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Modelling of Heat and Mass Transfer Involving Vapour Condensation in the Presence of Non-Condensable Gases

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Abstract

Mathematical models are employed to model water vapour condensation in an air-vapour mixture flow in a plane channel cooled by liquid water. A set of differential and algebraic equations is derived. The combination with models based on analogies between heat and mass transfer, these equations are solved simultaneously for the water vapour mixture, the condensate and the coolant flows. Numerical predictions are compared with experimental results from literature and good agreement has been found.

Introduction

Condensation of water vapour flowing over a cold surface is initiated and sustained when the temperature of the surface is maintained below the dew-point temperature of the vapour. In the process the latent energy of the vapour is released, heat is transferred to the surface, and a condensate is formed. The pioneering work in the field of condensation is due to Nusselt [1] who predicted, from a simplified theoretical analysis, the heat transfer coefficient of stationary pure vapour in film condensation on a vertical flat plate. Improvements and modifications to Nusselt's theoretical solution have been made by a number of researchers. For example, Bromley [2] assumed a linear temperature distribution in the liquid film. Rohsenow et al. [3] and others considered the effect of interfacial shear stress on both condensation flow and the effect of the vapour velocity diminishing along the length of tube condensers. Analyses that make use of the analogy between heat and mass transfer in situations involving condensation have been presented by many researchers. Colburn and Hougen [4] were the first to develop a theory for condensation mass transfer which was controlled by the mass concentration gradient through a non-condensable layer. They described the heat transfer process as the sum of sensible heat and latent heat flows. Dehbi et al. [5] derived a theoretical prediction of heat and mass transfer from a steam and non-condensable gas mixture in a vertical tube condenser. An algebraic equation for the film thickness was derived. The mass and heat transfer analogy was invoked to deduce the condensation rate. Che et al. [6] used the method of Colburn and Hougen [4] to analyze the heat and mass transfer process for the condensation of water vapour from moist air on a tube. Siddique et al. [7] conducted an analytical study using the analogy between heat and mass transfer. Their model included the effects of developing flow, condensate film roughness, and property variation in the gas phase. Lee and Kim [8] developed a theoretical model by improving Siddique et al.'s [7] analytical model and investigated steam condensation heat transfer in the presence of air or nitrogen gas in a vertical tube. Ambrosini et al. [9] conducted experiments in a plane channel to study the heat and mass transfer in the condensation of vapour from humid air. Recently, Li, Saraireh and Thorpe [10] have derived a set of differential and algebraic equations mode model the heat and mass transfer in tube condensers and the model predictions were compared with experimental results from the literature.

In this paper we derive equations for the heat and mass balance on heat exchangers involving water vapour condensation from gas-vapour mixture flows with water as the coolant in plane channels. The equations, in combination with many theoretical models for heat and mass transfer for gas-vapour mixture, model the heat and mass transfer in the mixture channel, the condenser wall and the coolant channel, and will be solved numerically and the predictions will be compared with experimental results reported in the literature.

Heat and Mass Transfer analysis

The model studied in this work consists of two fluid streams separated by a solid wall. Cooling water flows vertically inside one of the channels, whilst a mixture of air and water vapour flows vertically in the other channel. The water vapour condenses on the cool wall and a liquid film forms that flows downwards under the influence of gravity. A schematic of a condenser showing the important states is presented in Fig. 1. The condenser is divided into a number of small elements of incremental length dx and each segment acts as a control volumes for the air-vapour mixture (1), film condensation (2), wall material (3) and the cold water stream (4) as shown in Fig. 1.

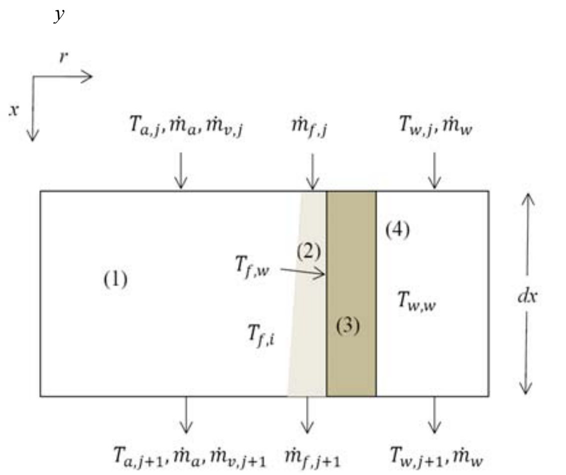

Figure 1: Schematic diagram of the small element in considering control volume analysis.

