline{\eta},-\tau_0)$, equations (6) – (8), imply $\mathcal{H}^*_{\nu j,\mu l}(\underline{y},\underline{\eta},\omega) = \mathcal{H}_{\mu l,\nu j}(\underline{y}+\underline{\eta},-\underline{\eta},\omega)$. The integration variables and index names can, therefore, be changed in the formula for the contribution of the second term in equation (10) to the integral $I_\omega(\underline{x}) = \int_{V_\infty(\underline{y})} I_\omega(\underline{x}\mid\underline{y})\,d\underline{y}$ to show that the third term makes the same contribution to this integral as the complex conjugate of the second term, which means that the acoustic spectrum $I_\omega(\underline{x}\mid\underline{y})$ due to a unit volume of turbulence at $\underline{y}$, can also be expressed as the sum

$$\frac{I_\omega(\underline{x}\mid\underline{y})}{(2\pi)^2} = I^{[1]}_\omega(\underline{x}\mid\underline{y}) + I^{[2]}_\omega(\underline{x}\mid\underline{y}) + I^{[3]}_\omega(\underline{x}\mid\underline{y}) \tag{11}$$

of the following three terms

(i). The momentum flux auto-covariance term:

$$I^{[1]}_\omega(\underline{x}\mid\underline{y}) =$$

$$G_{ij}(\underline{x}\mid\underline{y};\omega)\int_{V_\infty(\underline{\eta})}G^*_{kl}(\underline{x}\mid\underline{y}+\underline{\eta};\omega)\mathcal{H}_{ij,kl}(\underline{y},\underline{\eta},\omega)\,d\underline{\eta} \tag{12a}$$

(ii). The enthalpy flux – momentum flux covariance (or, the coupling term):

$$I^{[2]}_\omega(\underline{x}\mid\underline{y}) =$$

$$2\mathrm{Re}\left\{\Gamma_{4j}(\underline{x}\mid\underline{y};\omega)\int_{V_\infty(\underline{\eta})}G^*_{kl}(\underline{x}\mid\underline{y}+\underline{\eta};\omega)\mathcal{H}_{4j,kl}(\underline{y},\underline{\eta},\omega)\,d\underline{\eta}\right\} \tag{12b}$$

and (iii) The enthalpy flux auto-covariance term:

$$I^{[3]}_\omega(\underline{x}\mid\underline{y}) =$$

$$\Gamma_{4j}(\underline{x}\mid\underline{y};\omega)\int_{V_\infty(\underline{\eta})}\Gamma^*_{4l}(\underline{x}\mid\underline{y}+\underline{\eta};\omega)\mathcal{H}_{4j,4l}(\underline{y},\underline{\eta},\omega)\,d\underline{\eta} \tag{12c}$$

where Re denotes the real part and we have introduced the symmetric tensor $G_{ij} = (\Gamma_{ij}+\Gamma_{ji})/2$ since $\mathcal{H}_{ij,kl}$ has pair symmetries in its first and second pairs of suffixes and $\mathcal{H}_{4j,kl}$ possesses one pair symmetry in its last pair. Notice that the enthalpy flux – momentum flux coupling term involves the covariance of the enthalpy flux at (at location, $\underline{y}$) with the momentum flux (at location, $\underline{y}+\underline{\eta}$) while the enthalpy flux term involves the auto-covariance of the enthalpy flux itself.

Spectral tensor formalism

The mean flow fields in jets involve two characteristic dimensions: a cross-stream dimension, say $D_{\rm jet}$, and a much longer streamwise dimension, say L. Since the axial correlation lengths are always small compared to the streamwise dimension ($L \gg D_{\rm jet}$) of the mean flow and since figure (10) in Pokora & McGuirk [10] shows that the transverse correlation lengths tend to be small compared to the transverse dimension of the mean flow, we now assume that the streamwise and transverse length scales of the turbulence are small compared to the corresponding

dimensions $D_{\rm jet}$ and L of the mean flow. In the absence of strong critical layer effects (see Goldstein & Leib [9]), the propagator $\Gamma^*_{\mu l}(\underline{x} \mid \underline{y}+\underline{\eta};\omega)$ will be a function of the two characteristic scales $D_{\rm jet}$ and L and the acoustic wavelength, c_∞/ω. Then the propagator $\Gamma^*_{\mu l}(\underline{x} \mid \underline{y}+\underline{\eta};\omega)$ can only change by a significant amount over the correlation volume $V_\infty(\underline{\eta})$ of the turbulence when $\omega D_{\rm jet}/c_\infty$ is large and the propagator $\Gamma^*_{\mu l}(\underline{x} \mid \underline{z};\omega)$ can then be represented by its high frequency, or WKBJ approximation (Khavaran [11]). Hence, for variations on the scale of the correlation volume,

