

## Experiments and Numerical Predictions with a Trapezoidal Sloshing Absorber for Structural Control

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### Abstract

Liquid sloshing is the relatively low frequency oscillation of the free surface of a partially full container. Intentionally induced sloshing in a container may be used as a means to limit excessive oscillations of light resonant structures, such as tall buildings and long-span suspension bridges. These devices are commonly called sloshing absorbers.

The objective of this investigation is to demonstrate the effect of different flow obstructions, to increase the rate of energy dissipation in a trapezoid shaped sloshing absorber. Numerical investigations involve using Smoothed Particle Hydrodynamics (SPH) method. In addition, experimental observations are included to support the validity of numerical predictions.

### Introduction

Sloshing is the low frequency oscillation of a liquid in a container with a free surface. Due to the ability of a sloshing liquid to dissipate energy, intentionally induced sloshing can be used as an effective energy sink in structural control applications to suppress excessive levels of vibration [9]. Such a controller simply consists of a container full of liquid with a free surface, attached on the structure, as suggested in Figure 1(a). In this figure, the simple oscillator with mass ( $m$ ), stiffness ( $k$ ) and damping ( $c$ ), represents the structure to be controlled. The principle of the design, is to designate (tune) the sloshing frequency to be the same as the critical frequency of the structure. This tuning ensures strong interaction between sloshing and structural oscillations, causing the kinetic energy to flow back-and-forth between the liquid and the structure to be controlled. The harmful energy extracted from the structure, as a consequence of tuning, is dissipated through shearing in the sloshing liquid.

Liquid sloshing in rectangular containers have been studied quite extensively due to their relative simplicity and effectiveness in control [2,8,9]. These studies have demonstrated that shallow depths are more effective at dissipating energy than deep liquid levels [5,9,11]. The effect of container shape, on the other hand, has not yet attracted much attention. An earlier work of the authors demonstrated that a trapezoid container possesses significant promise, as it is able to mobilise large amounts of liquid easily, and establish a tuning stronger than that of a rectangular container [4]. The problem with this strong interaction, and the large amounts of kinetic energy extracted from the structure as a result of the strong interaction, is the insufficient rate of energy dissipation. Poor dissipation causes the extracted energy being returned to the structure periodically, significantly deteriorating the control performance.

The motivation of the present work is to explore the possibility of increasing the rate of energy dissipation in a trapezoid container, by introducing obstructions in the way of the sloshing liquid. These designed obstructions are expected to increase the velocity gradients and to enhance the shear dissipation as a result. In numerical models, Smoothed Particle Hydrodynamics (SPH) is used due to its ability to predict the free surface behaviour closely [1,10]. An experimental study has also been conducted, using a slightly different structure

than that in the simulations. Hence, one-to-one comparisons could not be made with the simulations. But, all significant trends predicted numerically could be verified experimentally.

### Numerical model and summary of predictions

The Smoothed Particle Hydrodynamics (SPH) code has been developed by CSIRO's Mathematics, Informatics and Statistics Division. SPH has the ability to predict complex free surface behaviour accurately due to its grid-free nature. The container of a sloshing absorber is represented by a partially constrained moving boundary. The numerical model is identical to that in Figure 1(a), with a stiffness ( $k$ ), and mass ( $m$ ) of 4260 N/m and 60.5 kg respectively, to give a natural frequency of 1.33 Hz. A 1% critical viscous damping ratio is included to represent a lightly damped resonant structure. The sloshing fluid is water, relatively shallow and of 10% of the structure's mass.

A particle size of 0.8 mm by 0.8 mm has been found to be fine enough in a two-dimensional space [6]. Time stepping is explicit and is limited by the Courant condition modified for the presence of viscosity [6], with a time step of integration of  $10^{-5}$  s. The fluid within the container is allowed to settle under gravity for a period of 5 s to a state of rest. The structure is then subjected to an initial velocity of 0.5 m/s over one time step. The structure is allowed to respond freely, its motion exciting the water within.

