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Progress of Laminar-Turbulent Transition in a Two-Dimensional Mixing Layer (Quantitative Representation of Transition Process)

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Abstract

Effects of oscillating plates on the plane mixing layer, its developing region and jet were experimentally investigated. The flow was formed by the jet issued from a two-dimensional nozzle and surrounding quiescent air. Two plates oscillate perpendicularly in relation to the main flow. Mean and fluctuating velocity components were measured by hot-wire anemometers. With the oscillation disturbance of the plate, self-preservation was promoted and the width of the layer was enhanced. The transition process with the disturbance was enhanced by an interaction of the inherent periodic variation and the disturbance induced by the plate.

Introduction

A jet is a fundamental flow that can be seen widely in the natural world and industrial machinery. Therefore, it is important to understand its characteristics and properties. Many studies have been published on it over the years, and a great number of them introduced various kinds of disturbance in laminar jet, inducing the transition to turbulence.

In free shear flows such as a jet, in the transition process a periodic disturbance appears at first, followed by the harmonics and subharmonics, and finally an irregular disturbance dominates farther downstream. In this way, in power spectrum distribution, a large peak appears, then many peaks gradually appear, and finally disappear. The respective processes include a linear region when a disturbance amplifies exponentially, a nonlinear region where harmonics and subharmonics of the fundamental wave appear, and an irregular region where irregular fluctuation dominates, respectively. Exponentially increasing velocity fluctuation in space [8] and time [9] was analyzed by Michalke. Freymuth confirmed this analysis experimentally [2]. Due to the sensitivity of spatially-developing shear flow to the outer disturbance, outer forced disturbance was added to jets or mixing layers. Then many studies were conducted in which the disturbance increased instability or promoted transition.

In the study of forced transition, experiments have been performed to introduce a periodic disturbance with a frequency approximately equal to the fundamental frequency occurring in the natural transition process. An additional style of the disturbance is roughly classified into two; one in which a speaker produces a sound wave disturbance [1,2,5-7,10,12,13] and another in which a flap [3] or piezo film actuator [11] is oscillated. A successful method was introduced by which a forced disturbance whose frequency was approximately the same as the fundamental frequency of the flow promoted a turbulent transition. But few experiments have been conducted in which the disturbance frequency was considerably different from the fundamental one. Therefore, in the present study, a disturbance of 5 Hz is introduced two orders of magnitude lower than the fundamental frequency. To produce such a low-frequency disturbance, an oscillating device is more appropriate than a sound wave. Additionally, the oscillating device can add a local disturbance, although the sound induces instability in the whole field.

A method to measure the progress of the transition process for the mixing layer remains to be discovered. On the other hand, for the wake, Sato and Saito proposed the randomness factor as the measure [14]. It describes the degree of randomness of the velocity fluctuation, which is defined as the ratio of the energy contained in the continuous spectrum to the total energy. We used it for the present transition process of the mixing layer and discussed its validity.

Experimental Apparatus and Methods

A wind tunnel of blowing type was used in which air is blown into the measurement section from a two-dimensional nozzle exit of aspect ratio 31 (310 mm in width and 10 mm in height). The flow at the exit has a velocity gradient within 1.6 mm from the upper and lower nozzle walls, respectively, while the velocity is kept constant in the middle region of the height (7 mm). Two side walls 310 mm apart for a whole measurement section were installed to secure the two-dimensionality of the flow.

Two oscillation plates 2 mm in thickness were then equipped across the whole width to produce a disturbance. The plates oscillate sinusoidally in relation to the flow at a frequency, f_e, of 5 Hz. This frequency is two orders of magnitude smaller than the fundamental frequency in the natural transition process, which is, approximately, 500 Hz. The flow field and coordinate system are shown in Figure 1. The upper half of the flow field is shown. The bottom of the oscillating plate becomes flush with the nozzle wall surface when it rises to its highest point, and then descends by 0.25 mm from the surface at most. This value was chosen so that no additional disturbance occurred. The Strouhal number based on this value, plate oscillation frequency, f_e, and nozzle exit velocity, U_0, is about 1.7×10^{-4}. In addition, another oscillating plate is installed in the lower half of the flow field symmetrical with the nozzle centerline, y = 0. The plate oscillates symmetrically with the former plate. In other words, the nozzle height 10 mm is decreased 0.5 mm at the moment when the two plates protrude maximally from the nozzle respective surfaces. An rpm-controlled motor was used for the plate

