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# Covariance integration approach to the determination of influence coefficients of internal pressure in a low rise building

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## Abstract

A covariance integration approach coupled with Eigenvalue analysis technique is used to determine the influence coefficients of internal pressure in a low-rise building with a dominant opening. The approach involves determination of the Eigenvalues and Eigenvectors of the fluctuating external pressure field around the opening obtained from wind tunnel experiments in order to evaluate the influence coefficients of internal pressure for different angles of wind attack. The size of the dominant opening area and angles of wind attack at which the internal pressure resonates are found to significantly influence the coefficients. In other words, influence coefficients in excess of unity at certain angles of attack are due to relatively strong Helmholtz resonance of internal pressure response contributed mainly by the first mode of fluctuating external pressure at the opening. The increasing porosity of the building envelope in the form of background leakage has a damping influence on the internal pressure fluctuations such that the magnitudes of the influence coefficients are reduced.

## Introduction

Wind induced internal pressures constitute a major portion of the load on low rise buildings, which represent a high percentage of the engineer-designed structures. The consideration of appropriate estimates of internal pressure is especially important for designing buildings in cyclone-prone areas where the potential of envelope damage due to debris impact leading to the creation of a dominant opening remains high. The first mathematical treatment of the dynamics of internal pressure in buildings with a dominant opening was presented by Holmes in his seminal paper [1] in which the internal pressure was conceived of as a response to the wind induced turbulent external pressure fluctuations near the opening. The theory supported by wind tunnel experiments showed that an analogy based on the Helmholtz acoustic resonator can be used to describe the response of internal pressure in a rigid non-porous building (building being treated as a Helmholtz resonator) using a second order non-linear differential equation. The derivation assumed a "slug" of air of area $A_o$ and effective length $l_e = \sqrt{\pi A_o / 4}$ to oscillate at the opening under the forcing of external fluctuating pressure, the stiffness being provided by the internal volume ($V_o$) of air acting as a pneumatic spring and damped by the irrecoverable energy lost due to flow past the opening. Since then, important theoretical contributions from Liu and Saathoff [2], Vickery and Bloxham [3], Sharma and Richards [4], Oh et al. [5] supported by wind tunnel and some full scale studies by others (Ginger and Letchford [6]) have greatly led to the development of a sound theoretical basis of internal pressure dynamics. A second order ordinary differential equation with non-linear damping of the form

$$\frac{\rho_a l_e V_e}{\gamma c A_o P_a}\ddot{C}_{pi} + C_L \frac{\rho_a q V_e^2}{2(\gamma A_o P_a)^2}\left|\dot{C}_{pi}\right|\dot{C}_{pi} + C_{pi} = C_{pe} \qquad (1)$$

has been established by the researchers to model the wind induced internal pressure response of a building cavity with an opening. In this equation, $\rho_a$ is the density of fluid (air in this case) inside the building cavity; $V_e$ is the effective volume of the cavity, being equal to $V_o$ (the nominal cavity volume) for a building with rigid envelope, and equal to $V_o(1+b)$ for a building with quasi-statically flexible envelope ($b$ being the ratio of the bulk modulus of air inside the cavity to that of the building envelope); $\gamma = 1.4$ is the ratio of specific heat capacities; $P_a$ is the ambient pressure of air; $c$ and $C_L$ are the discharge and loss coefficients of flow through the opening; $q = 0.5\rho_a \overline{U}_h^2$ is the ridge height dynamic pressure; and $C_{pi} = p_i / q$ and $C_{pe} = p_e / q$ are the internal and external pressure co-efficient respectively ($p_i$ and $p_e$ being the internal and external pressure around the opening). It is worth to note that significant differences regarding appropriate values for the ill-defined parameters ($c$, $C_L$ and $l_e$) still exist to date.

Equation (1) implies that under favourable forcing by the external pressure (i.e. with enough turbulence energy) near the opening, the internal pressure can exhibit a significant resonating response at the Helmholtz frequency of the cavity-opening combination much like the dynamic response of a structure under the fluctuating wind load. While the strongest resonance of internal pressure due to turbulent buffeting is expected for an onset flow normal to the opening, Sharma and Richards [7] have shown using wind tunnel tests that an even stronger resonance of internal pressure driven by "eddy dynamics" is possible at oblique flow angle under certain conditions irrespective of whether or not, the Helmholtz frequency of the building-opening combination lies in the energy containing region of the turbulent velocity spectrum. Thus, at the condition of a resonating internal pressure driven by turbulent buffeting or "eddy dynamics", the net dynamic load on the building envelope as a whole or in parts (such as claddings) may increase considerably leading to its failure.