The temperature of the air-vapor mixture entering the element is designated by $T_{a,j}$, the inlet mass flow rate of water vapor by $\dot{m}_{v,j}$, the outlet temperature by $T_{a,j+1}$, an outlet mass flow rate of water vapor by $\dot{m}_{v,j+1}$, the mass flow rate of dry air is $\dot{m}_a$. The mass flow rates of the condensate film entering and leaving the element are designated $\dot{m}_{f,j}$, $\dot{m}_{f,j+1}$, a film thickness is δ, and the temperature of the film/vapor interface is assigned $T_{f,i}$. A solid wall separates the condensate film and the coolant and the temperatures of the wall adjacent to each of the fluids are $T_{f,w}$ and $T_{w,w}$ respectively. The cooling water entering the element is $T_{w,j}$, and $T_{w,j+1}$is the outlet temperature. The mass flow rate of the cooling water is $\dot{m}_w$.

Following the analysis by Li et al. [10], a heat balance of the air-vapor mixture channel can be expressed as:

$$-\left(\dot{m}_a C_{pa} + C_{pv}\dot{m}_v\right)\frac{dT_a}{dx} - \left(h_{fg,0} + C_{pv}T\right)\frac{d\dot{m}_v}{dx} = h_{av}(T_a - T_{f,i}) \quad (1)$$

where C_{pa} and C_{pv} are the specific heats of dry air, and water vapour respectively, and $h_{fg,0}$ is the latent heat of water vapour at 0^oC (the reference temperature), The total heat transfer from the air-vapour mixture to the condensate film comprises sensible heat Q_s and the latent heat Q_l, i.e.,

$$\begin{aligned} Q &= Q_s + Q_l \\ &= h_a dx(T_a - T_{f,i}) + (\dot{m}_{v,j} - \dot{m}_{v,j+1})h_v \\ &= h_{av} dx(T_a - T_{f,i}) \end{aligned} \quad (2)$$

which can be rearranged to give

$$h_{av} = h_a - \left[\frac{h_v}{T_a - T_{f,i}}\frac{d\dot{m}_v}{dx}\right] \quad (3)$$

where h_{av} is the overall heat transfer coefficient including the sensible and latent heat, and h_v is the heat transported by the water vapor to the condensate film as water vapor and which is subsequently condensed liquid water. The rate of condensation can be calculated from Fick's law, which can be expressed as

$$-d\dot{m}_v = (\dot{m}_{v,j} - \dot{m}_{v,j+1}) = Kdx(C_v - C_{v,i}) \quad (4)$$

in which K is the mass transfer coefficient that governs the rate of transport of water vapour, C_v is the average concentration of water vapour in the air-vapour mixture, $C_{v,i}$ is the water vapour concentration at the interface between the mixture and the condensation, and it is function of the interface temperature $T_{f,i}$. By assuming that the bulk density ρ of the air-vapour mixture is constant across the channel through which the mixture flows, the above equation can be written as

$$\frac{d\dot{m}_v}{dx} = -K\rho(w_v - w_{v,i}) \quad (5)$$

where w_v is the mass fraction of the water vapour in the air-vapour mixture, and it is related to specific humidity, ϖ , by

$$w_v = \frac{\dot{m}_v}{\dot{m}} = \frac{\dot{m}_v}{\dot{m}_a + \dot{m}_v} = 1 - \frac{1}{1+\varpi} \quad (6)$$

and $\dot{m}$ is the mass flow rate of the humid air.

After the analysis of Nusselt (1916) for the liquid film, the heat balance on the condensate film can be calculated as:

$$h_{av}(T_a - T_{f,i}) = k_f\frac{T_{f,i} - T_{f,w}}{\delta} + \frac{d(\dot{m}_f h_f)}{dx} \quad (7)$$

where h_f is the specific enthalpy of the condensate. The last term in the above equation takes into account the energy change of the condensate as it flows down the film.

The heat transfer through the solid wall can be simplified to:

$$\frac{k_f(T_{f,i} - T_{f,w})}{\delta} = \frac{k_s(T_{f,w} - T_{w,w})}{\delta_s} \quad (8)$$

where k_s and k_f are the thermal conductivities of the solid wall and liquid film, respectively, and δ_s is the thickness of the solid wall.

The heat transfer from the solid wall to the stream of cooling channel should be balanced by the heat transfer in the coolant. Hence

$$\frac{k_s(T_{f,w} - T_{w,w})}{\delta_s} = h_w(T_{w,w} - T_w) \quad (9)$$

where h_w is the convective heat transfer coefficient of the water in the coolant channel. A thermal energy balance on the cooling water shown in Fig. 1 results in the following expression

$$C_{pw}\dot{m}_w\frac{dT_w}{dx} = h_w(T_{w,w} - T_w) \quad (10)$$

i.e the heat transfer into the coolant will cause the water temperature to change.