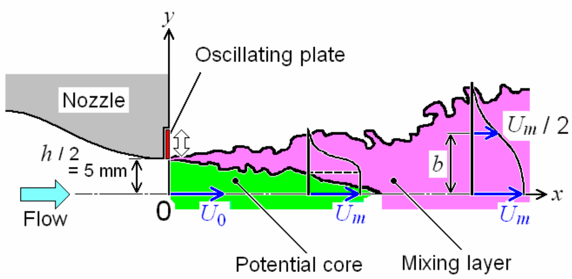

Figure 1. Schematic diagram of two-dimensional mixing layer and coordinate system.

oscillation. The revolution is transmitted to two cams by belts, and then the revolution of each cam is transmitted to the protrusion of each oscillating plate. The noise of the motor and cams were checked and confirmed not to affect the flow transition. A photo sensor signal corresponding to the angle of rotation of this cam is output, and the phase of the plate oscillation is detected with it. Two types of experiments were performed. In one, the plates remain stationary so that the plates do not narrow the nozzle exit section (stationary state). In another, the plate oscillated at a frequency of 5 Hz (oscillating state). In any case, the Reynolds number based on the nozzle exit velocity, $U_0 \simeq 7.5$ m/s, and nozzle exit height, h, was 5000.

X-shaped hot-wire probes with two tungsten sensing elements, each 5 μm in diameter and 1 mm in length, were used for the measurements. Output voltage was sampled at a sampling frequency of 5 kHz for about 52 seconds. This interval is equivalent to about 260 oscillations when the plates oscillate. The measurements were conducted in a range of $y \geqq 0$.

Results and Discussion

Effect of Disturbance on Fluctuating Velocity

First, we describe the effects of the disturbance from oscillating plates on the fluctuating velocity.

Figure 2 shows distributions of rms value of the fluctuating velocity in the streamwise component. The normal position, y, is normalized by half of the nozzle exit height, $h/2$, and rms value of the fluctuating velocity, u', is normalized by the local centerline velocity, U_m. Just after the nozzle (in the region where x/h is small), the fluctuating velocities are low in both cases. Downstream, the values increase and the position $y/(h/2)$ at which the value reaches maximum almost coincides with the position where mean velocity gradient $\partial U/\partial y$ becomes maximum in the mean velocity profiles (not shown here). In the region where the potential core exists (stationary state, $x/h \leqq 6$; oscillating state, $x/h \leqq 3$), the value reaches maximum at $y/(h/2) = 1$ since the region where $\partial U/\partial y$ exists is limited to the very narrow mixing layer near the tip of the nozzle. As the potential core becomes narrower farther downstream, the position where $\partial U/\partial y$ becomes maximum so the rms value reaches maximum shifts away from the center, $y/(h/2) = 0$. In the oscillating state, this department from the center side occurs farther upstream than in the stationary state. In addition, values in the oscillating state are larger than in the stationary state just after the nozzle. Hence, it is found that the phenomena resulting in the natural transition process appear farther upstream due to the periodic

