Numerical Simulation of Flow-Driven Flapping-Wing Turbines for Wind and Water Power Generation

J. Young¹, M.A. Ashraf², J.C.S. Lai¹ and M.F. Platzer²

¹School of Engineering and Information Technology
University of New South Wales
Australian Defence Force Academy, Canberra, ACT 2600, Australia

²AeroHydro Research & Technology Associates
Pebble Beach, California 93953, USA

Abstract

A fully flow-driven flapping wing turbine was simulated using a 2D Navier Stokes solver with two-way fluid-structure interaction, at a Reynolds number based on free stream flow of 1100. Efficiencies of up to 30% based on the Betz criterion were found, commensurate with values reported in the literature for prescribed-motion studies. Non-sinusoidal foil pitching and plunging motions were found superior to sinusoidal motions.

Introduction

Rivers, tidal flows and wind are important in the search for alternative renewable power sources that can reduce the carbon footprint of industry. Existing technologies typically use a rotary turbine, oriented with the axis of rotation aligned to the oncoming flow. In the wind turbine application, large blades and high rotation rates may cause excessive noise due to high tip speeds, and can also be a hazard to birds. In the tidal flow application, shallow water may preclude the use of large diameter blades, limiting the power that can be extracted by a single turbine.

In recent years flapping wings have been suggested as an innovative alternative strategy. Insects flapping their wings at low Reynolds numbers employ leading edge vortices to generate instantaneous forces several times greater than can be achieved with conventional attached aerodynamics [1]. Flapping wing turbines may be able to exploit the same techniques at the low Reynolds numbers associated with slow flow speeds. Indeed Kinsey and Dumas [2] have shown that leading edge vortex generation is an important part of optimal power generation at \( Re = 1100 \). Shallow water limitations may be overcome by employing a vertical flapping motion and extending the span as far as desired. Aniprop [3] demonstrated a small scale oscillating hydrofoil power generator developing 1 kW in a river, and the Engineering Business Limited Stingray generator produced 145 kW peak and 50 kW average in a 3.5 knot tidal flow [4].

Much of the analysis and experiment in flapping wing power generation has focussed on fully prescribed motions, or with one mode (e.g. pitch) prescribed and one (e.g. plunge) flow-driven. Kinsey and Dumas [2] mapped the power generation capacity of a pitching and plunging foil with both modes following prescribed sinusoidal motions, using a Navier-Stokes solver. They found an optimum pitch angle and flapping frequency combination, for a given plunge amplitude and phase angle between pitch and plunge, that gave an efficiency of 34% based on the Betz criterion and extracting power from both modes. Zhu et al [5-6] used inviscid and viscous flow solvers to study a foil with a prescribed sinusoidal pitch motion, and a flow-driven plunge motion. The efficiency, based on the power output from the plunge mode minus the power input into to the pitch mode, was comparable to Kinsey and Dumas. The Stingray full-scale demonstrator similarly drove the pitch angle of the wing with a prescribed motion, and power was extracted from the resulting flow-driven plunge motion. In contrast, a fully flow-driven turbine mechanism can be made much simpler than a device requiring power to be supplied to drive one of the motion modes. Platzer [7-8] has demonstrated a fully flow-driven turbine, and detailed water tunnel testing is currently underway. The present work presents a numerical study of fully flow-driven flapping wing motions.

Model Kinematics

The flapping wing turbine is modelled as a two-dimensional foil undergoing both pitching and plunging motions, and linked to a flywheel as shown schematically in figure 1.

Power extraction from the system is modelled as a rotational viscous damper attached to the flywheel. The equation of motion of the combined foil-flywheel system (ignoring the mass of the moving linkage elements) is determined via conservation of energy as:

\[
\frac{1}{2}m_a \dot{y}^2 + l_a \dot{\theta}^2 + l_b \dot{\beta}^2 = L \dot{y} + M \dot{\theta} - c_b (\dot{\beta})^2
\]

where \( y(t) \) = vertical position of the foil, \( \theta(t) \) = foil pitch angle, \( \beta(t) \) = flywheel rotation angle, \( m_a \) = foil mass, \( l_a \) = moment of inertia about the pivot point, \( l_b \) = moment of inertia of the flywheel, \( L \) = aerodynamic lift on the foil, \( M \) = aerodynamic moment on the foil about the pivot point, and \( c_b \) = the strength of the rotational damper on the flywheel.