This paper, drawing an analogy between the internal pressure response and the conventional dynamic response (such as moments, deflections etc) of structures driven by the fluctuating wind force, attempts to come up with appropriate influence coefficients (much like the structural influence coefficients used to weight the response of structures to dynamic loading) for internal pressure of low rise buildings with a single dominant opening from wind tunnel measurements. These influence coefficients are expected to incorporate the effect of true spatial properties of the fluctuating external pressure around the opening and hence can be used to determine the RMS (fluctuating) and peak internal pressures with more confidence. Out of a number of methods available for determining the spatial statistics of the fluctuating external pressure that forces the internal pressure response through the opening, the covariance integration method coupled with Eigenvalue analysis as described by Best and Holmes [8] has been used to determine the influence coefficients (and modal parameters) of the internal pressure coefficient of a low rise building with and without opening for different angles of wind attack. The approach involving analyses of the opening external and internal pressure records obtained from wind tunnel measurements reveal the importance of dominant opening size over which the external pressure is correlated and the angle of wind attack in determining the influence coefficients of the fluctuating internal pressures. It is found that the influence of the external pressure around the dominant opening in determining the extent and magnitude of internal pressure fluctuations is reduced with increasing leakage in the building envelope. Modal analysis of opening external pressure field at wind angles corresponding to high (resonating) internal pressure fluctuations in particular, also reveals the dominant contribution of the first mode of fluctuating external pressure at the opening.

## Covariance Integration method and Eigenvalue analysis

It is assumed for the purpose of this analysis that the pressures acting on the surface of the building around the opening are stationary, ergodic random processes. This will be close to reality when the mean velocity ($\overline{U}_h$) can be assumed to be nearly constant over the period of passage of storms.

The mean internal pressure $\overline{p}_i$ influenced by external pressure on the opening $\overline{p}_{e_j}$ acting over a tributary area $A_j$ with an influence coefficient $\beta_j$ is given by

$$\overline{p}_i = \frac{\sum_{j=1}^{n} A_j \beta_j \overline{p}_{e_j}}{\sum_{j=1}^{n} A_j} \tag{2}$$

where $n$ is the number of tributary areas representing the external pressure taps around the opening influencing the internal pressure. If $A_j = A_k \left(j \neq k\right)$, then equation (2) reduces to

$$\overline{p}_i = \frac{\sum_{j=1}^{n} \beta_j \overline{p}_{e_j}}{n} \tag{3}$$

Similarly the root mean square fluctuating internal pressure ($\tilde{p}_i$) can be shown to be given by

$$\tilde{p}_i = \left(\overline{p_i'^2}\right)^{1/2} = \left( \frac{\sum_{j=1}^{n}\sum_{k=1}^{n} A_j A_k \beta_j \beta_k \overline{p'_{e_j} p'_{e_k}}}{\sum_{j=1}^{n}\sum_{k=1}^{n} A_j A_k} \right)^{1/2} \tag{4}$$

Normalized by the ridge height dynamic pressure *q*, equation (4) can be expressed in coefficient form as

$$\tilde{C}_{pi} = \left( \frac{\sum_{j=1}^{n}\sum_{k=1}^{n} A_j A_k \beta_j \beta_k r_{jk} \tilde{C}_{pe_j} \tilde{C}_{pe_k}}{\sum_{j=1}^{n}\sum_{k=1}^{n} A_j A_k} \right)^{1/2} \tag{5}$$

where $r_{jk}$ is the correlation coefficient between the fluctuating pressures at taps $j$ and $k$ and $\tilde{C}_{pi}$ is the root mean square (RMS) value of fluctuating internal pressure. Assuming $A_j = A_k \left(j \neq k\right)$ and restating equation (5) in matrix form leads to

$$\tilde{C}_{pi} = \left( \frac{\{\beta\}^T [A]\left[\tilde{C}_{pe}\right][r]\left[\tilde{C}_{pe}\right][A]\{\beta\}}{n^2 [A]^2} \right)^{1/2} = \frac{1}{n}\left(\{\beta\}^T \left[\tilde{C}_{pe}\right][r]\left[\tilde{C}_{pe}\right]\{\beta\}\right)^{1/2} \tag{6}$$

where the RMS external pressure coefficient diagonal matrix [ $\tilde{C}_{pe}$ ] and the correlation coefficient matrix $[r]$, both of order *n* can be clubbed together to form the pressure coefficient covariance matrix ($[C_p]$) as

$$[C_p] = \left[\tilde{C}_{pe}\right][r]\left[\tilde{C}_{pe}\right] \tag{7}$$

The pressure coefficient covariance matrix [$C_p$] and the RMS internal pressure coefficient is evaluated from the sampled pressure data around the opening and inside the cavity respectively as obtained from wind tunnel measurements for each azimuth. Further if it is assumed that the influence coefficients $\beta_j, \left(j = 1:n\right)$ of external on internal pressure due to pressure measured at external taps equi-spaced along the opening are equal then the influence coefficient $\beta$ for a particular wind direction can be estimated using equations (6) and (7) as

$$\beta = \frac{n\tilde{C}_{pi}}{\sqrt{\sum_{j=1}^{n}\sum_{k=1}^{n} C_{p_{jk}}}} \tag{8}$$

