17th Australasian Fluid Mechanics Conference
Auckland, New Zealand
5-9 December 2010

Local Corotation of Line Segments and Vortex Identification

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Abstract

An easy-to-interpret local vortex intensity (i.e. near a point) is introduced and employed in 3D vortex identification. It is based on the maximum corotation of line segments found within a set of planar cross-sections going through the given point. The results obtained at higher thresholds for different vortical flows are promising. The proposed kinematic method remains applicable to compressible and variable-density flows.

Introduction

The need for efficient 3D vortex-identification schemes has become particularly important for the analysis of transitional and turbulent flows during last three decades. A variety of vortex definitions, vortex-identification methods, and vortex-core visualization techniques have been proposed, but none has been universally accepted as each has its own limitations.

Local corotation of line segments near a point is closely associated with the interpretation of the *residual* vorticity in 2D, see [8, 9] where different parts of vorticity characterize different parts of local motion: *shear* vorticity—shearing motion, *residual* vorticity—rigid-body rotation. The planar *residual* vorticity is just (double) the least-absolute-value angular velocity of all line segments within the flow plane going through the given point. This quantity is nonzero if all of the line segments corotate. In the present paper, the notion of local corotation is formulated for an arbitrary planar cross-section going through the given point in a 3D flow and it is quantitatively expressed in terms of the quasiplanar *residual* vorticity. It should be emphasized that to focus on planar aspects in 3D vortex identification is nothing new. For example, in [11] the second invariant Q_{2D} of a planar cross-section is applied.

The local intensity of a vortex (to be determined point by point) is sought as the maximum value of planar corotation over all planes going through the given point. Note that, contrary to the method proposed in [8], the strong coupling of the sought swirling motion with the motion in all other directions and planes is released. Consequently, in 3D, this method represents a computationally simpler algorithm than that of [8], though in planar flows both kinematic methods coincide. Two different flow situations—a hairpin vortex of boundary-layer transition [1] and the reconnection process of two Burgers vortices—are analyzed in terms of the local corotation. DNS data sets for this purpose have been provided by IAG Stuttgart. Time-consuming calculations of the maximum value of planar corotation over all possible planes at each mesh point have been considerably accelerated by means of GPU computing using NVIDIA CUDA. It will be shown that, at higher thresholds, the agreement of the present corotation method with the λ_2-method is very good.

Local Corotation of Line Segments

In 2D, there is a straightforward interpretation of the *residual* vorticity in terms of the least-absolute-value angular velocity of all line segments within the flow plane going through the given point. If all of the line segments corotate, the *residual* vorticity is nonzero: either positive or negative. The *residual* vorticity represents a direct corotation measure. Analogously, the notion of local corotation can be introduced for an *arbitrary* planar cross-section going through the given point in a 3D flow.

To quantify the local corotation in a 3D flow, the quasiplanar *residual* vorticity, ω_{RES}, is employed. Again, it is interpreted in terms of the least-absolute-value angular velocity of all line segments, however, now within an *arbitrary* planar cross-section going through the given point. The relevant quantities necessary for the determination of ω_{RES} are: (i) the vorticity component normal to the given planar cross-section ω and (ii) quasiplanar *deviatoric* strain-rate magnitude s_{D}.

The quantities ω, s_{D}, and ω_{RES} are unambiguously expressed in terms of angular velocities near a point depicted in figure 1 and read (cf. planar expressions from [8], the symbol ω_{SH} stands for the *shear* vorticity, the cross-section coordinates and velocity components are denoted x, y and u, v, respectively)

$$\omega_{\mathrm{RES}} = \omega - \omega_{\mathrm{SH}} = (\operatorname{sgn}\omega)\left[|\omega| - |s_{\mathrm{D}}|\right] = \Omega_{\mathrm{LOW}} \quad \textit{for} \; |\omega| \geq |s_{\mathrm{D}}| , \tag{1}$$

$$\omega_{\mathrm{RES}} = 0 \quad \textit{for} \; |\omega| \leq |s_{\mathrm{D}}| \tag{2}$$

where

$$\omega = \left(v_x - u_y\right)/2 = \left(\Omega_{\mathrm{HIGH}} + \Omega_{\mathrm{LOW}}\right)/2 = \Omega_{\mathrm{AVERAGE}} , \tag{3}$$

$$|s_{\mathrm{D}}| = \left(\sqrt{\left(u_x - v_y\right)^2 + \left(u_y + v_x\right)^2}\right)/2 . \tag{4}$$

In terms of angular velocities, the quantity s_{D} can be for (and only for) the corotation case, $|\omega| > |s_{\mathrm{D}}|$, expressed and viewed as (figure 1)

$$|s_{\mathrm{D}}| = \left(|\Omega_{\mathrm{HIGH}}| - |\Omega_{\mathrm{LOW}}|\right)/2 . \tag{5}$$

In figure 1, the term *contrarotation*, unlike *counterrotation*, indicates that rotating line segments share a common axis of rotation (going through the given point P).