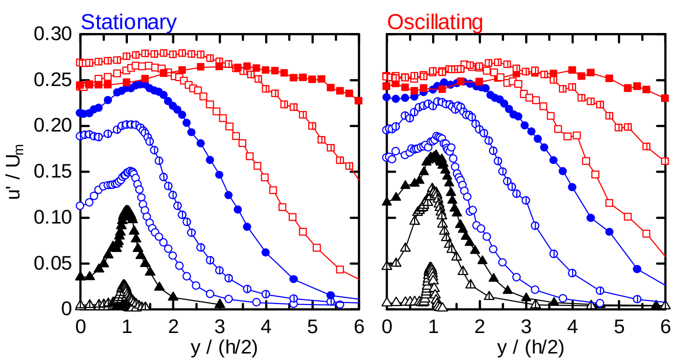

Figure 2. Distributions of streamwise fluctuating velocity: △, x/h = 0.5; ⧍, x/h = 2; ▲, x/h = 3; ○, x/h = 4; ⦶, x/h = 6; ●, x/h = 8; □, x/h = 10; ◫, x/h = 14; ■, x/h = 20.

disturbance by the oscillating plates. Furthermore, for $2.5 \leqq x/h \leqq 4$ in the stationary state and for $1 \leqq x/h \leqq 3$ in the oscillating state, there is another smaller maximum on the smaller side of $y/(h/2)$. This second maximum will be discussed in the last section.

Progress of Laminar-Turbulent Transition

In this section, the progress of the laminar-turbulent transition will be discussed from the spectral point of view.

Figure 3 shows the power spectrum distribution for the fluctuating velocity in the streamwise component at the position y where u' reaches maximum in each streamwise station. In the stationary state, just after the nozzle the spectrum at the fundamental frequency, f_0 = 400 ~ 500 Hz, indicates the peak value. Farther downstream the harmonics and subharmonics stand out due to the nonlinear interaction. Finally, the distribution turns into the continuous spectrum. Thus, the turbulent transition process typical of jet flow can be confirmed. In the oscillating state, just after the nozzle peaks at the plate oscillating frequency, f_e = 5 Hz, the subharmonic one, $f_e/2$, and the fundamental frequency also stands out. This plate oscillation promotes the transition into the continuous spectrum.

We also obtained the randomness factor as a measure of the progress of the laminar-turbulent transition. It was calculated as a ratio of the continuous spectrum area to the total area in the power spectrum distribution. Figure 4 shows the streamwise variation of the randomness factor. The factor was obtained from the power spectrum distribution at the position y where u' reaches maximum in each streamwise station as shown in Figure 3. In the stationary state, just after the nozzle, the randomness factor reaches approximately unity as there is no periodic variation. The values decrease downstream and reach minimum at the end of the linear region (stationary state, x/h = 3; oscillating state, x/h = 1.5) because of the appearance and amplification of periodic variations. Farther downstream, the values tend to increase due to the fact that the periodic variations attenuate and part of the continuous spectrum recovered. The values reach nearly unity at $x/h \simeq 16$ in the stationary state and $x/h \simeq 10$ in the oscillating state. The turbulent transition can be assumed to be completed in these stations. Thus, in the latter process, the factor is effective as a quantitative indicator of the progress of the transition process. However, Sato and Saito [14] had not reported the fact that in the former process the randomness factor decreases until the periodic variations appears in the laminar flow. Although the intermittency factor that is used for the wall boundary layer increases monotonically from 0 to 1 with the progress of transition, the randomness factor, as mentioned above, first decreases, then increases and reaches unity again. Consequently, there is an inconvenience that the randomness factor indicates the same value at two stramwise positions.

Next, in order to visualize the turbulent transition, Figure 5 shows the isocontour lines of the randomness factor in the x-y plane. Just after the nozzle, the region where $y/(h/2)$ is large was not measured. It is found that the randomness factor is not constant in y-direction as well as x-direction. In the stationary state, within the linear region $x/h \leqq 3$, the randomness factor reaches minimum at $y/(h/2) \simeq 1$ where the fluctuating velocity reaches maximum. Therefore, it is found that the periodic variation contributes to the increase of the fluctuating velocity. The minimum value within the whole region lies within the nonlinear region $4 \leqq x/h \leqq 5$ along the centerline, $y/(h/2) = 0$ where little of the potential core remains. The fact that the randomness factor does not reach minimum at $y/(h/2) \simeq 1$, where the fluctuating velocity reaches maximum, implies that the

