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Assessment of Stress-Sensitive Porous Media Properties from Fluid Flow Rate Transients

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Abstract

Stress-sensitive reservoirs are reservoirs whose permeability and porosity is pressure dependent. This dependence creates a nonlinearity in the equations that govern fluid flow. This work presents solutions for the transient behaviour of pressure (and therefore flow rate) in 1D reservoir models. These solutions are derived using the Cole-Hopf transform. The results show that early time flow rate data from a stress-sensitive model has the same behaviour as that of a non-stress sensitive model with a higher permeability. Analysis of late time flow rate data from a stress-sensitive reservoir would suggest a lower permeability than early time data. Solutions for stress-sensitive cases can be plotted using production data analysis plotting functions. This type curve plot implies that the parameters governing stress-sensitivity could be extracted from production data using a type curve approach.

Introduction

The ease with which fluid can flow through a porous medium (or a reservoir) is quantified by its permeability, which can be inferred from relationships between observed well flow rates and pressures. Major advances in the ability to infer permeability and other reservoir characteristics from routine well production data, as opposed to well test data, have been made over the past fifteen years (e.g. Doublet et al. 1994, Araya and Ozkan 2002). This study considers the analysis of flow rate transients from cases in which the permeability of the reservoir changes in response to pressure changes i.e. stress-sensitive reservoirs. In these reservoirs the effective permeability changes as fluid is withdrawn from the reservoir.

Modern production data analysis approaches are based on mathematical models which link the well rates and pressures. Interpretation of these data in terms of “material balance time” (Doublet et al., 1994) helps to account for variability in well production rates with time. Current approaches are able to handle vertical wells, horizontal wells and fractured wells in unfractured and fractured reservoirs. These approaches are developed from analytical solutions to the diffusivity equation. The rate or pressure response is non-dimensionalised and then both integrated and differentiated. Families of responses for varying reservoir parameters are plotted versus time to form sets of type curves which can be used to interpret field data.

Development of a production data analysis approach for stress-sensitive reservoirs requires a model to represent the variation of pressure and flow rate with time during production. The standard form of the diffusivity equation which describes pressure change in a reservoir due to well production assumes the reservoir permeability and porosity do not change in time as reservoir pressure changes. Allowing these parameters to change in time creates nonlinearities which pose challenges in deriving analytical solutions to the diffusivity equation. For stress-sensitive reservoirs a recent publication (Marshall, 2008) involving the Cole-Hopf transform provides valuable solutions for some relevant geometries.

Numerical experiments (Archer, 2008) have shown that if a reservoir is stress-sensitive its production response does not follow the type curves associated with a non-stress sensitive reservoir with the same geometry. That finding inspired the direction of this study. The current study uses an analytical representation of flow in stress-sensitive reservoirs to explore the behaviour of flow rate transients which could be used to diagnose stress sensitivity from the analysis of the production response of stress sensitive reservoirs.

Method

Governing Equation

In this section a mathematical model model governing transient pressure and flow rates responses in 1D porous media flows is derived. This model is a nonlinear PDE since it allows for the porosity and permeability of the medium, and the density of the fluid to all be pressure dependent.

Development of the model begins from Darcy’s which law links pressure gradients and velocity in porous medium flow.

$$\mathbf{v} = -\frac{k}{\mu}\nabla p \qquad (1)$$

where $\mathbf{v}$ is the fluid velocity, k is the permeability of the medium, μ is the fluid viscosity and p is pressure.

The continuity equation governs conservation of mass:

$$\frac{\partial}{\partial t}(\rho\phi) = -\nabla\cdot(\rho\mathbf{v}) \qquad (2)$$

where ρ is the fluid density, ϕ is the porosity of the porous medium and t is time.

Following the derivation presented by Marshall (2008) ρ, ϕ and k are assumed to depend exponentially on pressure.

$$\rho = \rho_o e^{\beta_l (p_i - p)} \quad \text{..........................(3)}$$

$$\phi = \phi_o e^{\beta_v (p_i - p)} \quad \text{..........................(4)}$$

$$k = k_o e^{\beta_K (p_i - p)} \quad \text{..........................(5)}$$

where p_i is the initial reservoir pressure.

When equations (1) to (5) are combined they result in the following nonlinear PDE.

$$\frac{\partial p}{\partial t} = D_o\left[\nabla^2 p + (\beta_l + \beta_K)\nabla p \cdot \nabla p\right] \text{........(6)}$$

where

$$D_o = \frac{K_o}{\mu\phi_o(\beta_l + \beta_v)} \quad \text{.....................(7)}$$

The nonlinear term in (6) is typically neglected by arguing that this term is insignificant in comparison to other terms. However in some reservoirs e.g. tight (low permeability) gas reservoirs the effects of stress sensitivity can mean that reservoir permeability may vary up to 90% (Warpinski and Teufel, 1992) between the permeability observed at the initial reservoir pressure and the permeability in the near well region (i.e. at a lower pressure).

