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Slip MHD Flow over Permeable Stretching Surface with Chemical Reaction

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Abstract

In this paper the magneto hydrodynamic (MHD) slip flow over a permeable stretching surface has been evaluated in the presence of a chemical reaction. Slip flow comes about if the characteristic size of the flow regime is small or the flow pressure is very low. By using appropriate similarity variables, the fundamental equations of the boundary layer are transformed to ordinary differential equations containing the Schmidt number, nonlinearity velocity of the surface and magnetic parameter which for a fixed values of slip coefficient (K) at the boundary conditions, local similarity solution would be valid. The ordinary differential equations of the problem are solved numerically using an explicit Runge-Kutta (4, 5) formula, the Dormand-Prince pair and shooting method. The velocity and concentration profiles in addition to the local skin-friction and the local Sherwood number for the various values of the involved parameters of the problem are presented and discussed in details.

Introduction

Microfluidics as a young research field plays a great role to develop control accuracy of small devices. In no-slip-flow, as a requirement of continuum physics, the flow velocity is zero at a solid-fluid interface. But in the existence of slip-flow, the flow velocity at the solid walls is nonzero [1]. Even if the separation of individual molecules is obvious at the nanoscales, it is still possible to explain the main transport phenomena in nanofluidic systems with a theory based on continuum and mean-field approaches [2, 3 and 4]. It is well known that flow past a permeable surface has practical applications especially in geophysical fluid dynamics. Examples of natural porous media are wood, beach sand, sandstone, limestone, the human lung and in small blood vessels. The magnetohydrodynamic (MHD) flow of a fluid in a micro/nanochannel is of interest in connection with certain problems of the movement of conductive physiological fluids, e.g., the blood, blood pump machines and with the need for theoretical research on the problem of the slip MHD flow along permeable surfaces. Thus, the micro-nano magnetohydrodynamic effects are recognized as a tool for controlling the micro-nanostructure of materials. Numerous investigations have been done analytically regarding to the slip flow regime. Martin and Boyd [5] have analyzed Blasius boundary layer problem with slip flow. Their results demonstrated that the boundary layer equations can be used to study flow at the MEMS scale and provide useful information to study the effects of rarefaction on the shear stress and structure of the flow. In another task [6] they have analyzed momentum and heat transfer in a laminar boundary layer with slip flow at constant wall temperature. Based on the boundary layer theory, non-equilibrium effects will cause a reduction in drag on airfoils. According to their studies of liquids over flat plate at constant wall temperature boundary conditions, there is no temperature jump. Recently, Matthews and Hill [7] have studied the effect of replacing the standard no-slip boundary condition with a nonlinear Navier boundary condition for the boundary layer equations. In another task they have investigated Newtonian flow with nonlinear Navier boundary condition for three simple pressure-driven flows through a pipe, a channel and an annulus [8]. The axisymmetric flow of a Newtonian fluid due to a stretching sheet with partial slip boundary condition has been investigated by Ariel [9]. Yazdi et al. [10] have investigated friction and heat transfer in the slip flow boundary layer at constant heat flux boundary conditions. In another task [11] they have studied liquid fluid past embedded open parallel microchannels within the surface. Wang [12] has studied the viscous flow due to a stretching sheet with partial slip and suction. Recently Yazdi et al [13] have analyzed convective heat transfer of the slip liquid flow past horizontal surface within the porous media at constant heat flux boundary conditions. Their results suggest that slip liquid flow can successfully reduce wall friction through slip-flow boundary conditions in convective heat transfer problems and increase heat transfer rate. It has been found that suction makes a significant effect on the velocity adjacent to the wall in the presence of slip. On the topic of MHD flow modeling, the boundary-layer equation of flow over a nonlinearly stretching sheet in the presence of a chemical reaction and a magnetic field has been investigated by Kechil and Hashim [14]. Recently Fang et al [15] have studied analytically hydrodynamic boundary layer of slip MHD viscous flow over a stretching sheet. Their investigation shows the velocity and shear stress profiles are influenced by the slip, magnetic and mass transfer parameters. They have illustrated that wall drag force increases with the increase of magnetic parameter. There have been many theoretical models developed to describe slip flow along the surface. To the best of our knowledge, no investigation has been made yet to analyze the slip MHD flow over permeable stretching surface with chemical reaction.