The above equations are derived for condensers of co-current flows. For those of counter current flows, these equations can still be used except that the mass flow rate of the cooling water needs to be taken as negative value. In this case, we are assuming that the cooling water in Fig. 1 flows upwards which is opposite to the flow direction of the gas-vapour mixture.

Models and Formulations

The heat transfer coefficients h_a and h_w for the air-vapour mixture and water flows can be obtained from the correlations for Nusselt number in channel flows

$$Nu = \frac{hd}{k} = f(Re_d, Pr) \quad (11)$$

Here Pr is the Prandtl number, and

$$Re_d = \frac{\rho U d}{\mu}$$

where d is the hydraulic diameter of the channel for air-vapour mixture or the water channel. The mass transfer coefficient K can be obtained by using correlation for the Sherwood number

$$Sh = \frac{Kd}{D} = f(Re_d, Sc) \quad (12)$$

Here Sc is the Schmidt number and D is the molecular diffusivity that can be calculated as a function of temperature as recommended by Rao et al. [11]

$$D = \frac{1.87\times10^{-10}\times T^{2.072}}{P} \quad (13)$$

where P is the total pressure in the air-vapour mixture. The saturation vapour pressure of water is a function of temperature one form of which is given by Rao et al. [11], namely

$$P_v = exp(77.3450 + 0.0057T - 7235/T)/T^{8.2} \quad (14)$$

where T is in Kelvin. The specific humidity of humid air can be calculated as

$$\varpi = \frac{m_v}{m_a} = \frac{0.622P_v}{P - P_v} \quad (15)$$

When the mass fraction of water vapour in the mixture is high, it is important to consider the effect of suction on heat and mass transfer which arises as a result of steep temperature and concentration gradients. Kays and Moffat [12] obtained the following correlation that accounts for for the suction effects:

$$Nu_x = \left[exp\left(\frac{\dot{m}_v'' Re Pr}{G_{mix} Nu_{o,x}}\right) - 1\right]^{-1}\left(\frac{G_{mix}}{\dot{m}_v'' Re Pr}\right)^{-1} \quad (16)$$

where $\dot{m}_v''$ is the condensate mass flux at the interface between the air-vapour mixture and the condensate film and G_{mix} is the mass flux of the mixture. From the analogy between heat and mass transfer, Equation (16) can be written as:

$$Sh_x = \left[exp\left(\frac{\dot{m}_v'' Re Sc}{G_{mix} Sh_{o,x}}\right) - 1\right]^{-1}\left(\frac{G_{mix}}{\dot{m}_v'' Re Sc}\right)^{-1} \quad (17)$$

After taking into account the suction effect on the mass transfer at the gas film interface, the mass flux at the interface can be calculated using the model of Bucci et al. [13]

$$\dot{m}'' = \rho K ln(1 + B_m) \quad (18)$$

where $\dot{m}''$ is the mass flux of water vapour and

$$B_m = \frac{w_{v,i} - w_{v,b}}{1 - w_{v,i}} \quad (19)$$

in which $w_{v,i}$ is the mass fraction of water vapour at the gas-liquid interface and $w_{v,b}$ is the bulk mass fraction of water vapour in the air-vapour mixture. In Eqs (16) and (17), $Nu_{o,x}$ and $Sh_{o,x}$ denote the respective local Nusselt and Sherwood numbers after taking into account the developing flow in the thermal entrance region as suggested by Reynolds et al. [14]

$$Nu_{o,x} = Nu_o\left[1+\frac{0.8(1+7\times10^4 Re^{-3/2})}{(x+x_0)/d}\right] \tag{20}$$

Using the heat and mass transfer analogy, the Sherwood number including the developing length effect can be written as:

$$Sh_{o,x} = Sh_o\left[1+\frac{0.8(1+7\times10^4 Re^{-3/2})}{(x+x_0)/d}\right] \tag{21}$$

where x_0 is an initial entrance length.

The Nusselt and Sherwood numbers for fully developed flows without suction effect can be calculated as, following Holman [15]:

$$Nu_o = 1.04\times0.0395Re^{0.75}Pr^{1/3} \tag{22}$$

$$Sh_o = 1.04\times0.0395Re^{0.75}Sc^{1/3} \tag{23}$$

The subscript *o* represents fully developed flows. As suggested in Li et al [10], the factor 1.04 is included for constant heat flux boundary condition even though the factor is small.