In Figure 1(b), displacement histories of the structure with (—) and without (----) the sloshing liquid is presented with the rectangular container shown in Figure 1(a). For the uncontrolled case, peak displacement of approximately 60 mm is sustained for a long time, due to light structural damping. The response of the structure is significantly improved with a tuned rectangular sloshing absorber. The controlled displacement history indicates an equivalent critical damping ratio of approximately 6%, a six-fold increase from the uncontrolled case. Simple logarithmic decrement is used for this damping estimate.

In Figures 2(a) and 2(b) the displacement histories with the rectangular and trapezoid containers are shown. The free surface length and the liquid depth of the rectangular container are 200 mm and 30 mm, to produce a 1.33 Hz fundamental sloshing frequency, same as the natural frequency of the structure [7]. For the trapezoid, the width of 200 mm is maintained, but the depth is adjusted to 34.9 mm to maintain the mass ratio of 10%. The walls of the trapezoid are angled out by  $30^\circ$  (from vertical) which results in a narrower width of 165 mm at the bottom.

In Figure 2(b), the diverging trapezoid case is drastically different than that of the rectangular container. For this case, the strong interaction due to large volume of easily mobilised liquid, causes a sharp rate of decay. This decay corresponds to about 15 % critical damping during the first two cycles, significantly larger than the 6% damping of the rectangular container. However, this impressive initial performance deteriorates, as the energy transferred to the sloshing liquid returns to the structure causing the displacement magnitudes to grow periodically at about every 2.5 s. Hence, if it

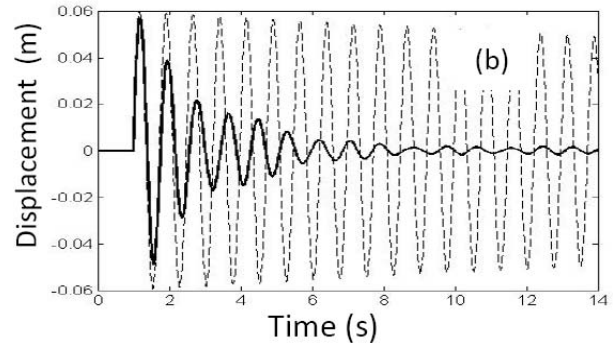
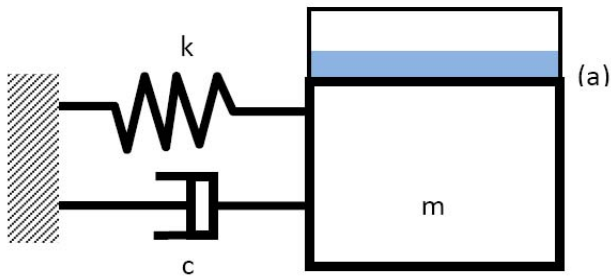


Figure 1. Showing (a) schematic of the structure with a rectangular absorber and (b) displacement histories of uncontrolled (----) and controlled (—) cases, from Reference 4.

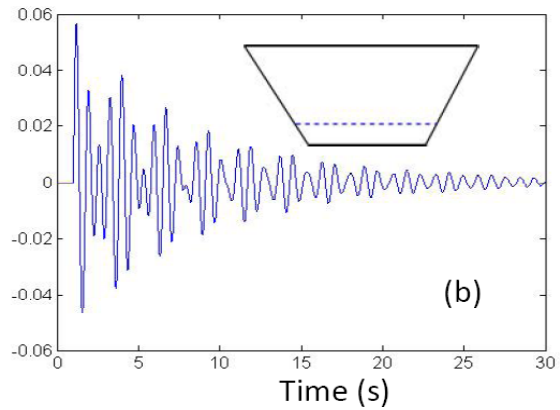
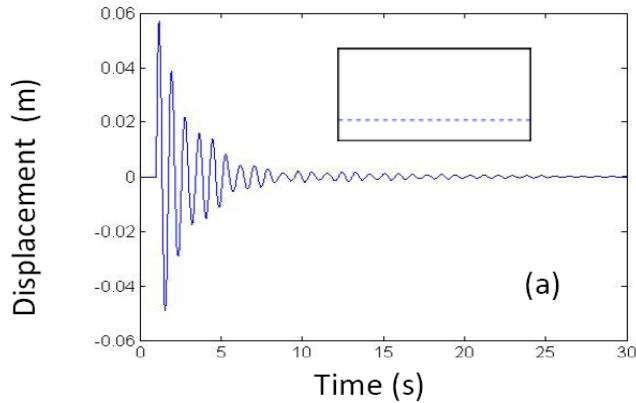


