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Numerical study of density ratio effects and compressibility of gas phase in sloshing

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Abstract

In this paper, we apply our two-phase Smoothed Particle Hydrodynamics (SPH) method for numerically simulating the sloshing phenomenon with varying ratio of densities between the liquid and gas. This method has recently been developed and validated on several benchmark problems. Here, we consider a closed rectangular tank subject to translational oscillation along its longitudinal axis. Simulations are conducted for water sloshing in an atmosphere of air, and with denser gas $SF_6 + N_2$. Global and local features of the flow are analyzed to show the effect of density ratio in numerically simulating the sloshing phenomenon.

Introduction

Sloshing is the well-known motion of a fluid in a partially filled tank due to tank excitation. The resonant condition in sloshing, which occurs when the frequency of tank motion is close to the natural frequency of the fluid, may cause large structural loads on the tank frame. This makes it crucial to identify the behaviour of the fluid flow during sloshing. In the past five decades, sloshing motion has been investigated by many researchers, using various techniques. Initial studies were based on mechanical models of the phenomenon by adjusting terms in the harmonic equation of motion [6, 9] when time–efficient and reasonably accurate results were deemed sufficient [1]. Some other researchers solved the potential flow problem with very complicated treatment of free–surface boundary conditions [4]. Although the method is accurate for applications with simple geometry, it cannot handle breaking waves and tanks with baffles. Besides the aforementioned techniques, numerical simulations are the most important and widely adopted technique for dealing with highly non–linear problems. Frandsen [5] used the Finite Difference method for solving the non–linear potential flow in a 2D tank. Celebi and Akyildiz [3] used the finite difference method along with Volume of Fluid technique (VoF) for tracking the free–surface. Sames et al. [15], applied a commercial VoF technique to rectangular and cylindrical tanks.

Generally in numerical studies the gas phase is neglected and flow is simulated as one phase. However, when sloshing is severe or when the filling depth of liquid is high, the gas phase can significantly influence the flow and consequently the impact pressure on the tank frames. Considering the physics of the flow, this would be expected, although modeling this effect theoretically or numerically might involve significant difficulties. In experimental studies, researchers have used gases of different densities to investigate the influence of the ambient medium. Maillard and Brosset [10] have used a combination of various gases to obtain different gas densities while using water as the sloshing liquid. A pressure reduction of $30\% - 50\%$ were observed in their results [10].

In this paper, we apply the recently developed two-phase Smoothed Particle Hydrodynamic (SPH) algorithm [14] for numerically simulating the sloshing phenomenon with different ratio of densities between the liquid and gas. We consider a closed rectangular tank subject to translational oscillation along its longitudinal axis. Simulations were conducted for water sloshing in an atmosphere of air, and with denser gases (such as $SF_6 + N_2$). Global and local features of the flow were analyzed to show the effect of density ratio on numerical simulations of sloshing.

The SPH method

The SPH method is based on the interpolation theory. The method allows any function to be expressed in terms of its values at a set of disordered points representing particle positions using a kernel function. The kernel function refers to a weighting function and specifies the contribution of a typical field variable, $A(r)$, at a certain position, r, in space. The SPH approximation of $A(r)$ is defined as [11, 12]

$$A(\mathbf{r}) = \sum_{j=1}^{N} \frac{m_j}{\rho(\mathbf{r}_j)} A(\mathbf{r}_j) W(\mathbf{r} - \mathbf{r}_j, h) \tag{1}$$

where the index j denotes the particle label and particle j carries a mass m_j at position $\mathbf{r}_j$ and density ρ_j. The summation is over particles which lie within a circle of radius $2h$ centered at $\mathbf{r}$, where the smoothing length h represents the effective width of the kernel. When the fluids have a density ratio > 2, the following form of SPH continuity equation

$$\left[\frac{d\rho}{dt}\right]_i = \rho_i \sum \frac{m_j}{\rho_j} \left(\mathbf{u}_i - \mathbf{u}_j\right) \cdot \nabla_i W_{ij}. \tag{2}$$