Mathematical formulation

For modeling fluid transport in slip boundary layer, the assumptions made for the derivation of the full Navier–Stokes equations have been examined. These assumptions are the fluid is assumed to be a continuum, the fluid is Newtonian. In addition, the fluid can be assumed to be incompressible. We will study the 2-D, steady, laminar flow in the presence of a transverse magnetic field with strength B(x) which is applied in the vertical direction, given by the special form.

$$B(x) = B_0 x^{\frac{n-1}{2}} \quad , B_0 \neq 0 \tag{1}$$

where x is the coordinate along the plate measured from the leading edge and *n* is a constant. The magnetic Reynolds number is assumed small so that the induced magnetic field is neglected. The positive y-coordinate is measured normal to the x-axis in the outward direction towards the fluid. The corresponding velocity components in the x and y directions are u and v, respectively. The surface velocity is given by:

$$u_w(x) = u_0 x^n \tag{2}$$

where u_0 is a constant parameter related to the surface stretching speed. The concentration adjust to the surface would be C_w and the solubility of A in B and the concentration of A far away from the plate would be C_∞. We assume that the reaction of a species A with B be the first order homogeneous chemical reaction with rate constant, κ. It is desired to analyse the system by a boundary layer method. It is assumed that the concentration of dissolved A is small enough and the related physical property D is constant in the fluid.The steady two-dimensional boundary layer equations for this problem, in the usual notation [16], are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu_\infty \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2 u}{\rho} \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + \kappa C \tag{5}$$

The associated boundary conditions are

$$\begin{aligned} y = 0 \Rightarrow\ & u = u_w + u_s \quad , \ v = \pm v_w, C = C_w \\ y \to \infty\ \Rightarrow & u = 0 \ , C = C_\infty \ , \end{aligned} \tag{6}$$

where ρ is the fluid density, σ is the electrical conductivity of the fluid, v_w is the wall mass transfer velocity and u_s is the velocity slip which is assumed to be proportional to the local shear stress as follows [1]:

$$u_s = l\left|\frac{\partial u}{\partial y}\right|_w \tag{7}$$

l is slip length as a proportional constant of the velocity slip. By using similarity transformation, the fundamental equations of the boundary layer are transformed to ordinary differential ones that are locally valid. Thus, the mathematical analysis of the problem can be simplified by introducing the following dimensionless coordinates:

$$\begin{aligned} & f'(\eta) = \frac{u}{u_w} = \frac{u}{u_0 x^n} \qquad \eta = y\sqrt{\frac{u_0(n+1)}{2\nu_\infty}}\,x^{\frac{n-1}{2}} \\ & v = -\sqrt{\frac{u_0 \nu_\infty (n+1)}{2}}\,x^{\frac{n-1}{2}}\left(f + \frac{n-1}{n+1}\eta f'\right) \\ & \phi = \frac{C - C_\infty}{C_w - C_\infty} \end{aligned} \tag{8}$$

It is helpful to introduce a slip coefficient using similarity variables:

$$f'(0) = 1 + Kf''(0) \tag{9}$$

where *K* is the slip coefficient defined for liquids by:

$$K = \frac{l}{x}\sqrt{\frac{u_w(n+1)}{2\nu_\infty}} \tag{10}$$

The fundamental partial differential equations (4) and (5) are transformed to ordinary differential equations substituting similarity variables (8) into Eqs. (4, 5) as follows:

$$f''' + ff'' - \frac{2M^2}{n+1}f' - \frac{2n}{n+1}f'^2 = 0 \tag{11}$$

$$\phi'' + Sc\left(f\phi' - \frac{2\,\mathrm{Re}\,\gamma}{(n+1)}\phi\right) = 0 \tag{12}$$

with associated boundary conditions:

$$\begin{aligned} & \eta = 0 \Rightarrow \begin{cases} f'(0) = 1 + Kf''(0) \\ f(0) = f_w \\ \\ \phi(0) = 1 \end{cases} \\ & \eta \to \infty \Rightarrow \quad f'(\infty) = 0, \phi(\infty) = 0 \end{aligned} \tag{13}$$

where f_w, Sc, Re, γ and M show the strength of the mass transfer at the sheet, Schmidt number, Reynolds number, non-dimensional chemical reaction parameter and magnetic parameter respectively:

$$\begin{aligned} & f_w = \frac{-x v_w}{\sqrt{\dfrac{u_0 x^n \nu_\infty (n+1)}{2}}}, M^2 = \frac{\sigma B_0^{\,2}}{\rho u_0} \\ & \gamma = \frac{\kappa \nu_\infty}{u_w^{\,2}}, \quad Sc = \frac{\nu_\infty}{D} \end{aligned} \tag{14}$$

f_w is negative for mass injection and positive at the presence of suction along the surface. It is obvious that the nonlinearity parameter of surface velocity, (n) and slip coefficient (K) which exists in the boundary condition tends to break down the similarity solution. Consequently the local similarity solution of the problem for the fixed values of the coordinate along the plate (x) and nonlinearity parameter (n) would be obtained properly for the momentum and concentration equations (11), (12). Therefore by using specified nonlinear parameter (n) in the specific location (x) on the surface, problem can be solved. These Nonlinear differential Eqs (11, 12) are solved numerically by using an explicit Runge-Kutta (4, 5) formula, the Dormand-Prince pair and shooting method subject to the slip-flow boundary conditions (13) which is locally valid. After solving this slip-flow problem numerically, the wall shear stress and the Sherwood number exhibits a dependence on the slip coefficient as follows:

$$\tau_w = \mu\left|\frac{\partial u}{\partial y}\right|_{y=0} = \mu u_0 \sqrt{\frac{u_0(n+1)}{2\nu_\infty}}\,x^{\frac{3n-1}{2}}\,f''(0) \tag{15}$$

$$Sh = \frac{-x}{C_w - C_\infty}\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{16}$$