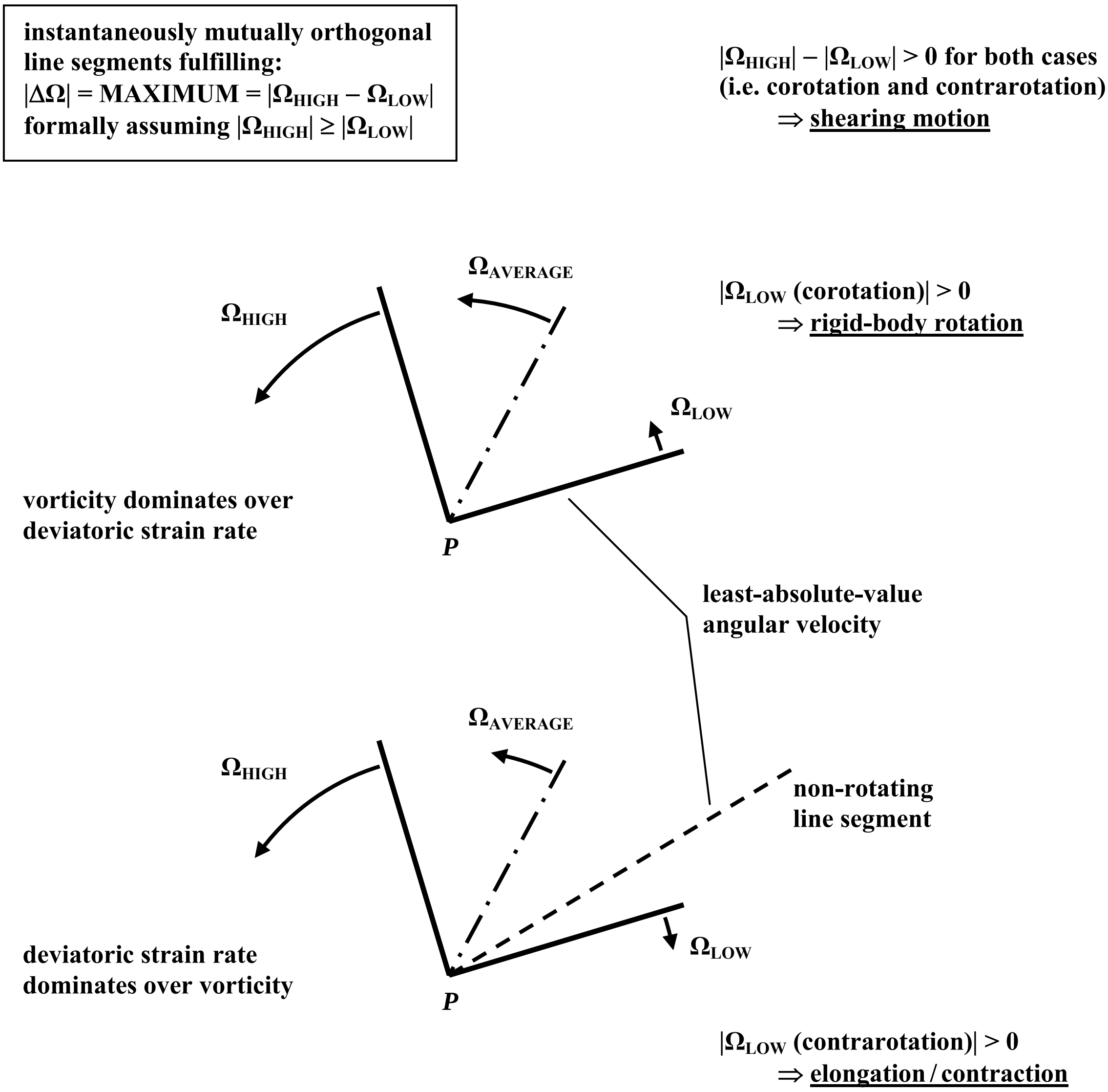

Figure 1. Interpretation of the *residual* vorticity in terms of the least-absolute-value angular velocity.

