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Sound generation by acceleration of density inhomogeneities and steady heat communication

N. Karimi[1], **M. J. Brear**[2]

[1]Institute of Reactive Flows and Diagnostics, Centre of Smart Interfaces
Technical University of Darmstadt, Petersenstr. 32, 64287 Darmstadt, Germany

[2]Department of Mechanical Engineering
University of Melbourne, Parkville, Victoria 3010, Australia

Abstract

This paper presents a theoretical and numerical study on the relative significance of two mechanisms of sound generation encountered in combusting flows. A non-diffusive, one-dimensional, steady heat communicating flow, subjected to downstream travelling acoustic excitations is considered. It is first shown numerically and analytically that even without unsteadiness in heat communication the investigated flow can generate sound. Existing theories of acoustic energy reveal that such generation of sound can be due to either of acceleration of entropy disturbances or interactions between acoustics and steady heat communication. An analytical scaling analysis is then put forward to estimate the relative significance of these sound generating mechanisms. The results are validated against the numerical simulations and include both high and low frequency limits. In particular, it is shown that at high forcing frequencies the sound generation is dominated by the steady heat communication.

Introduction

It has been known since Rayleigh [1] that unsteadiness in heat communication can generate sound. Later, Ffowcs Williams and Howe [2, 3] showed that acceleration of density inhomogeneities can be also a sound source. Since combusting flows are normally accelerating and feature heat addition unsteadiness, these two mechanisms are sometimes considered as the main sources of combustion noise [4, 5]. Nonetheless, Bloxidge et al. [6] showed theoretically that interaction of acoustics with steady heat communication can be also a sound source.

Recently Karimi et al. [7] investigated the acoustic and entropic fields of a simple one-dimensional heat communicating flow subjected to acoustic forcing. They showed analytically and numerically that even in the absence of unsteadiness in heat communication such a flow can generate sound for all forcing frequencies. However, the physical mechanisms responsible for this sound generation were not clear, since their investigated flow included both steady heat communication and convection of entropy disturbances by an accelerating mean flow. Importantly, the relative importance of each of these two mechanismss at varying forcing frequencies was unknown.

This paper therefore aims at estimating the relative significance of the two mechanisms of acceleration of density disturbances and steady heat communication in simple steady heat communicating flows. Fig. 1 shows schematically the problem under investigation. A non-diffusive, steady heat communicating flow, is excited by downstream travelling acoustic waves. This inhomogeneous flow features a linear increase (or decrease) in stagnation temperature and semi-infinite homogeneous regions. Thus, there is no sound reflection from these homogeneous ends while the inhomogeneous region can be reflective. The system is excited by an incident harmonic acoustic wave I with reflected and transmitted counterparts R and T. Interaction of

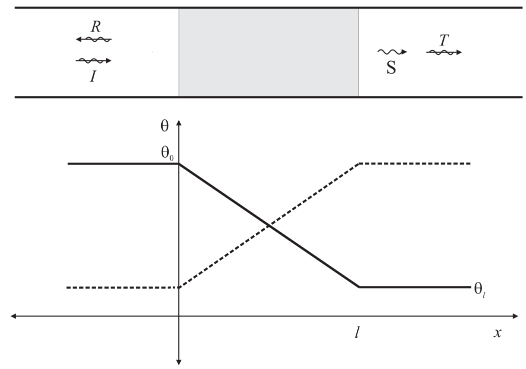

Figure 1: Schematic of the problem configuration, one dimensional accelerating flow, l=1m, heating case: θ_{t0}=600 K, θ_{tl}=1200 or 1800 K, cooling case: θ_{t0}=1200 K, θ_{tl}=600 K where θ_t is the stagnation temperature.

the acoustics with the heat communication flow also results in generation of an entropy wave S which convects downstream.