Eigenvalue analysis can be applied to the pressure coefficient covariance matrix [ $C_p$ ] given by equation (7) such that the Eigenvalues $\lambda_j \left(j = 1:n\right)$ sorted in order in magnitude can be arranged into a $n^{th}$ order diagonal matrix $[\lambda]$ and the corresponding matrix of normalized Eigenvectors $[E]$ can be used for reduction into the diagonal form as

$$[\lambda] = [E]^{-1}[C_p][E] \tag{9}$$

A vector of modal parameter $\{\alpha\}$ can be defined such as

$$\{\alpha\} = [E]^{-1}\{\beta\} = [E]^T\{\beta\} \tag{10}$$

Using equations (6), (9) and (10), the RMS value of fluctuating internal pressure coefficient ($\tilde{C}_{pi}$) can be evaluated as

$$\tilde{C}_{pi} = \frac{1}{n}\sqrt{\{\alpha\}^T [E]^T [C_p][E]\{\alpha\}} = \frac{1}{n}\sqrt{\{\alpha\}^T [\lambda]\{\alpha\}} = \frac{1}{n}\sqrt{\sum_{j=1}^{n} \alpha_j^2 \lambda_j} \tag{11}$$

Equation (11) can be used to serve as a check for $\widetilde{C}_{pi}$ against the measured value while the relative contribution of different modes of the fluctuating external pressure around the opening to fluctuations in internal pressure can be evaluated from the values of modal parameter $\{\alpha\}$ for each wind direction.

## Experimental Details

The aerodynamic information including records of internal pressure and external pressure around the opening are obtained from wind tunnel tests using a 1:100 scale model of the Twisted Flow Wind Tunnel (TFWT) building of The University of Auckland at Tamaki. A 1:100 scale category 3 (AS/NZ 1170.2:2002 [9]) boundary layer profile was developed for the purpose of wind tunnel simulations.

The TWFT building has a large hall 35.1m by 24.9m by 7m housing the wind tunnel with an adjoining office space. Since the purpose of this study was to accurately simulate the internal pressure response of the hall with a large (roller) door opening 5m by 4.2m, being forced by turbulence external pressure fluctuations, care was taken to model the internal volume cavity for a ridge height velocity ratio (model to full scale) of 1:4 by the way of internal volume exaggeration below the wind tunnel floor. This is necessary to maintain the correct relative position of the Helmholtz frequency of the opening-cavity configuration with respect to the frequencies in the onset turbulent velocity spectrum of the wind at model scale similar to full scale. The adjacent office space was however modelled without cavity scaling and hence represents a space of much smaller size in full scale.

A total of 64 channels of pressure data including 51 external taps distributed evenly on the face containing the opening, 8 external taps on the opposite face, 4 internal taps in the hall cavity, 1 internal tap in the adjoining space and 1 channel for dynamic pressure were sampled simultaneously at 600Hz for 120 secs for different angles of attack [θ in Figure 1(a)] varying from 0 to 360° in 20° increments. Out of the 51 external pressure taps on the face containing the opening of dimensions 5cm by 4.2cm, a total of 11 taps [marked as red in Figure 1(b)] were evenly distributed around the opening at equal distance from each other. Data of external and internal pressures acquired with the opening completely sealed as well as with openings of different sizes (100%, 80% and 50% of the maximum size) for each angle of wind attack are used in the present analysis. In addition, the same cavity-opening configuration but with uniformly distributed leakages (with holes of 1mm diameter) of porosity 0.1% and 0.2% of the total wall area respectively was also investigated to determine the effect of typical building leakages on the influence coefficients at different angles of attack.