Alternatively, s_D can be for (and only for) the contrarotation case, $|\omega| < |s_D|$, expressed as (cf. figure 1)

$$|s_D| = \left(|\Omega_{HIGH}| + |\Omega_{LOW}|\right)/2\,, \qquad (6)$$

or, alternatively, for $|\omega| = |s_D|$ describing pure shear, s_D is given by

$$|s_D| = |\Omega_{HIGH}|/2\ . \qquad (7)$$

As can be inferred from (3) and (4), a 3D uniform dilatation (volumetric deformation) does not affect both the above stated quasiplanar shape and rotational characteristics. However, the 3D shape changes near a point generally affect the quasiplanar deformation appearing in the cross-section and hence the calculation of s_D and, consequently, the calculation of ω_{RES}.

Vortex Identification

From the physical viewpoint, the application of the introduced corotation property to vortex identification has a very positive aspect. The vortex-identification scheme based on the quasiplanar *residual* vorticity ω_{RES} provides an easy-to-interpret local vortex intensity sought in the following manner. It is a plausible assumption of the proposed scheme that a vortex is locally (near a point) characterized by the maximum value of planar corotation over all planes going through the given point. The desired plane of swirling maximizes ω_{RES}, or equivalently, Ω_{LOW} (for the case of vorticity dominance, see figure 1). It should be recalled that the earlier widely used application of (total) vorticity to vortex visualization is nothing but a pointwise 3D search for a vorticity vector which is assumed to be perpendicular to the plane of swirling (vortex intensity being quantified by the vorticity magnitude). In other words, within the earlier (total) vorticity scheme, the plane of swirling maximizes an average angular velocity of all line segments, subsets of the plane, going through the given point.

In practice, though the number of planes to check near a point is infinite, the determination of the relevant cross-section should be based on a finite set of discrete representations. By considering a reasonable angle resolution (for example, one degree or less) we proceed to the approximation with reasonably high precision. The transformation matrix for an arbitrarily rotated cross-section can be obtained by a sequence of two rotational transformations. It should be noted that the method introduced in [8] needs three rotational transformations and, consequently, the corotation method represents a computationally simpler algorithm than that of [8]. Therefore, contrary to the method proposed in [8], the strong coupling of the sought swirling motion with the motion in all other directions and planes is released.

There is nothing new to focus on planar aspects in 3D vortex identification as shown, for example, in [11] where the second invariant Q_{2D} of a planar cross-section is employed (Q_{2D}-criterion). An analytical diagnosis of four popular local criteria, demonstrated by the Burgers and Sullivan vortex, indicates that—unlike the Δ-criterion [3]—the Q-criterion [5] and λ_2-criterion [6] may cut a connected vortex into broken segments at locations with strong axial stretching [11]. Similar to the vortex predicted by the Δ-criterion [3], the corotation-based vortex is resistant to strong axial stretching. However, the corotation criterion employed here as the region-type vortex-identification scheme fails at low threshold levels. The physical reason is as follows: with the exception of a few cases (especially purely 2D cases where strain-rate dominates over vorticity), there is always a plane of nonzero corotation with elliptical streamlines near a point. Simply said, for low thresholds there is a tendency of the corotation-based vortex identification to cover almost the entire examined region. The situation is similar for the Q_{2D}-criterion [11], used earlier in [7] as the swirl condition alongside the sectional-pressure-minimum condition. Also note that practical applications of the most popular criteria employ a nonzero threshold. The vortex surface with a positive threshold appears significantly smoother [12]. Moreover, the study of the relationship between local identification schemes [2] shows that all of the popular local criteria, given the proposed usage of threshold, result in a remarkable vortex similarity.

As mentioned in [11], owing to their universality, kinematic criteria are preferred if they work well. The kinematic Δ-criterion [3] and λ_{ci}-criterion [2, 12] are easily extendable to compressible flows [10] unlike some other vortex-identification schemes. The widely used λ_2-method [6], although it employs the eigenvalues of the kinematic quantity $\mathbf{S}^2+\mathbf{\Omega}^2$ ($\nabla\mathbf{u}=\mathbf{S}+\mathbf{\Omega}$), is formulated on dynamic considerations (namely on the search for a pressure minimum across the vortex by the requirement of two positive eigenvalues of the pressure Hessian). Consequently, this criterion is not extendable to compressible flows due to nonzero divergence and nonzero density gradients [4]. The kinematic methods based on vorticity decomposition and local corotation (the present case) clearly remain applicable to compressible and variable-density flows.