Figure 2. Displacement histories of the (a) rectangular and (b) diverging trapezoid containers from Reference 4.

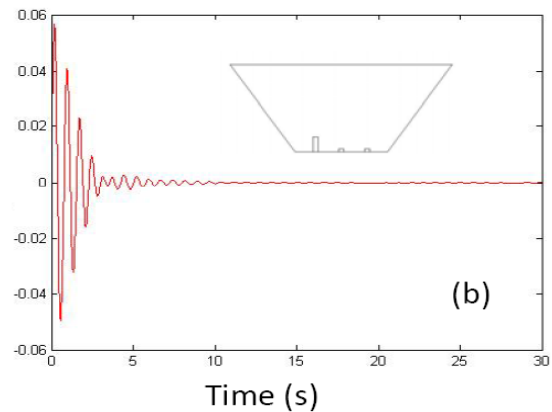
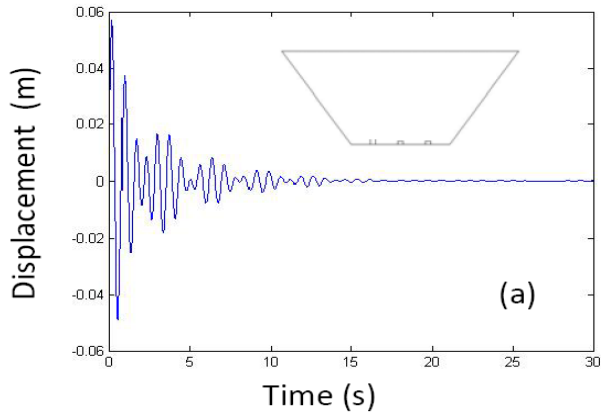


Figure 3. Displacement histories for (a) 3 obstructions of 4 mm,  $t/h = 0.11$  (obstruction height to liquid depth ratio), and (b) when the left-most is increased to 20 mm ( $t/h = 0.57$ ) from Reference 3.

were possible to dissipate the large amount of energy transferred to the sloshing liquid rapidly, the impressive initial control performance could be retained. Roughness elements, in the form of simple obstructions, could be helpful by creating additional steep velocity gradients in the flow.

In Figure 3(a), the displacement history of the structure is given where three equally spaced obstructions are placed at the bottom. These are square obstructions with 4 mm height ( $t$ ), corresponding to 0.11 ( $t/h = 4/34.9$ ) of the static liquid depth. The spacing between the obstructions is 45 mm which is 0.23 of the 200 mm free surface length. Two observations may be made here. First, the initial rapid decay and the following strong beat are largely maintained. Secondly, the small obstructions are quite effective to rapidly dissipate the remnant kinetic energy, once the oscillations decay below about 20 mm. The case in Figure 3(a), is only marginally poorer than that of the rectangular container.

In Figure 3(b), displacement histories of a new case is shown when height of the left obstruction is increased to 20 mm,  $t/h=0.57$ ,

maintaining the middle and the right obstructions at the small  $t/h$  of 0.11. The case seems to possess the ideal combination of rapid initial decay with suppressed beat, practically eliminating oscillations within 3 cycles (about 3 s). The marginal amount of energy left after this initial period of rapid decay, is absorbed quickly by the small obstructions. The initial decay corresponds to approximately 10% critical damping, some loss from the 15% critical damping of the trapezoid container with no obstructions, but far superior to all other cases examined.