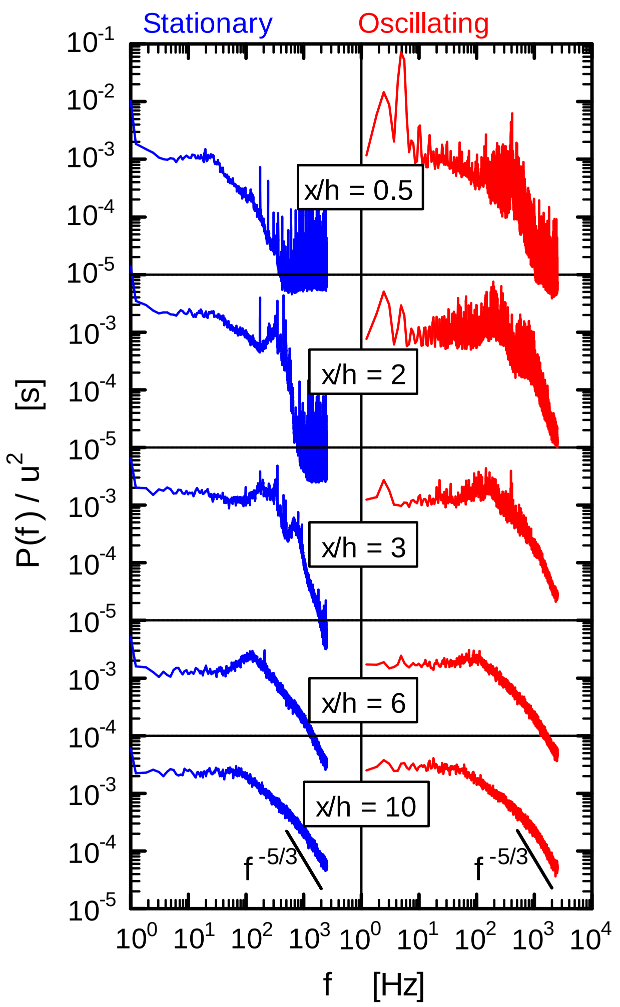

Figure 3. Power spectrum distribution for the fluctuating velocity in the streamwise component.

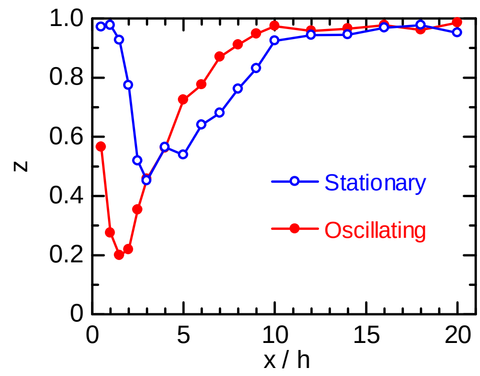

Figure 4. Streamwise variation of randomness factor.

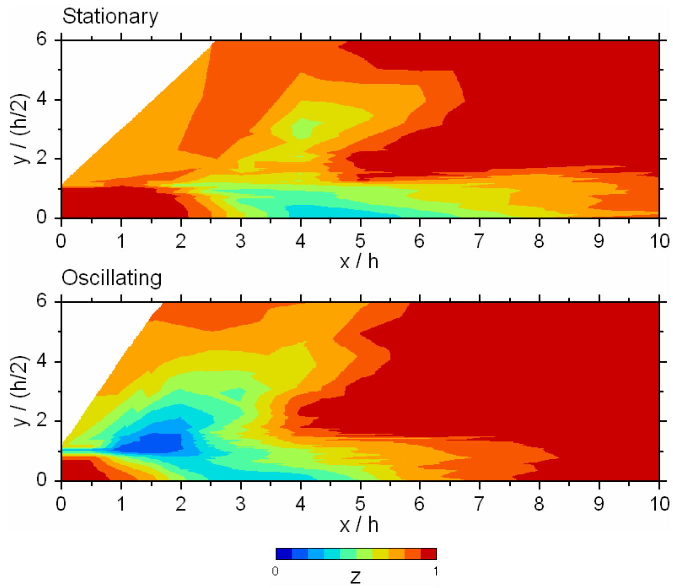

Figure 5. Isocontour areas of randomness factor.