Result and Discussion

Table .1 shows Comparison of the velocity gradient at the wall f''(0) between the present code results and that obtained before. This table indicates that our results are compatible with the previous works of Cortell [17] and Fang [15]. It's obvious that accelerating velocity tends to increase the skin friction at the wall. On the other hand, the slip coefficient tends to decrease wall shear stress.

n	f_w	K	M	Cortell [17] (2007)	Fang [15] (2009)	Presented Results
0	0	0	0	0.6275		0.6275
0.2				0.7667		0.7667
0.5				0.8895		0.8896
0.75				0.9538		0.9540
1	0	0	0.5		1.1180	1.1180
		0.5			0.6495	0.6495
		1			0.4691	0.4691

Table 1. Comparison of the velocity gradient at the wall $f''(0)$ between the present code results and that obtained before

Fig.1 and Fig.2 show variation of the velocity profiles and velocity gradient respectively as a function of η for various values of magnetic parameter (M) at f_w=0.2, n=0.5 and K=0.5. Fig.1 indicates that the Lorentz force changes the velocity profile such that the velocity distribution decreases with increasing M. At the presence of the Lorenz force, the skin friction coefficient increases with the increase in magnetic parameter.

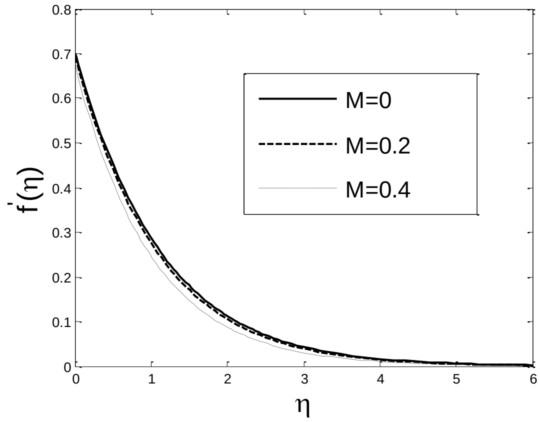

Figure 1. Velocity distribution as a function of η for various values of M at f_w=0.2, n=0.5, K=0.5

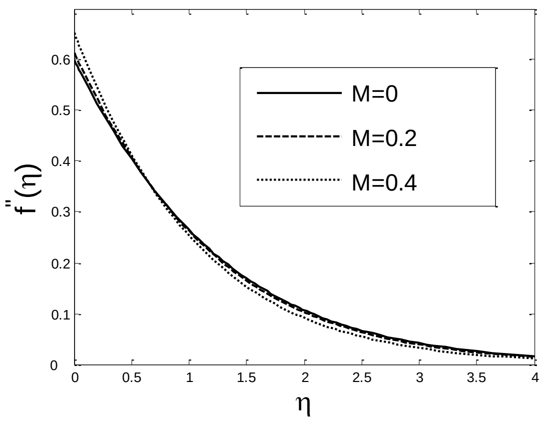

Figure 2. Velocity gradient as a function of η for various values of M at f_w=0.2, n=0.5, K=0.5

Fig.3 shows variation of the velocity gradient as function of η at various values of suction/injection. It's clear that the wall shear stress would be increased with the application of suction whereas injection tends to decrease wall shear stress. It is understandable that the velocity of the fluid on the wall increases by increasing suction parameter along the surface and decreases by injection at specific slip coefficient.

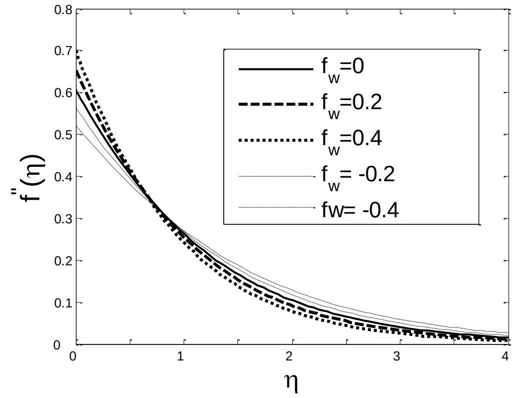

Figure 3. Velocity gradient as a function of η for various values of f_w at M=0.4, n=0.5, K=0.5

Fig.4 and Fig.5 illustrate variation of the concentration profiles and concentration gradient respectively as a function of η for various values of Schmidt number (Sc) at n=0, f_w=0.2, γ=0.5, K=0.5, Re=1, M=0.1. Schmidt number tends to increase Sherwood number by increasing concentration gradient. The reason for this trend is that the concentration boundary layer becomes thin for large Sc number.