### Experiments

Experiments performed with rectilinear translation (using helical springs and lumped mass), as suggested in Figure 1(a), and using a cantilevered beam and a platform, have revealed that the second configuration possesses the superior dissipation characteristics. The difference between the two is that a cantilevered beam, in its first bending mode, does provide a curvilinear translation, curvature becoming increasingly predominant at the peak displacements, as indicated (in red) in Figure 4(a). This curvature and the resulting

rotation imposed on the container of the absorber, enhances the rate of dissipation. Having observed such a difference, the experimental efforts were made exclusively with the cantilevered structure.

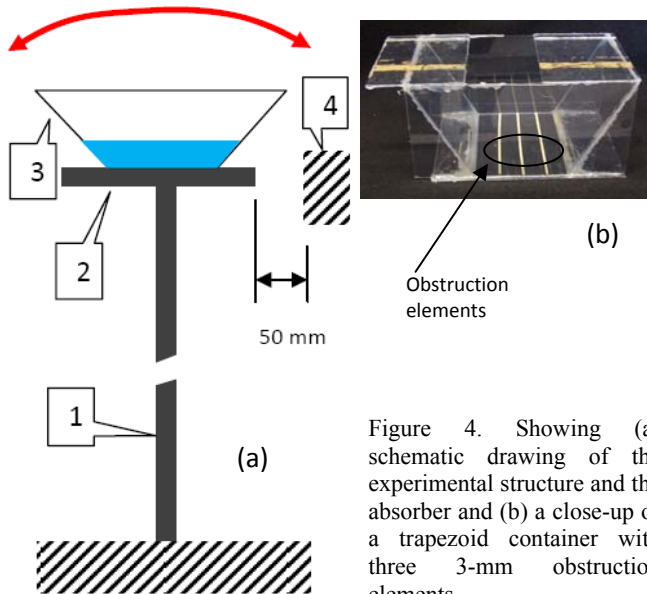


Figure 4. Showing (a) schematic drawing of the experimental structure and the absorber and (b) a close-up of a trapezoid container with three 3-mm obstruction elements.

The experimental structure shown in Figure 4(a), consists of a flat profile mild steel beam (50 mm by 3 mm cross section, and 412 mm length) and a platform (Items 1 and 2) to accommodate the absorber (Item 3). The equivalent stiffness and the mass of the structure are 965 N/m and 13.8 kg. The natural frequency and the equivalent viscous critical damping ratio have been measured to be 1.3 Hz and 0.6%, respectively. Measured values are the averages of 6 repetitions (with marginal variations of well within 10% in each sample).

The rectangular container has a free surface length of 200 mm, and a width of 220 mm. A water depth of 10 mm, provided the best tuning for the 1.3 Hz structural frequency, resulting in a mass ratio of 0.036 between the mass of the water and that of the structure. It may be worthwhile to notice that this mass ratio is approximately 3 times smaller than that of the numerical simulations (with rectilinear oscillations). The trapezoid container maintained the same mass ratio, width and the free surface length, resulting in a water depth of 13 mm. The taper angle is the same as that in the numerical model, 30° from the vertical plane. Containers are made of 3 mm thick plexiglass sheets. A close up picture of the trapezoid container with 3-mm obstructions is shown in Figure 4(b).

The structure is given an initial displacement of 50 mm, using a reference stop block (Item 4). The oscillations of the free surface and the structure are captured with a digital camera at 20 frames/s. Displacement histories of the structure are then obtained by examining the video record.