A few authors, (e.g. Kikani and Pedrosa, 1991) have attempted to handle this nonlinearity via a perturbation expansion approach, while other authors (e.g. Osorio et al., 1997) have developed numerical models which include stress sensitivity. In the relatively small body of work available on modelling the flow behaviour of stress sensitive reservoirs, pressure transient behaviour has been the usual focus. The current study addresses rate transient behaviour to assess whether rate transient behaviour (i.e. variations in a well flow rate in time, while the well flows at constant pressure) can be used to diagnose the presence of stress sensitivity in a reservoir's production response.

Solution Strategy

Marshall (2008) provides solutions to equation (6) via the Cole-Hopf transform for two cases of a 1D system – one with a fixed pressure at both the inner and outer boundary and also one case with a fixed flowrate at the inner boundary in conjunction with a fixed pressure at the outer boundary. This choice of outer boundary is sometimes relevant to petroleum reservoir flows e.g. when reservoirs are in hydraulic communication with large aquifers. However a more typical outer boundary condition in petroleum reservoir engineering is a "no-flow" outer boundary condition which requires a zero pressure gradient at the far end of the system.

To derive solutions to equation (6) Marshall casts the equation in terms of dimensionless variables as:

$$\frac{\partial P}{\partial \tau} = \frac{\partial^2 P}{\partial X^2} + \beta^*\left(\frac{\partial P}{\partial X}\right)^2 \quad \text{.................(8)}$$

where

$$P = \frac{(p_i - p)}{(p_i - p_{well})}, \quad X = \frac{x}{L}, \quad \tau = \frac{D_o t}{L^2} \text{ ....(9)}$$

in which L is the length of the 1D reservoir.

The Cole-Hopf (Cole, 1951 and Hopf, 1950) transform allows solutions to equation (8) to be found by linearising the equation by rewriting it in terms of a new variable, Y. Under the transform

$$P = \frac{1}{\beta^*}\ln(B^* Y) \quad \text{......................(10)}$$

with $\beta^* = (p_i - p_{well})(\beta_l + \beta_K)$ equation 8 becomes

$$\frac{\partial Y}{\partial \tau} = \frac{\partial^2 Y}{\partial X^2} \quad \text{...........................(11)}$$

Production from petroleum wells at constant bottom hole pressure is very common, so the following inner boundary condition was used in this study:

$$Y(0,\tau) = Y_o \quad \text{..........................(12)}$$

where $Y_o = \frac{1}{\beta^*} e^{\beta^*}$

The initial condition for all cases represents a constant initial pressure:

$$Y(X,0) = Y_o \quad \text{..........................(13)}$$

Two outer boundary conditions were considered in this study. The first is a constant pressure boundary condition (as per Marshall, 2008):

$$Y(1,\tau) = Y_1 \quad \text{...........................(14)}$$

where $Y_1 = \frac{1}{\beta^*}$.

Marshall presents the solution to equation (8) with boundary and initial conditions defined by (12) to (14) as:

$$L(Y(X,\tau)) = \frac{Y_1}{s} + \frac{Y_o - Y_1}{s}\frac{\sinh(\sqrt{s}(1-X))}{\sinh(\sqrt{s})} \text{ .(15)}$$

Marshall also applies the Laplace transform inversion integral to invert this solution.

The current study also considers an alternative outer boundary condition more typical of petroleum reservoir condition. This is a zero pressure gradient (i.e. no fluid flow) at the outer boundary. This boundary condition is defined as:

$$\frac{\partial Y}{\partial X}(1,\tau) = 0 \quad \text{.........................(16)}$$

With this boundary condition the following solution to the transformed pressure was derived:

$$L(Y(X,\tau)) = \frac{Y_1}{s} + \frac{Y_o - Y_1}{s}\frac{\cosh(\sqrt{s}(1-X))}{\cosh(\sqrt{s})} \quad (17)$$

Production Data Analysis

Doublet et al. 1994 presented an innovative approach to the analysis of production data (i.e. well pressures and rates). That work is based on presentation and interpretation of production data in terms of three key "plotting functions" which take on characteristic shapes. The first of these plotting functions is "material balance time":

$$\bar{t} = \frac{N_p}{q} \quad \text{.............................(18)}$$

where q is the well production rate (derived from the Darcy velocity at X=0), and N_p is the cumulative production from the well. The flow rate and rate integral functions which are plotted against material balance time in Doublet et al.'s production data analysis approach area:

$$\left(\frac{q}{p_i - p_{well}}\right) = \frac{q}{\Delta p} \quad \text{....................(19)}$$

$$\left(\frac{q}{p_i - p_{well}}\right)_i = \frac{1}{\bar{t}}\int_0^{\bar{t}} \frac{q}{\Delta p} d\tau \quad \text{...............(20)}$$

$$\left(\frac{q}{p_i - p_{well}}\right)_{id} = -\frac{d(q/\Delta p)_i}{d\ln \bar{t}} = -\bar{t}\,\frac{d(q/\Delta p)_i}{d\bar{t}} \quad \text{.(21)}$$