$$\Gamma^*_{\mu l}(\underline{x} \mid \underline{y}+\underline{\eta};\omega) \approx \Gamma^*_{\mu l}(\underline{x} \mid \underline{y};\omega)\exp\left[i\frac{\omega}{c_\infty}\underline{\eta}.\underline{\nabla}_{\underline{y}}S(\underline{x} \mid \underline{y})\right] \qquad (13)$$

where $\Gamma^*_{\mu l}(\underline{x} \mid \underline{y};\omega) = (\omega/c_\infty)\, A^{(0)}_{\mu l}(\underline{x} \mid \underline{y})\exp\left[i\frac{\omega}{c_\infty}S(\underline{x} \mid \underline{y})\right]$. The term $A^{(0)}_{\mu l}(\underline{x} \mid \underline{y})$ is the first term in the amplitude series, and $S(\underline{x} \mid \underline{y})$ is the phase function, which satisfies the usual Eikonal equation.

Using this result, the three components of the far-field acoustic spectrum $I_\omega(\underline{x} \mid \underline{y})$ are given by the purely algebraic result:

$$I^{[1]}_\omega\,(\underline{x} \mid \underline{y}) \approx$$

$$G_{ij}(\underline{x} \mid \underline{y};\omega)G^*_{kl}(\underline{x} \mid \underline{y};\omega)\Phi^*_{ij,kl}\left(\underline{y}, \frac{\omega}{c_\infty}\underline{\eta}.\underline{\nabla}_{\underline{y}}S(\underline{x} \mid \underline{y}), \omega\right) \qquad (14a)$$

$$I^{[2]}_\omega\,(\underline{x} \mid \underline{y}) \approx$$

$$2\mathrm{Re}\left\{\Gamma_{4j}(\underline{x} \mid \underline{y};\omega)G^*_{kl}(\underline{x} \mid \underline{y};\omega)\Phi^*_{4j,kl}\left(\underline{y}, \frac{\omega}{c_\infty}\underline{\eta}.\underline{\nabla}_{\underline{y}}S(\underline{x} \mid \underline{y}), \omega\right)\right\} \qquad (14b)$$

$$I^{[3]}_\omega\,(\underline{x} \mid \underline{y}) \approx$$

$$\Gamma_{4j}(\underline{x} \mid \underline{y};\omega)\Gamma^*_{4l}(\underline{x} \mid \underline{y};\omega)\Phi^*_{4j,4l}\left(\underline{y}, \frac{\omega}{c_\infty}\underline{\eta}.\underline{\nabla}_{\underline{y}}S(\underline{x} \mid \underline{y}), \omega\right) \qquad (14c)$$

Equation (14) depends on the turbulence correlations only through the complex conjugate of the fourth rank spectral tensor $\Phi_{\nu j,\mu l}(\underline{y}, k_1, \underline{k}_\perp, \omega)$, which is related to the Reynolds stress auto-covariance tensor (5) by (6), (7) and the Fourier transform

$$\Phi_{\nu j,\mu l}(\underline{y}, k_1, \underline{k}_\perp, \omega) = \int_{\underline{\eta}} \mathcal{H}_{\nu j,\mu l}(\underline{y}, \underline{\eta}, \omega)e^{-i\underline{k}.\underline{\eta}}\, d\underline{\eta}. \qquad (15)$$

The wave vector $\underline{k}$ is defined by its Cartesian components $\underline{k} = (k_1, \underline{k}_\perp)$, where k_1 is the axial component and $\underline{k}_\perp$ is in the transverse direction $\mathcal{H}_{\nu j,\mu l}(\underline{y}, \underline{\eta}, \omega)$ is related to the Reynolds stress auto-covariance through equations (5) - (7), and $\underline{k}_\perp$ can be identified with $\underline{\nabla}_\perp S(\underline{x} \mid \underline{y})$. The momentum flux term (equation 14a) was analysed by Afsar [12]. Our aim is to now analyse the symmetry properties of the coupling term, equation (14b) which has not received attention.