As condensation proceeds along the channel, the flow of the condensate changes from laminar flow to turbulent flow, and the film surface becomes rough and wavy. This roughness effect is modelled using the correlations suggested by Norris [16] for the heat transfer over rough surfaces

$$Nu_{o,r} = Nu_{o,s}\left(\frac{f_r}{f_s}\right)^{nh} \tag{24}$$

Using the heat and mass transfer analogy, one may write:

$$Sh_{o,r} = Sh_{o,s}\left(\frac{f_r}{f_s}\right)^{nm} \tag{25}$$

where the subscripts r and s refer to the rough and smooth conditions, $nh = 0.68Pr^{0.215}$, $nm = 0.68Sc^{0.215}$ and f_r/f_s represents the ratio of the Fanning friction factor for a rough wall to that for a smooth wall as suggested by Norris [16]. For smooth walls the friction factor is calculated from

$$f_s = 0.316\times Re^{-0.25} \tag{26}$$

The friction factor for a rough wall can be found using Haaland [17] correlation

$$\frac{1}{f_r^{1/2}} = -1.8log_{10}\left[\frac{6.9}{Re}+\left(\frac{\varepsilon/d}{3.7}\right)^{1.11}\right] \tag{27}$$

where Re is air-vapour mixture Reynolds number. The friction factor f_r increases as the surface roughness increases. A conservative assumption of that the roughness height ε is equal to half the film thickness δ is chosen, as suggested by Siddique [7]. In this paper, we apply the corrections due to film roughness starting from $x = 0$ and we assuming that all the flows are turbulent.

The interfacial shear stress is calculated as

$$\tau_g = f\rho_g\frac{(U_b-u_{fb})^2}{2} \tag{28}$$

where U_b is the bulk velocity of the air-vapour mixture, u_{fb} is the mean velocity of the water liquid in the condensate film, and ρ_g is the density of the air-vapour mixture.

Solution Procedure

The full system of seven differential and algebraic equations describing the heat and mass transfer for condensation from air-vapour mixture in a tube condenser are given by Eqs. (1), (3), (5), (7), (8), (9) and (10). A computer program has been written in MATLAB® to solve these equations. In the program, the differential equations were discretized into finite difference equations using the first order forward Euler method, and the calculations were performed using the fourth-order Runge-Kutta method to solve the differential and algebraic equations to obtain the heat flux, condensation rate, and outlet water temperature. For detail procedures of the numerical solution, see Li et al [10]

Results and Discussion

The present theoretical work is validated with the experimental data of Ambrosini et al. [9] who conducted experiments in a 2 m long square cross sectional plane channel (0.34 × 0.34) m to study the heat and mass transfer in the condensation of water vapour from humid air. They obtained experimental data from five tests at various inlet conditions for heat fluxes along the channel, the total condensation rate, and the outlet water temperature. These data were related to five operating conditions characterised by a nominal value of the secondary coolant temperature close to 30 °C, a steam generator power of 10 kW and mixture velocities from 1.5 to 3.5 m/s, which are shown in Table 1. Numerical results are obtained from the present work for heat flux, condensation rate and outlet water temperature $T_{w,out}$, for the same conditions of inlet water temperature $T_{w,in}$, inlet mixture velocity V, inlet air temperature T_a, and the relative humidity ϕ of the air at the inlet as those of Ambrosini et al. [9]. The heat fluxes, condensation rate and outlet water temperature computed from the present model are shown in Figs 2, 3 and 4, respectively, which show satisfactory agreement with the experimental data of Ambrosini et al. [9].

Test	$T_{w,in}[^oC]$	$V_{mix}[m/s]$	ϕ	$T_{a,in}$
1	31.24	1.46	100	82.66
2	31.10	2.02	100	80.61
3	31.07	2.52	97.83	79.13
4	30.90	3.01	87.35	78.73
5	30.71	3.59	96.55	75.02

Table 1: Experimental conditions from Ambrosini et al. [9].

Fig. 2 shows the comparison between the heat flux estimated by the theoretical model $Q=h_{av}(T_a-T_{f,i})$ and the experimental data of Ambrosini et al. [9]. It can be seen from the figure that the trend predicted by the current models agree with the experimental data and there is some discrepancy between the model predictions and the experimental data. The predicted heat flux shows a sharp decrease at $x = 0$ and this sharp increase occurs over a shorter distance than that from the experimental data. This means that Eqs (20) and (21) may not fully taken into account the effects resulting from the developing flow in the entrance region of the mixture channel. In general, the figure shows that for x < 0.4 m from the vapor-air mixture inlet, the model predictions are less than the experimental results but for x > 0.4 m, the model predictions are higher than the experimental results. The discrepancy may be due to some possible experimental errors or the simple 1D model we have used here. Ambrosini et al. [9] made the predictions using CFD simulation and found better agreement between the experimental results and the CFD results.