When plotted as a family of curves (between which a key parameter such as reservoir size varies) equations (18) to (21) form "type curve" sets which can be overlain with field data from wells. The process of matching the field data to the underlying theoretical models allows reservoir permeability and size to be determined

The current study considers whether the behaviour of the well flow rates from the theoretical model defined by equations (15) and (17) has characteristic features which can be used to diagnose the presence of stress-sensitive behaviour and to quantify this effect.

Results

A base case was considered which has parameters defined in table 1. Since the petroleum industry internationally does not use SI units these parameters are listed with both "field units" and SI units.

Parameter	Value, field units	Value, SI units
Permeability, k_o, at $p = p_i$	100 md	9.869×10^{-14} m^2
Viscosity, μ	1 cp	0.001 Pa.s
Initial reservoir pressure, p_i	5000 psi	3.44×10^{7} Pa
Well reservoir pressure, p_{well}	1000 psi	6.8944×10^{6} Pa
Porosity, ϕ_o at $p = p_i$	0.25	0.25
β_l	2×10^{-5} psi^{-1}	2.9×10^{-9} Pa^{-1}
Length of reservoir, L	2000 ft	609.6 m
Cross sectional area of reservoir, A	2500 ft^2	232.25 m^2

Table 1.Base case model parameters

Constant Pressure Outer Boundary

To assess how the degree of stress-sensitivity in the base case model affects flow rates at the well when a constant pressure outer boundary condition is imposed, cases were run with β* varying from 0 (no stress sensitivity) to 1 (very significant stress sensitivity), as shown in Figure 1. The curves that result are essentially parallel to one and other at all times, with the more stress sensitive cases producing higher flow rates since equation (5) implies that these cases will have higher effective permeabilities near the wellbore (i.e. where pressure is lowest).

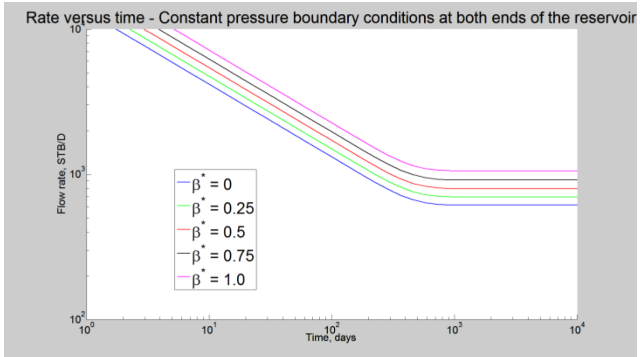

Figure 1. Flow rate versus time, varying β*, constant pressure outer boundary condition.

For comparison Figure 2 shows rate transient data for non-stress sensitive cases with varying levels of permeability (homogeneous, non-pressure dependent). It is evident that when the base permeability, k_o, controls the timing of the change in the flow rate profile. This change reflects a change from "transient" flow to "boundary dominated" flow in which the constant pressure outer boundary condition prevents further decline in the well flow rate.

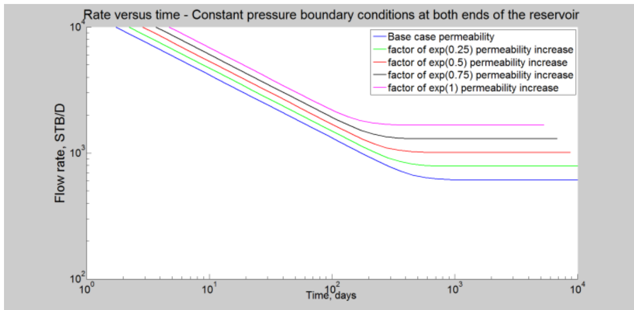

Figure 2. Flow rate versus time, non-stress sensitive cases with varying permeabilities.

Figure 3 compares a stress-sensitive case with β* = 0.5 to two non-stress sensitive cases. The non-stress sensitive cases have increases in the base case permeability of 1.3 and 1.7 respectively. These increases were chosen in order to match the early time (transient) and late time (boundary dominated). If the early time were interpreted using a non-stress sensitive model the base permeability would be overestimated by 70% (when compared to the permeability at initial reservoir pressure). If the late time data were interpreted using a non-stress sensitive model the base permeability would be over-estimated by 30%.