Theoretical and numerical methods

Equations of motion

Consider the one-dimensional Euler equations applied to a calorifically perfect, ideal gas,

$$\frac{\partial}{\partial t}(\rho)+\frac{\partial}{\partial x}(\rho u)=0, \qquad (1)$$

$$\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}\left(p+\rho u^2\right)=0, \qquad (2)$$

$$\frac{\partial}{\partial t}\left(\frac{p}{\gamma-1}+\rho u^2\right)+\frac{\partial}{\partial x}\left(\left[\frac{\gamma p}{\gamma-1}+\rho u^2\right]u\right)=q, \qquad (3)$$

where p (Pa), ρ $(\mathrm{kg/m^3})$, s (kJ/kg.K) and u (m/s) are respectively the static pressure, static density, entropy and flow velocity, q $(\mathrm{W/m^3})$ is the heat communication per unit volume and γ is the ratio of specific heats ($\gamma = 1.4$). Throughout this paper symbols $\overline{()}$ and $()'$ respectively refer to mean value and first order disturbance.

Acoustic energy

The acoustic energy balance for heat communicating flows was derived by Bloxsidge et al. [6] who extended Morfey's earlier work [8].

$$\frac{\partial(eA)}{\partial t}+\frac{\partial(EA)}{\partial x}=D, \qquad (4)$$

where

$$e=p'^2/2\bar{\rho}\bar{c}^2+\bar{\rho}u'^2/2+\bar{u}p'u'/\bar{c}^2, \qquad (5)$$

is the acoustic energy density $(\mathrm{J/m^3})$ and

$$E=p'u'+\bar{\rho}\bar{u}u'^2+\bar{u}^2p'u'/\bar{c}^2+\bar{u}p'^2/\bar{\rho}\bar{c}^2, \qquad (6)$$

is the acoustic energy flux (W/m^2). Bloxsidge et al. [6] expressed the acoustic energy source term D (W/m) as

$$D = \left(\bar{\rho}\bar{u}u' + p'\right)\frac{\bar{q}(\gamma-1)}{\gamma\bar{p}}\left(\frac{q'}{\bar{q}} - \frac{p'}{\bar{p}} - \frac{u'}{\bar{u}}\right) + \left(u' + \frac{\bar{u}p'}{\bar{\rho}\bar{c}^2}\right)\left[\left(\frac{\bar{\rho}s'}{c_p} - \frac{p'}{\bar{c}^2}\right)\bar{u}\frac{d\bar{u}}{dx} - \frac{p'}{\bar{\rho}}\frac{d\bar{\rho}}{dx}\right], \tag{7}$$

which can be rearranged in the following way

$$D = D_{uhc} + D_{shc} + D_{mfa}, \tag{8}$$

where

$$D_{uhc} = \frac{\bar{q}(\gamma-1)}{\gamma\bar{p}}\left(\bar{\rho}\bar{u}u' + p'\right)\frac{q'}{\bar{q}}, \tag{9}$$

$$\begin{aligned} D_{shc} = \quad & - \quad \frac{\bar{q}(\gamma-1)}{(\gamma)\,\bar{p}}\left(\bar{\rho}\bar{u}u' + p'\right)\left(\frac{p'}{\bar{p}} + \frac{u'}{\bar{u}}\right) \\ & - \quad \frac{1}{\bar{\rho}}\frac{d\bar{\rho}}{dx}p'\left(u' + \frac{\bar{u}p'}{\bar{\rho}\bar{c}^2}\right), \end{aligned} \tag{10}$$

$$D_{mfa} = -\bar{u}\frac{d\bar{u}}{dx}\left(u' + \frac{\bar{u}p'}{\bar{\rho}\bar{c}^2}\right)\rho'. \tag{11}$$

are respectively the sources terms due to unsteady heat communication, steady heat communication and mean flow acceleration. With the arrangement in Eqs. (8) to (11) we can consider two mechanisms which can generate sound in the absence of unsteady heat communication, i.e. $D_{uhc} = 0$. First D_{mfa} describes sound generation by the acceleration of entropy inhomogeneities. The remaining terms then only depend on the steady heat communication and so are grouped as D_{shc}. It is not clear how significant D_{shc} is compared to the well-established mechanism of acceleration of entropy inhomogeneities [2].