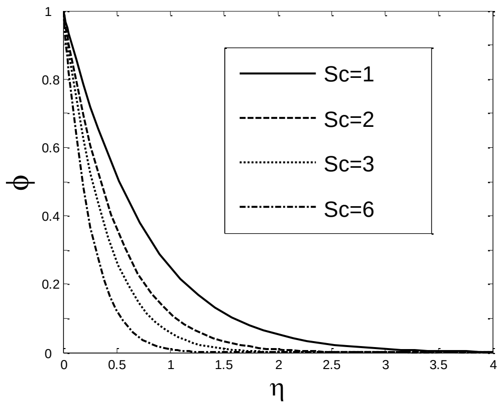

Figure 4. Concentration profiles as a function of η for various values of Schmidt number (Sc) at n=0, f_w=0.2, γ=0.5, K=0.5, Re=1, M=0.1

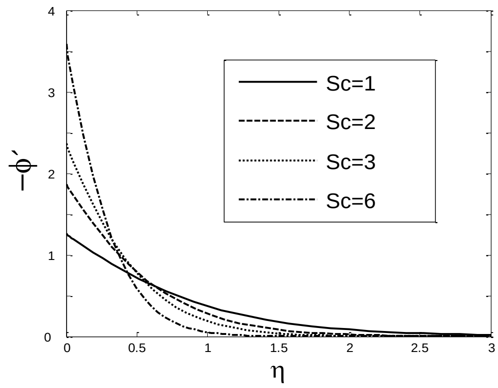

Figure 5. Concentration gradient as a function of η for various values of Schmidt number (Sc) at f_w=0.2, γ=0.5, K=0.5, M=0.1

Fig.6 and Fig.7 illustrates variation of the concentration profiles and concentration gradient respectively as a function of η for various values of slip coefficient (K) at n=0, f_w=0.2, γ=0.5, Sc=1, Re=1, M=0.1. Increasing slip coefficient increases the velocity of the fluid on top of the surface. Hence it increases concentration profile and decreases concentration gradient at the surface (η=0) respectively.

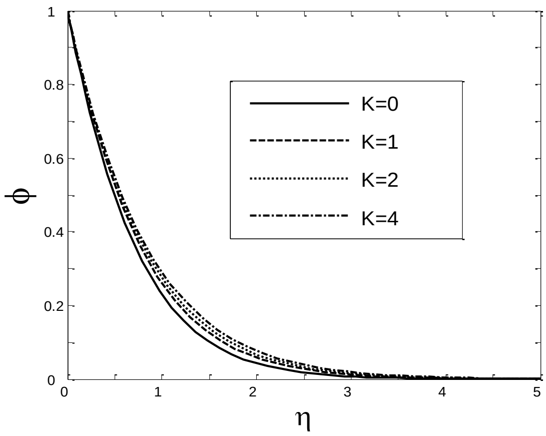

Figure 6.Concentration profiles as a function of η for various values of slip coefficient (K) at f_w=0.2, γ=0.5, Sc=1, M=0.1

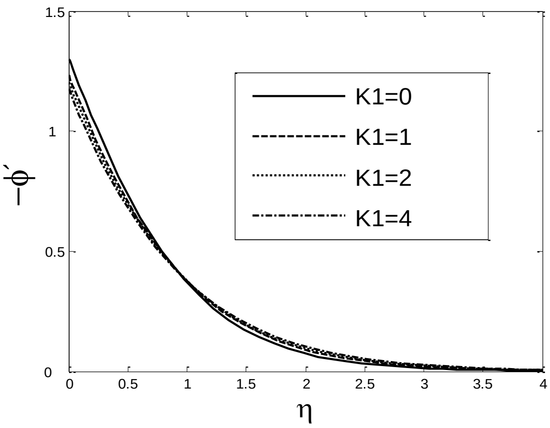

Figure 7.Concentration gradient as a function of η for various values of slip coefficient (K) at f_w=0.2, γ=0.5, Sc=1, M=0.1

Conclusions

The problem of the MHD slip flow over nonlinear permeable stretching surface in the presence of chemical reaction is evaluated analytically using a similarity solution. The slip boundary condition along the surface is considered. The results suggest that in the presence of the Lorenz force, the skin friction coefficient increases with the increase in the magnetic parameter. It has been found that Schmidt number tends to increase the Sherwood number whereas the slip coefficient decreases the concentration gradient at the surface.

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