The differential transducers (range ~± 650 Pa, XSCL series, Honeywell Inc.) used for the study were referenced to the static pressure measured using a pitot tube placed 500mm (50m in full scale) above the wind tunnel floor approximately 12 building heights upstream of the model. The total pressure also measured simultaneously using the same Pitot tube was used to obtain the dynamic pressure (hence velocity) at that height. A suitable correction factor used to obtain the mean ridge height dynamic pressure for normalizing the internal and external pressure records and the frequency dependant transfer functions used to convert the spectral characteristics of fluctuating velocity (and dynamic pressure) from reference to ridge height was generated from separate set of tests earlier. A schematic of the model with the opening location and the arrangement of exaggerated volume attached below the wind tunnel floor are shown in Figures 1(a) and (b) respectively.

## Results and Discussion

Influence coefficients of opening external pressure on fluctuating internal pressure for different angles of attack calculated using Equation (8) are plotted for nominally sealed (0% opening) building and with dominant openings of different sizes in Figure 2(a). Figure 2(b) plots the same for 0 and 100% opening but with uniformly distributed background leakage holes (1mm dia.) of porosity approximately 0.1% concentrated on roof.

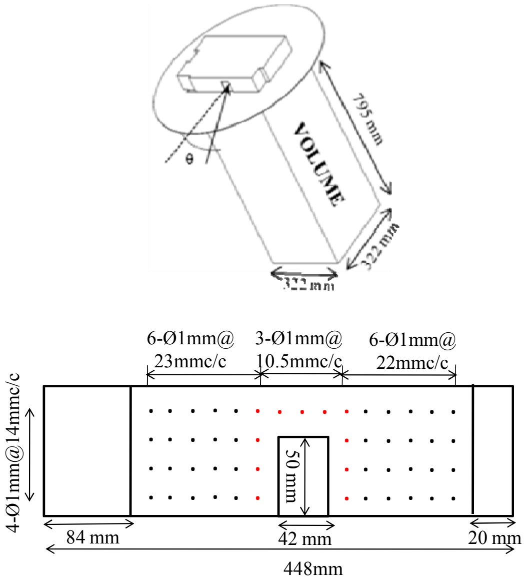

Figure 1. (a) Schematic of the wind tunnel model along with volume scaling and (b) Layout of external pressure taps on the windward wall with the opening (All dimensions in mm)

The influence coefficients for internal pressure vary significantly with the size of the opening. In fact the influence coefficients for 100% opening case are consistently 2-3 times higher than that for the sealed configuration implying that the size of the opening over which the external pressure acts is the most important factor influencing the internal pressure fluctuations.

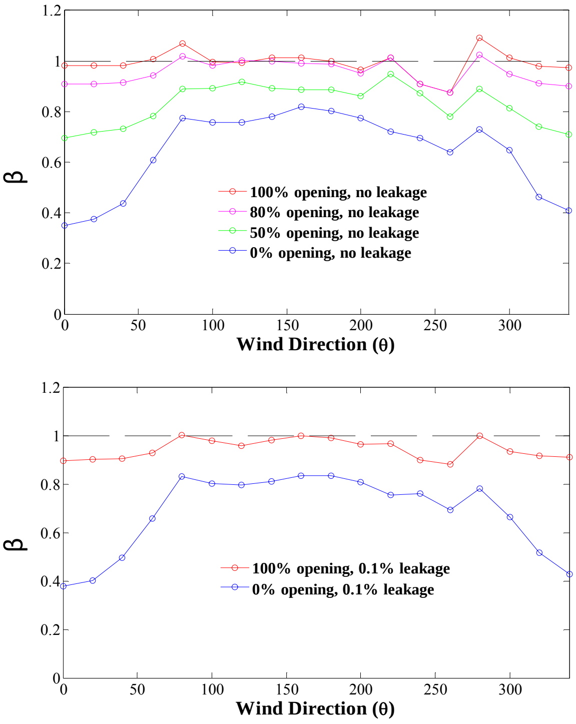

Figure 2. Influence coefficient for a building with and without dominant opening with (a) no background leakage and (b) 0.1% leakage for different angles of attack

The influence coefficients at ±80° and 220° in excess of unity for the building with dominant opening without leakage indicate possibility of increased fluctuations of internal pressure compared to the external pressures at these angles. This is further demonstrated in Figure 3 which shows the gain of fluctuating internal pressure over the area averaged external pressure at these angles of wind attack (θ). Higher gains of internal pressure at the frequency of 33Hz corresponding to the theoretical Helmholtz frequency given by Equation (1) support the occurrence of oblique flow Helmholtz resonance of internal pressure due to "eddy dynamics" mentioned earlier.

Typical porosity of modern buildings in Australia/New Zealand range from 0.01% to 0.2% and hence representative leakages of 0.1% and 0.2% concentrated on roof was chosen to determine its effect on the influence coefficients of external pressure on fluctuating internal pressure of a building with a dominant opening.