Symmetries of the coupling term: Extension of Afsar *et al* [8] to heated flows

The tensor $\Phi^*_{4j,kl}$ in equation (14b) has 18 independent components owing to the single pair symmetry in suffixes (k, l). We can achieve an "irreducible representation" of $\Phi^*_{4j,kl}$ using the statistical axi-symmetry model in Afsar *et al* [8] for the Reynolds stress auto-covariance tensor, which was verified against turbulence data from a large eddy simulation (LES) of a cold jet flow. The results showed that the model was fairly accurate and we therefore extend it to the tensor $\Phi^*_{4j,kl}$ by making the following two approximations consistent with the real space approximations in Afsar *et al* [8]:

Approximation 1: $\Phi^*_{4j,kl}(\underline{y}, k_1, \underline{k}_\perp, \omega) \approx \Phi^*_{4j,kl}(\underline{y}, k_1, k^2_\perp, \omega)$, (where $k^2_\perp = k^2_2 + k^2_3$).

Approximation 2: $\Phi^*_{4j,kl}(\underline{y}, k_1, k^2_\perp, \omega)$ is an axi-symmetric tensor with respect to the wave number vector $\underline{k}$ in the sense that the trilinear scalar form defined by the tensor $\Phi^*_{4j,kl}$ remains invariant under the full rotation group with respect to the k_1-direction.

A more general definition of axi-symmetry could be obtained by insisting trilinear form is invariant to proper rotations only (the SO[3] Lie group) since $\Phi^*_{4j,kl}$ is a tensor with odd parity and, in general, its scalar form may not be invariant to reflections in the plane perpendicular to the axis of symmetry (see Afsar *et al*, [13]). However, invariance to the full rotation group *is* a reasonable starting point if the additional terms (without insisting mirror symmetry) are small, and notwithstanding that, since this is first time analysis of the coupling term has been conducted, it makes sense to proceed with the simplest order or approximations.

Approximations 1 & 2 (which, following Afsar *et al* [8], we also refer to as *statistical axi-symmetry*) give an irreducible representation of $\Phi^*_{4j,kl}$ that possesses only 3 independent components, viz.:

$$\Phi^*_{4j,kl}(\underline{y}, k_1, k^2_\perp, \omega) = (\delta_{j1}\delta_{kl} - \delta_{j1}\delta_{k1}\delta_{l1})\,\Phi^*_{41,22}(\underline{y}, k_1, k^2_\perp, \omega)$$

$$+ (\delta_{k1}\delta_{jl} + \delta_{l1}\delta_{jk} - 2\delta_{j1}\delta_{k1}\delta_{l1})\,\Phi^*_{42,21}(\underline{y}, k_1, k^2_\perp, \omega)$$

$$+\delta_{j1}\delta_{k1}\delta_{l1}\Phi^*_{41,11}(\underline{y}, k_1, k^2_\perp, \omega) \qquad (15)$$

which shows that $\Phi^*_{4j,kl}(\underline{y}, k_1, k^2_\perp, \omega)$ depends upon the 3 components: $\Phi^*_{41,11}$, $\Phi^*_{41,22}$ & $\Phi^*_{42,21}$ of $\Phi^*_{\nu j,\mu l}(\underline{y}, k_1, k^2_\perp, \omega)$. This result also implies that $\Phi^*_{41,22} = \Phi^*_{41,33}$ and $\Phi^*_{42,21} = \Phi^*_{43,31} = \Phi^*_{42,12} = \Phi^*_{43,13}$.

Equation (15) allows the acoustic spectrum of the coupling term to be expressed as:

$$I^{[2]}_\omega\,(\underline{x} \mid \underline{y}) \approx$$

$$2\mathrm{Re}\left\{\Gamma_{41}G^*_0\Phi^*_{41,22}+\Gamma_{41}G^*_{11}\Phi^*_{41,11}+2\left[\Gamma_{42}G^*_{12} + \Gamma_{43}G^*_{13}\right]\Phi^*_{42,21}\right\} \qquad (16)$$

Cancellation effects introduced by the coupling term

Measured turbulence data for the coupling term only exists for the axial component, $R_{41,11}$ (Mielke *et al*, [14]). Figure (1) shows that the coupling term is only likely to be important at the higher transonic and supersonic acoustic Mach numbers where $R_{41,11}$ is positive. However, $R_{41,11}$ enters equation (16) through,