### Experimental observations

The history of the uncontrolled displacements is shown in Figure 5(a), with marginal decay from the initial displacement of 50 mm over the first 5 s. The uncontrolled oscillations require about 90 s to dissipate the initial potential energy and to come to stop. Histories with the rectangular and the trapezoid containers are remarkably similar to the predicted ones in Figures 2(a) and 2(b). The rectangular absorber, in Figure 5(b), produces an equivalent critical damping ratio of about 5%, an approximately 10 fold increase from the uncontrolled case. The trapezoid absorber, in Figure 5(c), achieves an impressive rate of energy extraction from the structure during the first 2 s, with an equivalent damping ratio of about 12%, far superior

to that of the rectangular container. However, the rate of dissipation of the extracted energy is insufficient. And, oscillation magnitudes grow with a beat, significantly deteriorating the control performance.

Displacement history with three equally spaced 1-mm obstructions ( $t/h = 1/13 = 0.08-0.08-0.08$ ), is given in Figure 5(d). The effect of these small obstructions is similar to the earlier numerical predictions, practically eliminating the oscillations after the first beat. Increasing the left obstruction to 6 mm ( $t/h = 0.46-0.08-0.08$ ), shown in Figure 5(e), maintains the effectiveness of the small obstructions and eliminates the prominent beat. The equivalent critical damping ratio in Figure 5(e) is approximately 11.2%, the most effective control performance observed experimentally. The settling time for this case is little over 3 s, representing a rather impressive 95% improvement over the uncontrolled case (of 90 s).

Increasing the size of the smaller obstructions to 3 mm ( $t/h = 0.46-0.23-0.23$ ) in Figure 5(f), maintains the effective characteristics of the best controller, but marginally increases the settling time. In Figure 5(g), with 9 mm larger obstructions facing the travelling surface waves, ( $t/h = 0.69-0.23-0.69$ ), the rapid rate of energy dissipation is lost. The control is comparable to that of the rectangular absorber.

One inherent feature of nonlinear systems (such as the one discussed here with strong fluid-structure interactions), is dependence of dynamic response upon the magnitude of external disturbance. This dependence could be observed with the rectangular container when the initial displacement magnitude was doubled and halved, around the nominal 50 mm. Although smaller displacement retained about the same settling time, the larger initial displacement caused 50% deterioration. With the inclusion of obstructions, however, virtually no detectable variation could be observed with different initial displacements. Hence, even when the performance is comparable to that of the rectangular absorber, such as the one in Figure 5(g), there may be significant benefits of the suggested controller for events when the external disturbance is uncertain.

### Conclusions

A summary of numerical simulations is presented to improve the control performance of a trapezoid shaped sloshing absorber with designed obstructions. The objective is to quickly stop transient oscillations of a resonant structure. These predictions indicate that three equally-spaced obstructions sized to be 57%, 11%, 11% of the static liquid depth ( $t/h$ ), produce the best performance. Such a case is twice as effective as the more frequently used rectangular absorber.

Experimental observations confirm the numerical predictions, with the most effective obstruction sizes as 46%, 8%, 8% of the liquid depth. Experiments have a curvilinear displacement trajectory, as opposed to the rectilinear displacements of the numerical predictions. The advantage of the curvilinear trajectory over the rectilinear one, is to produce a comparable control performance with 1/3<sup>rd</sup> of the added controller mass. In addition, performance of the proposed configuration seems to be relatively insensitive to the varying disturbance magnitudes which may be a great benefit for practical implementation.