Numerical solver

The present work solves Eqs. (1) to (3) in conservation form by using the Dispersion-Relation-Preserving (DRP) scheme of Tam and Webb [9]. The specific DRP scheme chosen uses an optimised, four level, time marching scheme and seven point stencil for spatial differentiation. Further details and validation of the solver can be found in [7, 10].

Results and discussion

Sound generation in steady heat communicating flow

Consider the balance of acoustic energy as defined in Eqs. (4) to (6). The acoustic energy reflection and transmission coefficients are defined respectively as

$$\Sigma_R = \frac{(1-\bar{M}_0)^2}{(1+\bar{M}_0)^2}\left|\frac{R}{I}\right|^2, \quad \Sigma_T = \frac{\bar{\rho}_0\bar{c}_0}{\bar{\rho}_l\bar{c}_l}\frac{(1+\bar{M}_l)^2}{(1+\bar{M}_0)^2}\left|\frac{T}{I}\right|^2. \tag{12}$$

Figs. 2 and 3 show the numerically calculated reflection and transmission coefficients at finite Mach numbers for heated and cooled flows respectively. These figures also include the analytically developed acoustic energy reflection and transmission coefficients for zero mean flow case [7]. As can be seen, there is a good agreement between the low Mach number numerical simulations and the analytical calculations. However, as the inlet Mach number increases there is a deviation from the zero mean flow coefficients. This results in a net generation or dissipation of acoustic energy in cooled and heated flows respectively. Further simulations show that generation (or dissipation) of acoustic energy becomes more significant as the inlet Mach number and the overall temperature ratio increase [10].

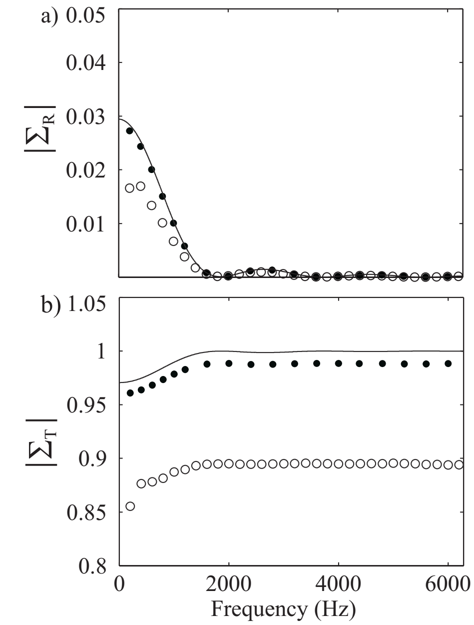

Figure 2: Amplitude of the acoustic energy a) reflection and b) transmission coefficients, heating case $\theta_l/\theta_0 = 2$. Solid lines: zero mean flow analytical result, dots: simulations for $\bar{M}_0$=0.01, circles: simulations for $\bar{M}_0$=0.1.

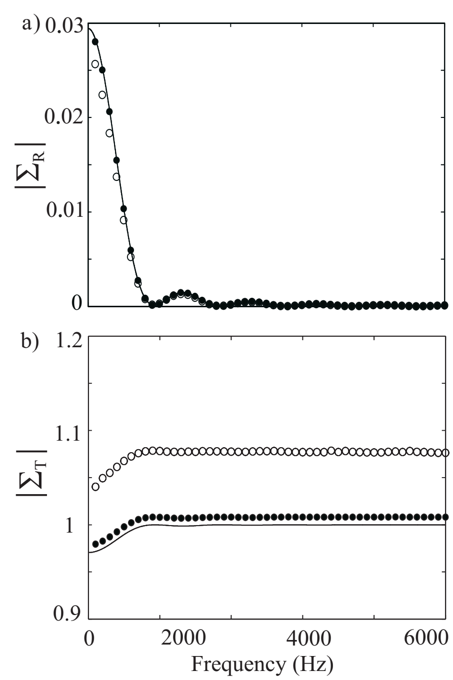

Figure 3: Amplitude of the acoustic energy a) reflection and b) transmission coefficients, cooling case $\theta_l/\theta_0 = 0.5$. Solid lines: zero mean flow analytical result, dots: simulations for $\bar{M}_0$=0.01, circles: simulations for $\bar{M}_0$=0.1.