Data Processing Method

The value of planar corotation in terms of the quasiplanar *residual* vorticity ω_{RES} is maximized over all admissible planes individually at each mesh point. This leads to a large set of independent optimization problems. Two design variables are represented by angular rotations of the plane. The design space is uniformly sampled with a chosen step size of these angles. While smaller angle step size improves the resolution of the search, it can lead to large number of objective function evaluations, which depends on angle step size $\Delta\alpha$ as $1/(\Delta\alpha)^2$. However, according to our experience with the method, fine resolution often improves the quality of the identification considerably, and a step size of one degree was chosen for the present calculations. This choice leads to 32761 objective function evaluations at each mesh point.

Since the optimization problems are completely independent, and the application has a large ratio of arithmetic operations and memory transfers (such problems are sometimes called arithmetic intensive), it is a good candidate for parallel computing. For its simple accessibility, we have chosen the CUDA platform by NVIDIA to accelerate the computation by a Graphical Processing Unit (GPU). This platform allows simultaneous optimization at multiple mesh points using multiple cores of GPUs, that are present in common personal computers as well as in special computing servers of GPUs. Using this technique, we have been able to reduce the overall computational time for evaluation of the maximum ω_{RES} at a point to times comparable to file input/output operations with the data.

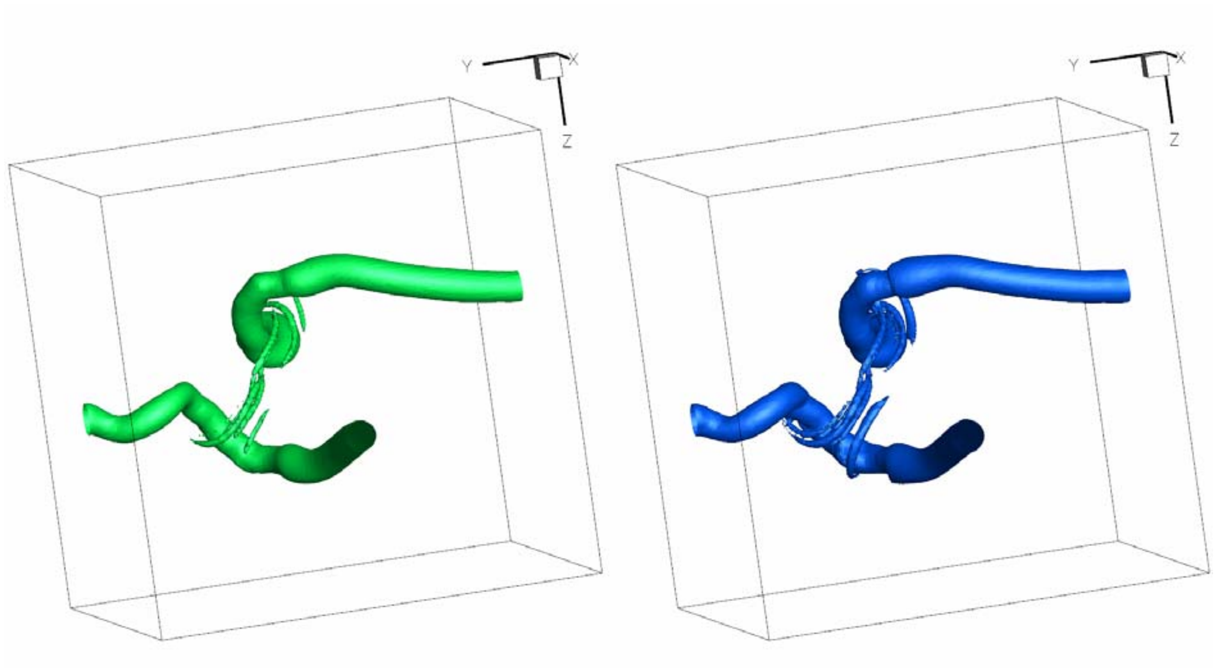

Figure 2. Reconnection process of two Burgers vortices: λ_2-method (left), present corotation method (right).