is more accurate [13]. If this equation is used as a constraint on the SPH lagrangian [13, 14], the Euler equation becomes

$$\left[\frac{d\mathbf{u}}{dt}\right]_i = -\sum m_j \left(\frac{P_i + P_j}{\rho_i \rho_j}\right) \nabla_i W_{ij} \tag{3}$$

where P, m and ρ are pressure, mass and density, respectively. Following Grenier *et al.* [7] we include a repulsive term, R_{ij} to improve stability

$$R_{ij} = \varepsilon \left| \frac{\rho_{0i} - \rho_{0j}}{\rho_{0i} + \rho_{0j}} \frac{P_i + P_j}{\rho_i \rho_j} \right|, \tag{4}$$

To model the Navier–Stokes equation a viscous term Π_{ij} is added to Euler equation which becomes

$$\Pi_{ij} = -\frac{16}{\rho_i \rho_j} \frac{\mu_i \mu_j}{\mu_i + \mu_j} \frac{\mathbf{u}_{ij} \cdot \mathbf{r}_{ij}}{\mathbf{r}_{ij}^2 + 0.01 h^2} \tag{5}$$

where μ is the dynamic viscosity of the fluids. The Navier–Stokes equation is then written as

$$\left[\frac{d\mathbf{u}}{dt}\right]_i = -\sum m_j \left(\frac{P_i + P_j}{\rho_i \rho_j} + \Pi_{ij} + R_{ij}\right) \nabla_i W_{ij}. \tag{6}$$

Note that R_{ij} is only non–zero for interactions between particles from different fluids. It therefore only acts on an interface layer which is ≈ 4 particle spacings thick. Gerenier *et. al*, chose ε to be 0.08 in Eq. 4, but we found it possible to simulate the system with $\varepsilon = 0.01$. we have not explored the effect of different values of ε.

Since the quasi–incompressible form of SPH is adopted in this work, the pressure is evaluated through the following equation [2]

$$P_i = \frac{\rho_{0i} {c_i}^2}{\gamma} \left((\frac{\rho_i}{\rho_{0i}})^\gamma - 1 \right) \tag{7}$$

where γ is the polytropic constant. Table 1 shows the properties of the fluids used in this work. To ensure incompressibility in

Fluid	ρ $[kg/m^3]$	γ
Water	1000	7
Air	1	1.4
$SF_6 + N_2$	4	1.13

Table 1: characteristics of fluids

the liquid, the speed of sound should be $c_{liquid} = 10V$ where V is the maximum velocity inside the flow. This assumption ensures that the Mach number is sufficiently small to approximate a constant density fluid. It should be taken into consideration that this speed of sound is typically much smaller than the real speed of sound inside the liquid to allow the use of larger time–steps. However, it is found that for stability purpose the speed of sound inside the gas phase should be closer to the real speed of sound of the gas (typically ≈ 3 times larger than c_{liquid}), which leads to a larger value for the gas phase than the liquid phase in the numerical simulations. Grenier *et al.* [7] choose the ratio of the speed of sound of the low density fluids $c_{\ell d}$ to that of the high density fluid c_{hd} according to $\sqrt{\gamma_{ld}\rho_{hd}/(\gamma_{hd}\rho_{\ell d})}$. For the present problem this means that for the air water combination, the speed of sound for the air is approximately 14 times that of the water.

SPH Simulations

Figure 1 shows the schematic of the problem. In all SPH calculations, the particles were placed on a grid of squares with initial spacing of $l_0 = 6 \times 10^{-3} m$. The smoothing length was set to $h = 1.5 l_0$. The maximum speed inside the flow used for estimation of the speed of sound is considered to be

$$V_{max} = \sqrt{gD} \tag{8}$$

This choice of V_{max} is to ensure that the pressure inside the fluid can hold the column of liquid with height D. The simulations were run at the Reynolds number of $Re = 1000$, where $Re = \rho_{liquid}(2g)^{1/2} D^{3/2}/\mu_{liquid}$. The viscosity ratio was set to $\mu_{gas}/\mu_{liquid} = 0.01$. The kernel used in the simulations is one of the Wendland kernels [16], and is given by