Scaling analysis

In the present problem the unsteadiness in heat communication, q' has been set to zero, i.e. $D_{uhc}=0$. Thus, sound generation observed in Figs. 2 and 3 is due to acceleration of entropy disturbances by the mean flow D_{mfa}, or mean heat communication D_{shc}. This section presents an approximate comparison of the source terms in Eq. (8) associated with the steady heat communication and mean flow acceleration at high frequencies. The results are applicable to low mean Mach numbers only.

Equations (1) and (2) can be linearised and combined to give

$$\frac{\partial p'}{\partial x}+\bar{\rho}\bar{u}\frac{\partial u'}{\partial x}+\left(\bar{\rho}u'+\bar{u}\rho'\right)\frac{d\bar{u}}{dx}+\bar{\rho}\frac{\partial u'}{\partial t}=0. \qquad (13)$$

By expressing density disturbances as entropy and pressure disturbances, see [7], Eq. (13) can be expressed as

$$\bar{\rho}\frac{\partial u'}{\partial t}+\frac{\partial p'}{\partial x}+\bar{\rho}\bar{u}\frac{\partial u'}{\partial x}-\left(\frac{\bar{\rho}\bar{u}}{c_p}s'-\bar{\rho}u'-\frac{\bar{M}}{\bar{c}}p'\right)\frac{d\bar{u}}{dx}=0. \qquad (14)$$

In the limit of infinite forcing frequency it has been shown in reference [7] that $|s'|\rightarrow 0$ in all cases studied here. Consider high forcing frequencies satisfying $\lambda_a \ll l$ where λ_a is the acoustic wavelength. It can be then argued that

$$\frac{\partial(\;\;)}{\partial x}\simeq\frac{(\;\;)}{\lambda_a}\simeq\frac{\omega}{\bar{c}}(\;\;), \quad \frac{\partial(\;\;)}{\partial t}\simeq\omega(\;\;). \qquad (15)$$

Further, for mean quantities

$$\frac{d\overline{(\;\;)}}{dx}\simeq\frac{\Delta\overline{(\;\;)}}{l}, \qquad (16)$$

Substituting expressions (15) and (16) into (14) then results in

$$p'\simeq\frac{\bar{\rho}\bar{c}[(\bar{M}+1)\,\omega+\Delta\bar{u}/l]}{\omega+M\Delta\bar{u}/l}u'+\frac{\bar{\rho}\bar{c}\bar{u}\Delta u/c_p l}{\omega+M\Delta\bar{u}/l}s'. \qquad (17)$$

It is clear from Relation (17) that the second term on the right hand side approaches zero at infinite frequency. In this limit Relation (17) then simplifies at low Mach numbers to

$$\frac{p'}{\bar{p}}\simeq\gamma\bar{M}\frac{u'}{\bar{u}}. \qquad (18)$$

Substitution of Relation (18) into Eq. (25) reveals after some algebra that

$$D_{mfa}\simeq-\frac{A\Delta\bar{u}\bar{\rho}}{l}\bar{M}u'^2. \qquad (19)$$

Similarly, assuming a low Mach number mean flow, relation (25) becomes approximately

$$D_{shc}\simeq-\gamma\frac{A\Delta\bar{\theta}}{\bar{\theta}l}\bar{\rho}\bar{c}\bar{M}u'^2. \qquad (20)$$

Further, in the low Mach number limit, Eq. (2) indicates that the mean pressure can be assumed constant. Therefore it follows from the ideal gas law that $\bar{\rho}d\bar{\rho}/d\bar{x}\simeq-\bar{\theta}d\bar{\theta}/d\bar{x}$. Combining this with continuity of mass and using relation (15) gives

$$\frac{\Delta\bar{u}}{\bar{u}}\simeq\frac{\Delta\bar{\theta}}{\bar{\theta}} \qquad (21)$$