increase in the fluctuating velocity within the nonlinear region is affected more by the mean velocity gradient $\partial U/\partial y$ than the periodic variation. In the oscillating state, within the linear region $x/h \leqq 1.5$, the randomness factor reaches minimum at $y/(h/2) \simeq 1$ where the fluctuating velocity reaches maximum. The minimum value within the whole region lies in $1 \leqq x/h \leqq 2$ along $y/(h/2) \simeq 1$, within the nonlinear region. In the nonlinear region the value along the centerline, $y/(h/2) = 0$, is smaller than the one along $y/(h/2) \simeq 1$. This is the same as in the stationary state, where the nonlinear region of the minimum value is along the centerline, $y/(h/2) = 0$. Hence, at the streamwise positions in which the potential core barely remains, the randomness factor indicates small values in the potential core also in the oscillating state. Just as in Figure 2, it is seen from Figure 4 and 5, that the laminar-turbulent transition is promoted by the disturbance induced by the oscillating plate.

Relation between Periodicity and Fluctuation

Finally, the relationship between the two maximums in the fluctuating velocity distribution and the periodic variation is examined.

The distributions of the fluctuating velocity and the randomness factor are shown in Figure 6. In Fig. 6(a) and Fig. 6(c), the power spectra for the fundamental frequency, f_0, and the subharmonic frequency, $f_0/2$, and particularly in Fig. 6(c) for the plate oscillating frequency, f_e, are also plotted. In the stationary state, Fig. 6(a) shows the result at $x/h = 2.5$ where the second maximum of the fluctuating velocity appears on the smaller side of $y/(h/2)$. At $y/(h/2) \simeq 0.95$, where the fluctuating velocity reaches maximum, the randomness factor reaches minimum and the power spectrum for the subharmonic frequency, $f_0/2$, is large. On the other hand, at $y/(h/2) \simeq 0.7$, where the fluctuating velocity reaches the second maximum, the randomness factor reaches the local minimum and the power spectrum for the fundamental frequency, f_0, is large. Hence, the fluctuating velocity becomes large at positions where the fundamental or subharmonic wave becomes dominant, so the periodic variation contributes to the two maximums of the fluctuating velocity. Fig. 6(b) shows the result at $x/h = 6$, within the nonlinear region where the regular variations largely disappear. The second maximum of the fluctuating velocity on the center side of the nozzle disappears. So the fluctuating velocity reaches maximum at only one position, $y/(h/2) \simeq 1$, where the randomness factor does not become low. Thus, the periodic variations no longer contribute to the maximum of the fluctuations in the nonlinear region.

In the oscillating state, Fig. 6(c) shows the result at $x/h = 1.5$ where the second maximum of the fluctuating velocity appears at $y/(h/2) \simeq 0.7$. At this position, although the randomness factor does not reach minimum, the power spectrum for the fundamental frequency, f_0, indicates a small maximum. At $y/(h/2) \simeq 0.95$, where the