The overall condensation rates computed from the present model for the different experiments conditions as listed in Table 1 are shown in Fig. 3. The predicted overall condensation rates were calculated by summing $-d\dot{m}_v$ along the channel and these can also be calculated by the inlet and outlet mass flow rates of the water vapour in the air-vapour mixture channel. The results show a good agreement between the calculated and experiments data, with the maximum relative error being less than 9% between the predictions and the experimental results.

Fig. 4 shows the comparison between the predicted outlet water temperature and the experimental data of Ambrosini et al. [9]. It can be seen from the figure that the predicted and experimental results are in very good agreement.

Conclusions

A mathematical model has been developed to study condensation of water vapour in the presence of air, where the vapour mixture is flowing downward and the cold water flows upward. A comparative study of heat flux, condensation rate, and the outlet water temperature with experimental data has been made. The results show that the mathematical model in this study can predict the heat and mass transfer phenomena in the condensing heat exchangers with good accuracy.

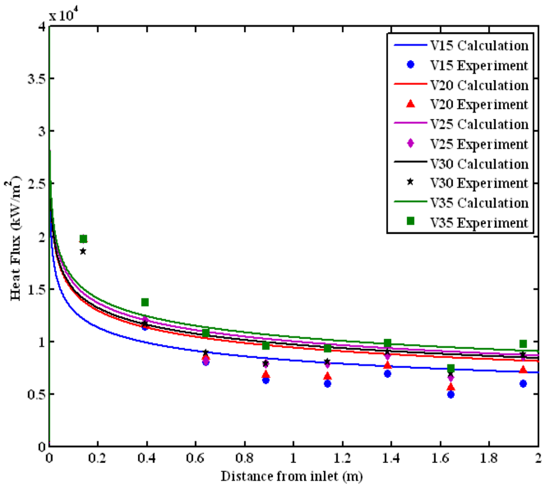

Figure 2. Comparison of the predicted and experimental heat flux for all the tests of Ambrosini et al. [9].

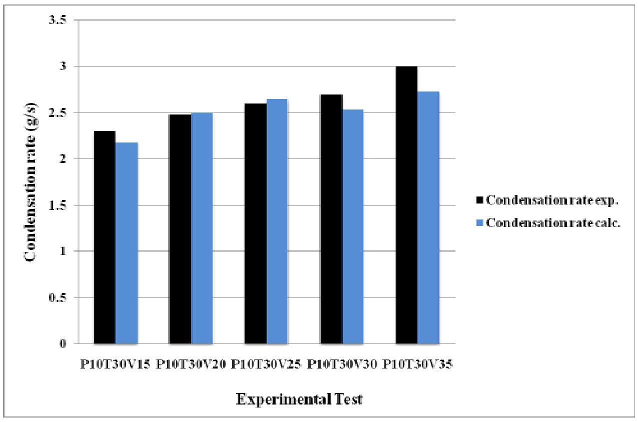

Figure 3. Comparison of the predicteded and experimental values of condensation rate for all the tests of Ambrosini et al. [9].

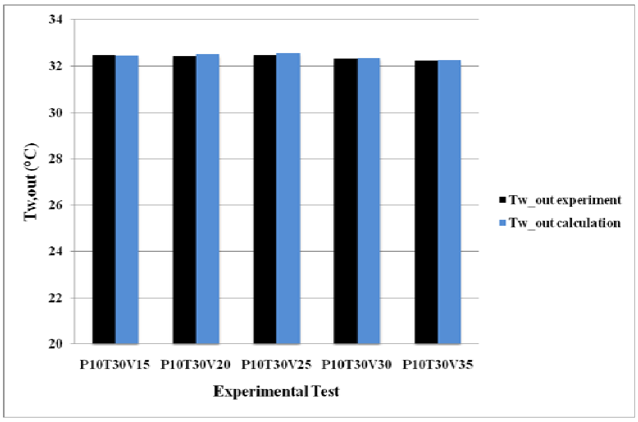

Figure 4. Comparison of the predictedted and experimental values of outlet water temperature $T_{w,out}$ for all the tests of Ambrosini et al. [9].

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