Applying relation (21) to relations (19) and (20) then results in

$$\left|\frac{D_{shc}}{D_{mfa}}\right|\simeq\frac{\gamma}{\bar{M}}, \qquad (22)$$

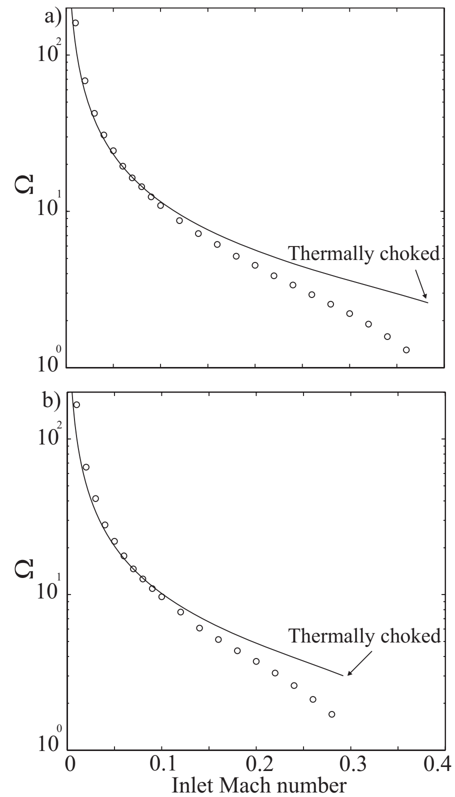

Figure 4: Comparison between the analytical scaling and numerical simulation of Ω at high frequency for a) stagnation temperature ratio of 2 and b) stagnation temperature ratio of 3.

which should hold at any given streamwise position in the inhomogeneous region.

Fig. 4 presents the spatially averaged value of the right hand side of Relation (22), $1/l\int_0^l \gamma/\bar{M}dx$. It also compares this theoretical result with $\Omega=1/l\int_0^l \overline{D_{shc}}/\overline{D_{mfa}}dx$ from the simulations. The forcing frequency in each numerical simulation was chosen such that $\lambda_a/l<0.2$ always. Further, entropy generation was negligible at these frequencies. As can be seen in Fig. 4, theory and simulation are in very good agreement over the lower Mach number region, as was assumed in deriving the theoretical results. It follows that at low Mach numbers and high forcing frequencies, the dominant source terms involve steady heat communication, and the effect of the source terms related to the acceleration of the mean flow is small. However, at close to sonic conditions the two sets of source terms have comparable strength.

As $\omega\rightarrow 0$ the generation of entropy within the heat communicating region is significant [7]. Acceleration of these entropy disturbances then generates sound though the D_{mfa} source term and also greatly complicates the scaling analysis. Further, the numerical simulation takes longer to converge as $\omega\rightarrow 0$. Therefore, in this paper we only present numerical results for Ω at frequencies such that λ_c/l=2 always, where λ_c is the convective wavelength. It is noted that further decreases in the forcing frequency does not change the result significantly. Fig. 5 shows that in the low frequency limit the two sets of source terms D_{mfa} and D_{shc} have similar strength.

Alternative arrangement of the source terms

Equation (8) can be rewritten in the following form

$$D=D_{uhc}+D^*_{shc}+D^*_{mfa}, \qquad (23)$$

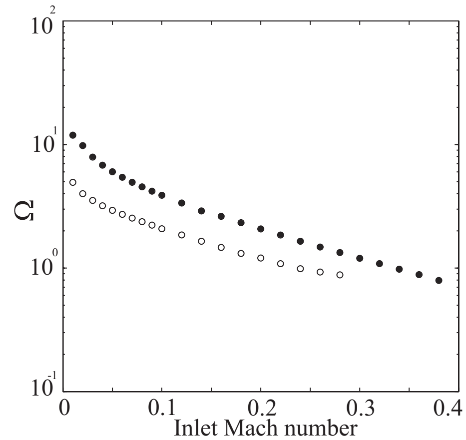

Figure 5: Values of Ω at low forcing frequency, calculated by numerical simulation. Dots and circles: stagnation temperature ratios of 2 and 3 respectively.