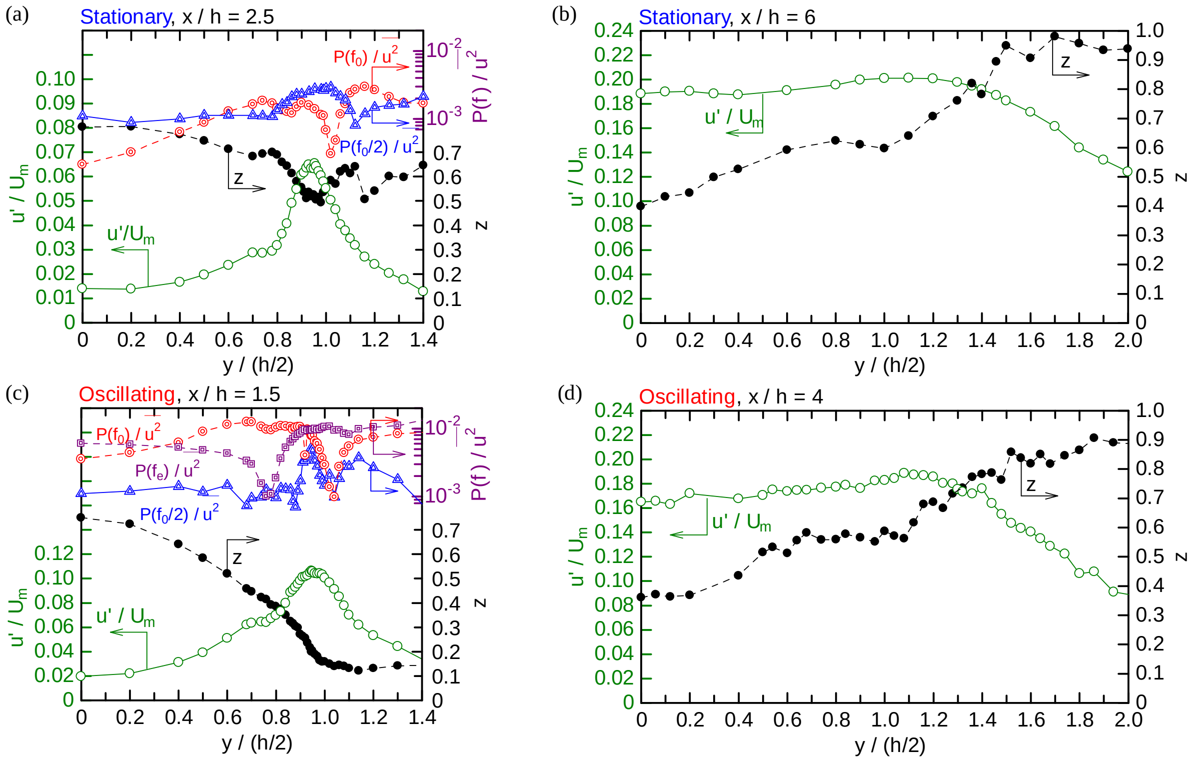

Figure 6. Distributions of fluctuating velocity, randomness factor and power spectrum: (a) and (b), stationary; (c) and (d), oscillating.

fluctuating velocity reaches maximum, the randomness factor is low and the power spectra for subharmonic frequency, $f_0/2$, and plate oscillating frequency, f_e, are large. Hence, in the oscillating state, periodic variation of not only harmonic and subharmonic frequencies but also the one induced by plate oscillation contributes to two maximums of the fluctuating velocity in the linear region. Fig. 6(d) shows the result at x/h = 4, within the nonlinear region where the regular variations largely disappear. The second maximum of the fluctuating velocity on the center side of the nozzle disappears. So the fluctuating velocity reaches maximum at only one position, $y/(h/2)$ = 1.1, where the randomness factor does not become low. Therefore, just in the stationary state, the periodic variations no longer contribute to the maximum fluctuations in the nonlinear region.

Conclusions

In the oscillating state, the phenomena that occur in the natural transition process appear farther upstream so that the laminar-turbulent transition is promoted by the disturbance induced by the oscillating plate.

In the linear region, periodic variation contributes to the maximums of the fluctuating velocity, whereas in the nonlinear region, the periodic variation is attenuated and does not contribute to the maximum of fluctuating velocity.

As an indicator of the progress of the transition process, the randomness factor proposed by Sato and Saito [14] is applied to the present study. In the process in which the periodic variation in velocity turns into an irregular variation, the randomness factor increases monotonically, and is regarded as a quantitative indicator of the progress of transition process. On the other hand, farther upstream, in the process in which the periodic variations appear in the laminar flow, the randomness factor decreases monotonically. Thus, the fact that the randomness factor indicates the same value at two streamwise positions makes for difficult handling.

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