$$W(r,h) = W_0 \times \begin{cases} (1+2s)(2-s)^4 & 0 \le s \le 2 \\ 0 & 2 \le s \end{cases} \tag{9}$$

where $s = |\mathbf{r}|/h$, and in two-dimensional problems the normalization factor takes the value $W_0 = 7/(64\pi h^2)$. The impact pressure was measured at three locations on the walls, (see Fig. 1). These locations will be referred throughout the text as $P1$ on the top right corner of the right, $P2$ and $P3$ the first and second point on the top wall from right, respectively.

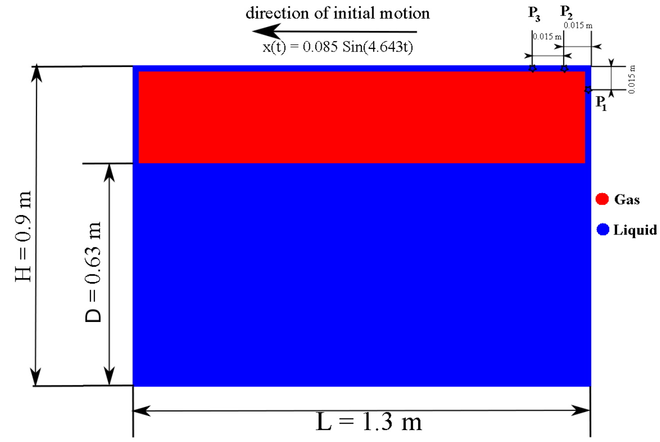

Figure 1: Geometry of the problem.

Figure 2 shows the shape of the interface for both air–water and $SF_6 + N_2$–water combinations. The flow pattern is generally similar in both simulations, however slight differences in the formation of sprays and splashing of water jet can be observed. In order to compare two cases more accurately, the time history of impact pressure is illustrated in Fig. 3 for the three sensors from both cases. It can be seen that the impact pressure in the air–water simulation is larger than the $SF_6 + N_2$–water simulation by a factor of ≈ 3. This is associated to the larger density of $SF_6 + N_2$ than density of air. The effect of the entrapped gas during the impact can also be seen in terms of oscillations of the pressure in the decay time. The oscillations are more pronounced in the $SF_6 + N_2$–water simulation. The is associated with the compressibility and isentropic constant (γ) of the gas phase as $SF_6 + N_2$ is more compressible than air. Lee et al. [8] investigated the effect of gas phase compressibility in sloshing by introducing the so–called compressibility number. This is a non–dimensional parameter and defined as

$$comp = \frac{\omega A}{\sqrt{\frac{E_g}{\rho_g}}} \tag{10}$$

where ω is the oscillation frequency, A is amplitude of oscillation and E_g, ρ_g are the bulk modulus and density of the gas, respectively. Using the properties of air and $SF_6 + N_2$ in Eq. 10 leads to

$$comp_a = 0.5024 comp_{SF_6+N_2} \tag{11}$$

which verifies that air is less compressible compared to $SF_6 + N_2$. However, in the present numerical simulations same speed of sound is used for air and $SF_6 + N_2$ *i.e.* ≈ 3 times larger than c_{liquid}. Therefore the difference in compressibility of two gases is only associated with their γ ratios. It appears that the effect of density ratio is important when the flow is impacting, but the compressibility plays a bigger role in the oscillatory part that follows the impact. Further studies are needed with the real speed of sound of gases for better understanding of compressibility effects in sloshing.