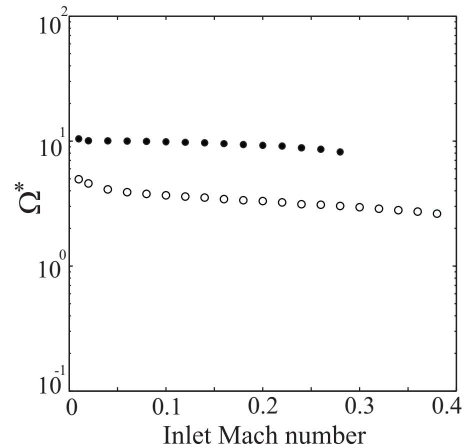

Figure 6: Calculated value of Ω for stagnation temperature ratios of 2 (circles) and 3 (dots).

where

$$D^*_{shc} = -\frac{\bar{q}A(\gamma-1)}{\gamma\bar{p}} \times \left[(\bar{\rho}\bar{u}u' + p')\left(\frac{p'}{\bar{p}} + \frac{u'}{\bar{u}}\right) - p'\left(\frac{u'}{\bar{u}} + \frac{p'}{\bar{\rho}\bar{c}^2}\right)\right], \quad (24)$$

$$D^*_{mfa} = A\bar{u}\frac{\mathrm{d}\bar{u}}{\mathrm{d}x}\left(u' + \frac{\bar{u}p'}{\bar{\rho}\bar{c}^2}\right)\frac{\bar{\rho}s'}{c_p}. \quad (25)$$

Karimi et al. [7] showed that $|s'| \to 0$ as $\omega \to \infty$. Thus, when $\lambda_c/l \ll 1$ and under acoustic excitation there are negligible entropy disturbances in any steady heat communicating flow. It follows that in this limit $D_{mfa} \to 0$ and the *only* existing sound generating mechanism is that by steady heat communication D^*_{shc}.

At the limit of zero frequency the generation of entropy is significant [7]. Thus, D^*_{mfa} is expected to be large at this limit. To compare the relative significance of D^*_{mfa} and D^*_{shc} at low frequencies the parameter Ω^* is defined as $\Omega^* = 1/l\int_0^l \overline{D^*_{shc}}/\overline{D^*_{mfa}}\mathrm{d}x$. Fig. 6 shows the values of Ω^* calculated for a range of inlet Mach numbers such that $\lambda_c/l = 2$ always. At low frequencies the two source terms D^*_{mfa} and D^*_{shc} have comparable strength.

It should be noted that sound generation by acceleration of the mean flow is characterised in terms of either entropy disturbances or density disturbances [2]. When the acoustic and entropic fields are decoupled the acoustic part of the density inhomogeneity $\rho' = p'/\bar{c}^2 - \bar{\rho}s'/c_p$ does not contribute to the generation of sound, and therefore the two mechanisms are equivalent. However, the entropic and acoustic fields are coupled in heat communicating flows.

Conclusions

A simple, one-dimensional, non-diffusive, heat-communicating flow under acoustic excitation was considered. It was shown theoretically and numerically that such a flow can generate sound, even though there is no unsteadiness in heat communication. Based on existing acoustic energy equations, it was argued that this sound generation can be due to acceleration of density/entropy disturbances by the mean flow or mean heat communication. A scaling analysis was then put-forward to characterise the relative significance of these two mechanisms at different forcing frequencies. It was shown that considering either density or entropy disturbance acceleration, at high frequencies the sound generation is dominated by the steady heat communication. However, at low frequencies and for the mean temperature ratios investigated in this paper, the strength of the two mechanisms was comparable.

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