No Flow Outer Boundary

The rate transient behaviour of the stress sensitive solution derived for a case with a no flow outer boundary condition and a constant pressure inner boundary condition is shown in Figure 4 for β * values between zero and one. The lower curve is for β* equal to zero and the upper curve is for β * equal to one. Unlike Figure 1 the curves are not exactly parallel to each other.

Figure 5 compares a non-stress sensitive case to a stress-sensitive case with β* equal to 0.5. The early time flow in this stress-sensitive case can be matched by the solution for flow in a non-stress sensitive reservoir with permeability 70% greater than the base permeability in the stress-sensitive case.

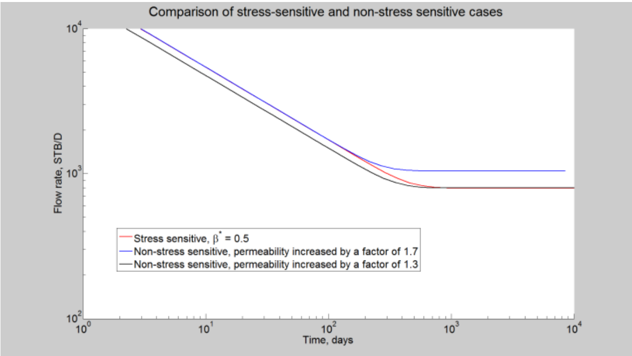

Figure 3. Comparison of stress-sensitive case (β* = 0.5) and two non-stress sensitive cases.

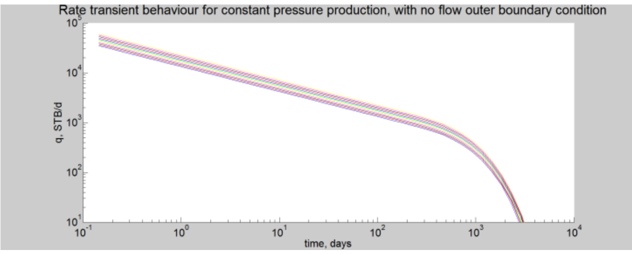

Figure 4. Rate transient behaviour for various β* values for no flow outer boundary condition case.

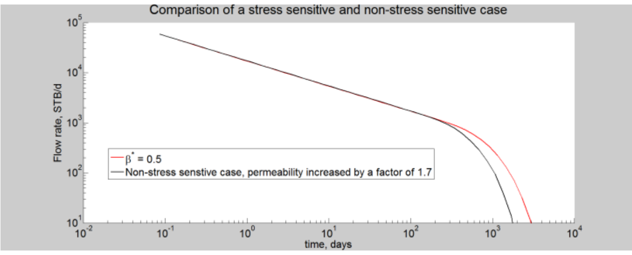

Figure 5. Comparison of a stress-sensitive case and a non-stress sensitive case.

The plotting functions defined in equations (18) to (21) are plotted in Figure 6 for the base case parameters and β* varying from 0 to 1. The blue curves represent equation (19), the red curves equation (20) and the black curves depict equation (21). In each set of curves the upper curve is for β* (i.e. the most extreme stress sensitivity). This family of curves implies that β * could be estimated from production data by considering the relationship between the normalised rates (equation 19) and the rate integral derivative function (equation 21).

Conclusions

This paper analyses the transient flow rate behaviour of stress-sensitive reservoirs and compares this behaviour to that of non-stress sensitive reservoirs. Production at constant well pressure was considered. With a constant pressure boundary outer condition results show that the early time data and late time data would both imply overall permeabilities that are much higher than the base permeability, k_0, if interpreted as if this data were from a non-stress sensitive case.

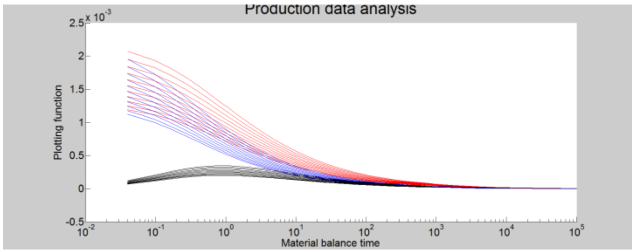

Figure 6. Production data analysis type curves for stress sensitive 1D reservoir with no flow outer boundary condition.

A new solution with a no-flow outer boundary condition was derived and presented in a type curve format. If other parameters can be fixed this type curve implies that β* star values could be extracted from production data (even in early time) by considering the separation of data plotted in the form of equations 19 and 20, from data plotted in the form of equation 21.

Extension to radial flow in polar coordinates would be attractive however a suitable analytical solution is not available. The Cole-Hopf transform could though be integrated with other numerical methods for this purpose.

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