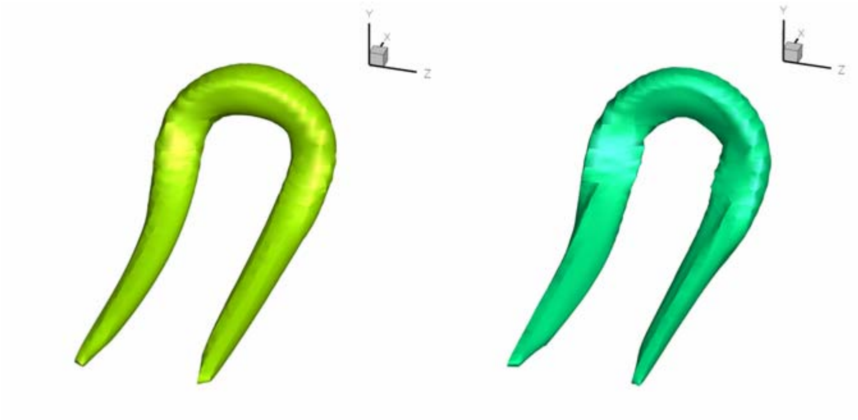

Figure 3. A hairpin vortex of boundary-layer transition: λ_2-method (left), present corotation method (right).

Results

Two types of vortices (figures 2, 3) are analyzed in terms of the maximum ω_{RES} at a point. While the boundary-layer transition [1] required the threshold of about 16.7% of λ_2 maximum value to capture well what λ_2-criterion predicts for a hairpin vortex (figure 3), for Burgers vortices (Ma=0.3) the threshold was set much lower, at about 3.9% of λ_2 maximum value, to capture the identification outcome of λ_2-criterion (figure 2) quite well.

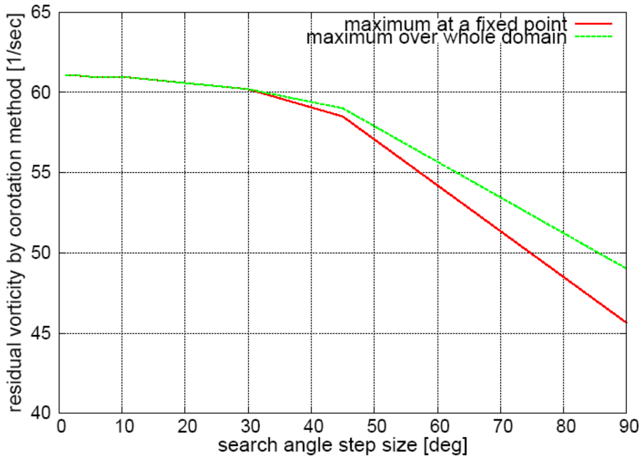

Figure 4. Residual vorticity vs. angle step size.

For the case of Burgers vortices it is shown in figure 4 how the angle step size (1, 2, 5, 10, 30, 45, 90 deg) affects the final value of ω_{RES} at the fixed point corresponding to the position of the *global* maximum for the finest one-degree resolution. There is no significant difference in the value of ω_{RES} up to more than 10 deg. For more than 30 deg the *global* maximum moves from the original position obtained for one-degree resolution. For the range from 1 to 10 deg the processing time (by NVIDIA CUDA) reasonably ranges from tens of seconds to several minutes.

Conclusions

An easy-to-interpret local vortex intensity is introduced and applied to the identification of vortical structures. The vortex intensity is locally (near a point) characterized by the maximum value of planar corotation over all planar cross-sections going through the given point. The corotation measure for an arbitrary planar cross-section is quantified by the quasiplanar *residual* vorticity ω_{RES} which has to do with the rigid-body rotation after the extraction of the quasiplanar shearing motion (near a point). ω_{RES} is nothing but the least-absolute-value angular velocity of all line segments within the examined flow plane going through the given point as shown in figure 1. This method clearly remains applicable to compressible and variable-density flows.

Acknowledgments

The authors are very grateful to Prof. Ulrich Rist and Dr. Kudret Baysal, IAG Stuttgart, for providing the DNS data sets used in the present paper. This work was supported by the Grant Agency of the Academy of Sciences of the Czech Republic through grant IAA200600801, and by the Academy of Sciences of the Czech Republic through AV0Z20600510 and AV0Z10190503.

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