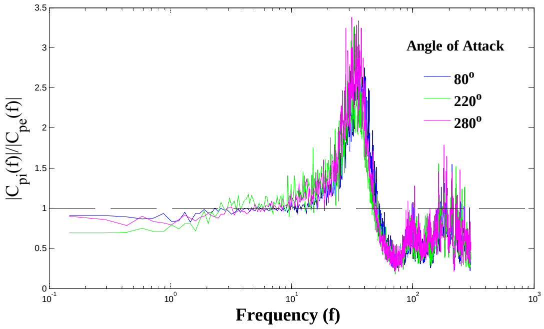

Figure 3. Frequency dependant gain of internal over external pressure

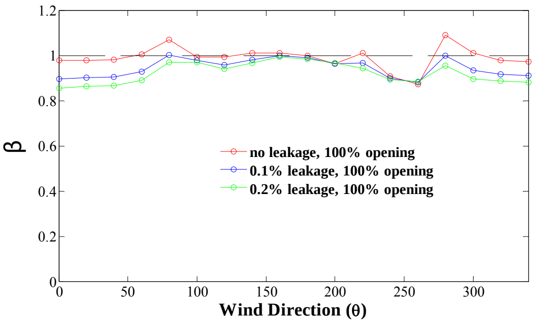

Figure 4. Effect of background leakage (porosity) on the influence coefficients of internal pressure of a building with a dominant opening

Figure 4 plots the influence coefficients with and without background leakage. The effect of background leakage on fluctuating internal pressure is such that the influence of the external pressure around the dominant opening in determining the magnitude of internal pressure fluctuations is reduced due to increasing influence of external pressure around individual leakage holes with increasing size (i.e. porosity ratio).

The relative contributions of each of the 11 modes calculated as per Equation (10) to internal pressure fluctuations for 0, 80, 220 and 280 degree angles of attack corresponding to relatively higher values of influence coefficient $\beta$ are plotted in Figure 5(a) for completely sealed building and Figure 5(b) for a building with a dominant opening without any leakage respectively.

The first modal parameter $\{\alpha\}$ for the given wind directions is much higher in magnitude compared to the other modal parameters and is expected to have contributed the maximum to internal pressure fluctuations. The magnitude of the first modal parameter can also be seen to be greater for the building with dominant opening than for the completely sealed building reflecting the effect of opening area size on the internal pressure response of a building.

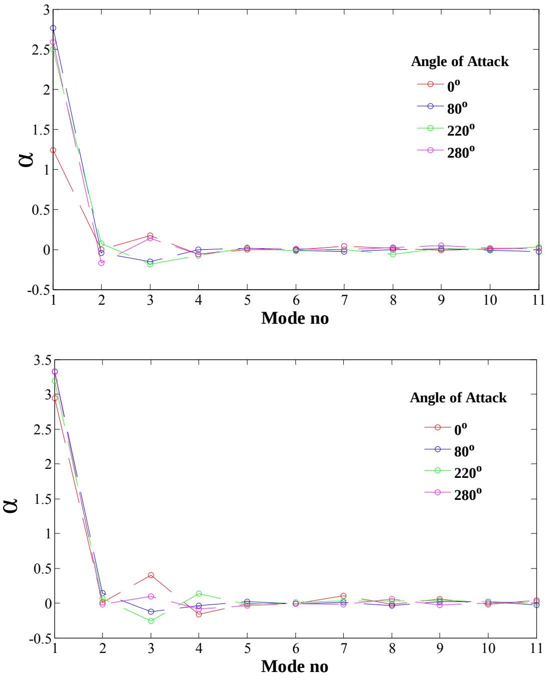

Figure 5. Relative contribution of different modes on $\alpha$ for (a) completely sealed and (b) building with a dominant opening at different angle of wind attack

## Conclusion

Influence coefficients of the internal pressure for a building with and without dominant opening are estimated for different angles of wind attack. The influence coefficients are found to be 2-3 times higher for the case of a building with a dominant opening compared to that for a well sealed building; implying that the size of the opening over which the external pressure acts is the single most important factor influencing the internal pressure fluctuations. In particular, values of influence coefficients in excess of unity observed at oblique angle(s) of attack (±80°) for the building with dominant opening are due to Helmholtz resonance through "eddy dynamics" rather than turbulent buffeting. It is also found that the influence of the external pressure around the dominant opening in determining the extent and magnitude of internal pressure fluctuations is reduced with increasing leakage porosity of the building envelope. Eigenvalue analysis of the opening external pressure covariance matrix indicates that 80% of internal pressure fluctuations are contributed by the first mode while the contributions of higher (standing wave) modes are less significant.

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