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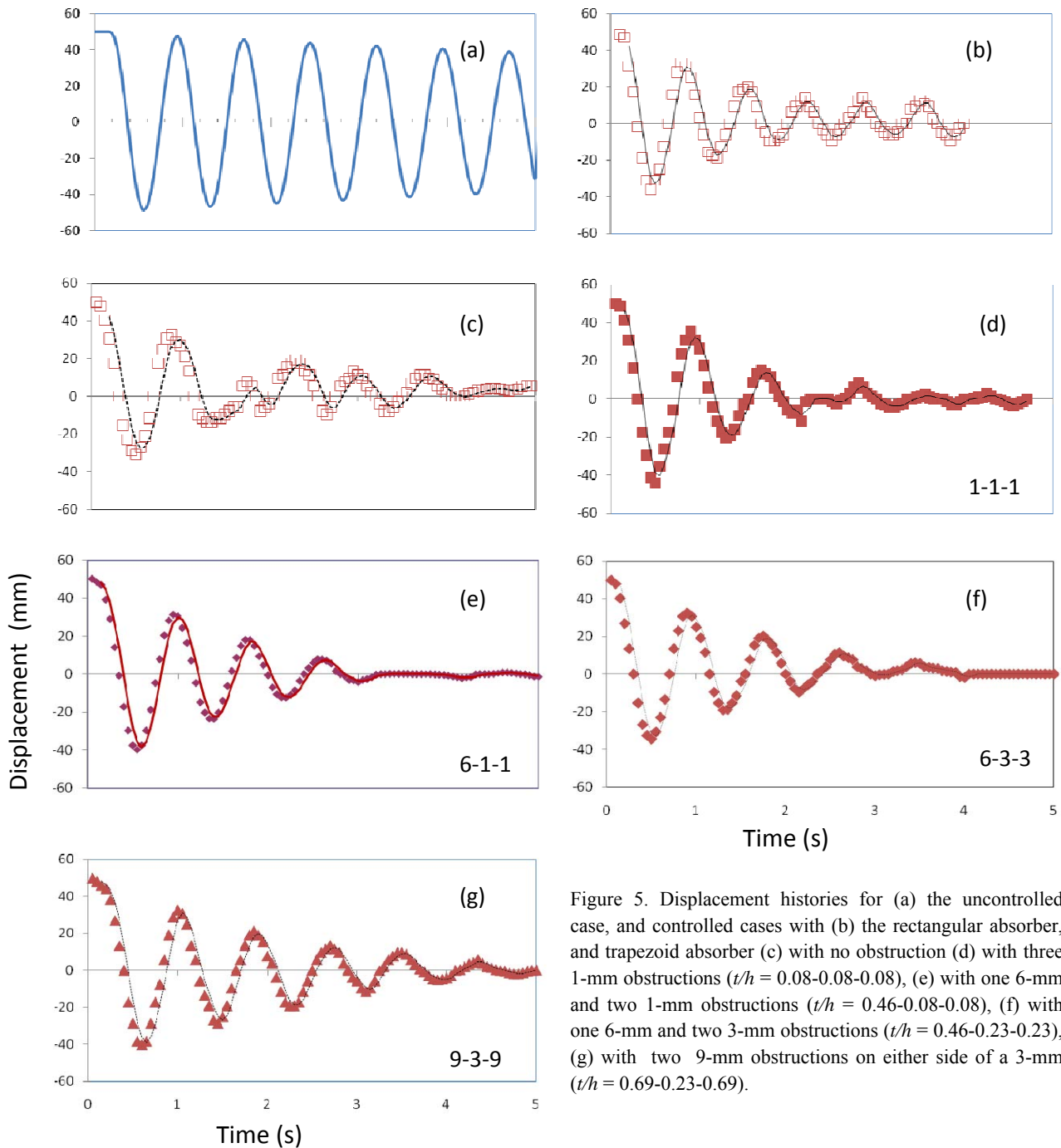


Figure 5. Displacement histories for (a) the uncontrolled case, and controlled cases with (b) the rectangular absorber, and trapezoid absorber (c) with no obstruction (d) with three 1-mm obstructions ( $t/h = 0.08-0.08-0.08$ ), (e) with one 6-mm and two 1-mm obstructions ( $t/h = 0.46-0.08-0.08$ ), (f) with one 6-mm and two 3-mm obstructions ( $t/h = 0.46-0.23-0.23$ ), (g) with two 9-mm obstructions on either side of a 3-mm ( $t/h = 0.69-0.23-0.69$ ).