Conclusions

In this paper, a set of numerical simulations were carried out for studying the sloshing phenomenon in a rectangular tank and the effect of density ratio and compressibility of the gas phase in the numerical simulations. Simulations were conducted for water sloshing in an atmosphere of air, and with denser gases $SF_6 + N_2$. Although the shape of interface and so the flow pattern are quite similar in both cases, the density ratio of liquid and gas along with the compressibility of the gas has a significant

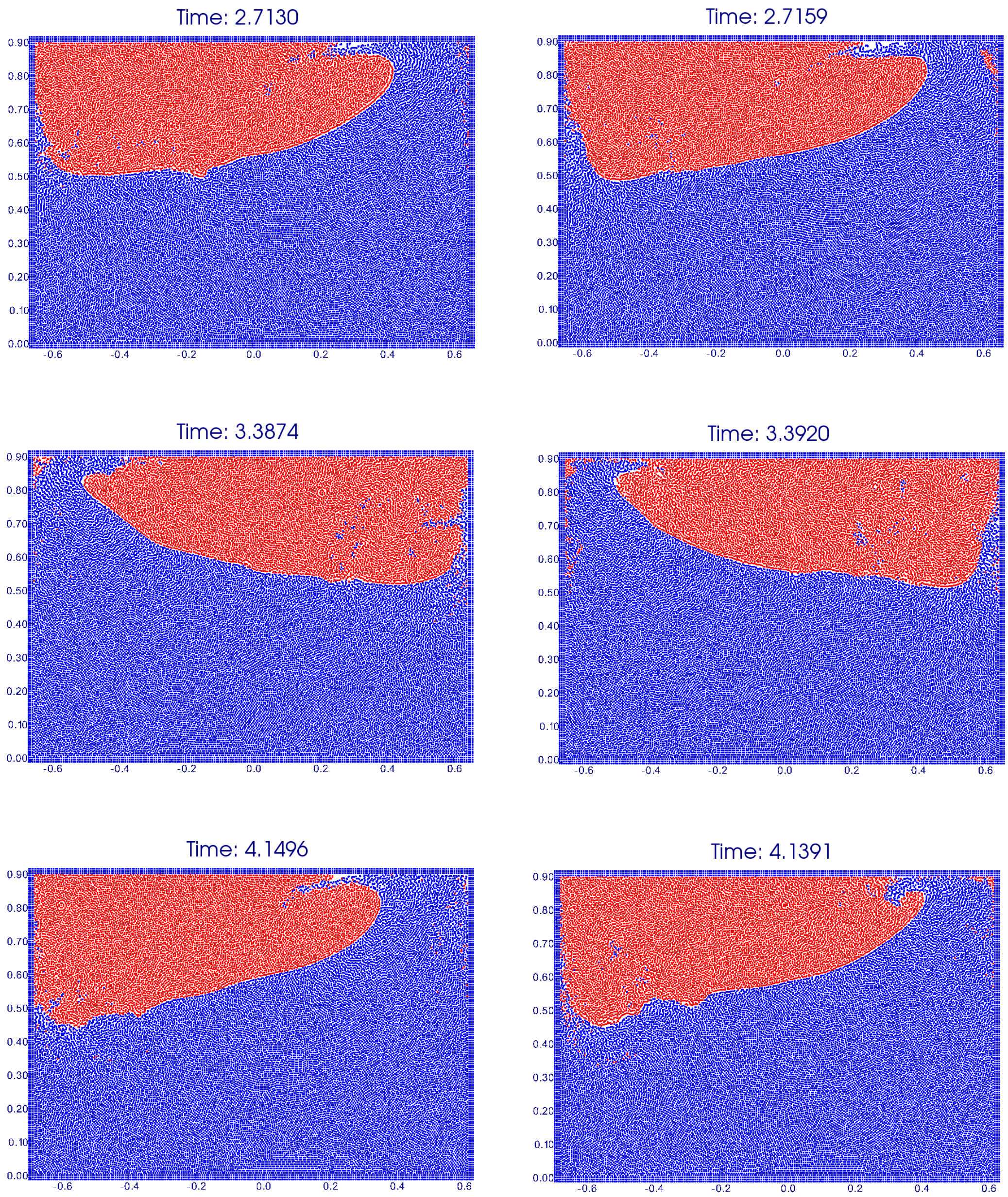

Figure 2: SPH results for sloshing flow at different times for air–water (left column) and $SF_6 + N_2$ (right column).

influence on the impact pressure. It is observed that the impact pressure in the air–water simulation is larger than the $SF_6 + N_2$–water simulation by a factor of ≈ 3. It has also been observed that since $SF_6 + N_2$ has a smaller polytropic constant than air and is so more compressible therefore the oscillations of the pressure in the decay time are more pronounced than the air–water simulation.