$\Phi^*_{41,11}$, which is defined by equations (6), (7) & (15), and under statistical axi-symmetry is given by:

$$\int_{\underline{\eta}} \left[\frac{(3-\gamma)}{2} H_{41,11} - (\gamma - 1) H_{41,22} \right] (\underline{y}, \underline{\eta}, \omega)\, e^{-i\underline{k}.\underline{\eta}}\, d\underline{\eta} \quad (17)$$

There is no measured data for the amplitude of $R_{41,22}$. However, as in auto-covariance of the Reynolds stress, where the lateral correlation function is about one-third of the longitudinal correlation at the end of the potential core (Leib & Goldstein [15]), it may be reasonable to suppose a similar result for $R_{41,22}$ relative to $R_{41,11}$. This would then imply the square brackets in equation (17) is proportional to a positive constant times $H_{41,11}$ (the Fourier transform of $R_{41,11}$). And that $\Phi^*_{41,11}$ is, therefore, expected to be positive.

Now, equation (5.20) of Goldstein & Leib [9]) shows that

$$I^{[2]}_{\omega}\ (\underline{x} \mid \underline{y}) \approx \left(\frac{\widetilde{c^2}}{c^2_\infty} \right) \frac{4\cos^3\theta}{\left[1 - M(\underline{y}_\perp)\cos\theta\right]^3} \frac{|\widetilde{\mathcal{G}}|^2}{c_\infty} \Phi^*_{41,11} \quad (18)$$

for a parallel mean flow, where $(\widetilde{\mathcal{G}})$ denotes the scaled adjoint Lilley Green's function (defined by their equations 4.20 and 5.22). Equation (18) shows that for a parallel shear layer, the propagator $\Gamma_{41} G^*_{11}$ is proportional to 3 inverse Doppler factors, and, therefore, changes sign when the acoustic Mach number is supersonic and the observation point is close to the downstream jet axis (within the 'zone of silence' of the parallel shear layer Green's function).

At high subsonic Mach numbers the propagator $\Gamma_{41} G^*_{11}$ is always positive and this component of the coupling term makes a positive (enhancement) contribution to the acoustic spectrum. When the Mach number is supersonic, the propagator $\Gamma_{41} G^*_{11}$ changes sign at small observation angles to the jet axis, so that the contribution made by the component (18) to the coupling term $I^{[2]}_{\omega}$ is negative. This is the first time a ***cancellation effect*** brought about through the coupling between enthalpy flux and momentum flux has been highlighted. This cancellation effect has the potential of directly reducing the peak jet noise since the sign change in the propagator occurs at angles within the 'zone of silence' of $|\widetilde{\mathcal{G}}|^2$.

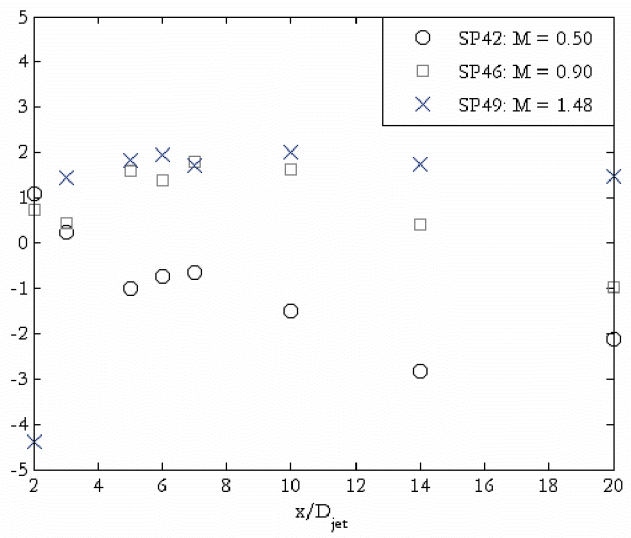

Figure 1: Amplitude of $R_{41,11}/(c_\infty R_{11,11})$ for heated jets defined in Tanna [2] at a temperature ratio of 2.7 (data courtesy of A. Fagan, NASA Glenn Research Centre. The Experiments are described in Mielke *et al* [14]).

Conclusions

In this paper we have shown the coupling between enthalpy flux and momentum flux could play an important role in the acoustic spectrum of heated jet flows. The mathematical structure of the coupling term indicates it can provide a cancellation, if the acoustic Mach number is supersonic and the observation angle is close to the downstream jet axis. This result could be exploited to provide a simple means for jet noise reduction.

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