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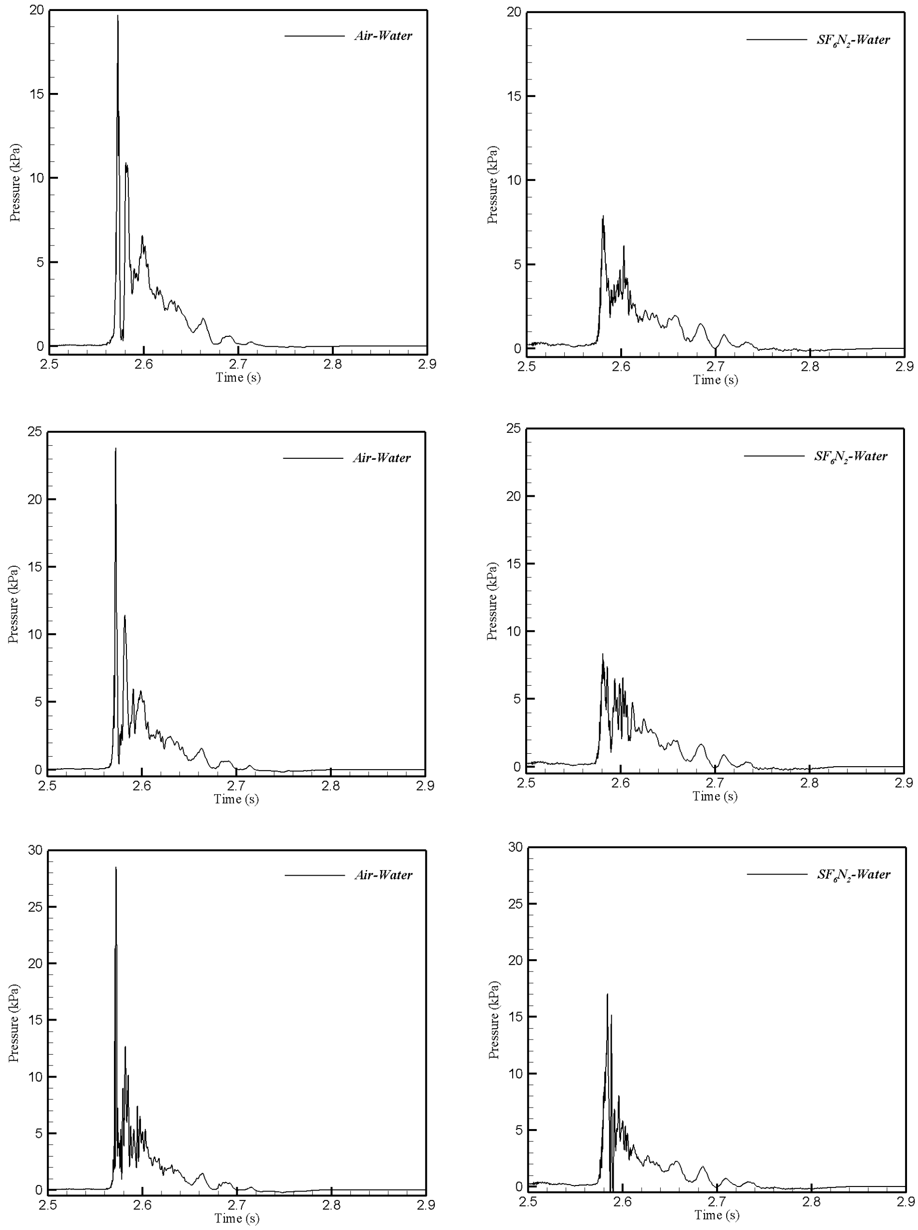

Figure 3: Comparison between gas phase effects on impact pressure for sensors $P1$ (first row),$P2$ (second row) and $P3$ (third row).

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