The power output of the turbine is measured as the time-average of the rate of energy dissipation in the damper:

\[
P = \frac{1}{T} \int_{t}^{t+T} c_b (\dot{\beta})^2 \, dt
\]

The efficiency is determined via comparison with the available power in the flow passing through the frontal area swept by the foil:
\[ P_a = \frac{1}{2} \rho U_\infty^3 ds \]  
\[ \eta = P/P_a \]  
(3)  
(4)

where \( \rho \) = fluid density, \( U_\infty \) = free stream velocity, \( d \) = the total extent of motion of the trailing edge, and a unit span \( s \) is assumed. Analysis is simplified by reducing the system from two degrees of freedom (foil position \( y \) and angle \( \theta \)) to a single degree of freedom (flywheel angle \( \beta \)) as follows. The plunge and pitch motions are modelled as arbitrary functions of \( \beta \), so that \( y = f(\beta) \) and \( \theta = g(\beta) \) (hereafter referred to as ‘linkage functions’). The equation of motion of the foil is then solved in terms of \( \beta \):

\[ \ddot{\beta} = \left[ \frac{\left( m_a(f_\beta)^2 + l_a(g_\beta)^2 + l_b \right)}{\left( m_a f_\beta + l_a g_\beta g_\beta \right) f(\beta)^2} \right. \]  
\[ \left. - \frac{(m_a l_b f_\beta + l_a g_\beta g_\beta)(\dot{\beta})^2}{\left( m_a f_\beta + l_a g_\beta g_\beta \right) f(\beta)^2} \right] \]  
(5)

where \( f_\beta = \partial f / \partial \beta \), \( g_\beta = \partial g / \partial \beta \), etc.

The relationship between plunge and pitch motions is encompassed within the definition of the linkage functions \( f(\beta) \) and \( g(\beta) \). For example, a simple definition would be \( y = h \sin(\beta) \) and \( \theta = \theta_0 \sin(\beta + \phi) \), where \( h \) = non-dimensional plunge amplitude, \( \theta_0 \) = pitch amplitude and \( \phi \) = phase angle between pitch and plunge. In this case if the flywheel rotated at a constant rate, the plunge and pitch motions would be sinusoidal in time, but this is not guaranteed by the solution of equation (5), and hence non-sinusoidal motions may be achieved even with sinusoidal linkage functions.

The generality of the formulation allows a much wider range of flapping kinematics to be considered, however. The motion of the foil in the water-tunnel experimental model is characterised by translation of the foil at a constant pitch angle, with periods of rapid rotation of the foil at the top and bottom of the stroke. This may be incorporated into the pitch angle linkage function via a ‘stroke reversal fraction’ \( \Delta \beta_{\text{rev}} \) representing the fraction of the total flapping cycle over which the foil reverses pitch angle, where \( \Delta \beta_{\text{rev}} = 0.1 \) represents rapid pitch reversal, and \( \Delta \beta_{\text{rev}} = 0.5 \) represents fully sinusoidal pitch variation. For plunge motion given by \( y = h \sin(\beta) \), the pitch motion at the top of the upstroke is given by:

\[ \theta = -\theta_0 \sin \left( \frac{\beta + \phi - \Delta \beta_{\text{rev}}}{2\Delta \beta_{\text{rev}}} \right) \]  
for \( 1 - \Delta \beta_{\text{rev}} \pi - \phi \leq \beta \leq (1 + \Delta \beta_{\text{rev}}) \pi - \phi \)  
(6)

and similarly for the bottom of the downstroke, with constant pitch angle in between. This is shown in figure 2 for \( \phi = 90^\circ \).

![Figure 2. Pitch motion as a function of flywheel angle, for three different values of stroke reversal fraction \( \Delta \beta_{\text{rev}} \).](image)

The use of generalised linkage functions allows determination of whether the motions and frequencies prescribed in previous studies can be achieved in practice when fully flow-driven.

**Numerical Method**

**Algorithm**

The 2D unsteady incompressible Navier Stokes equations, coupled to the flywheel equation of motion, are solved using two-way fluid-structure-interaction (FSI) within the commercial CFD package Fluent 12.0. A second-order upwind spatial discretisation is used, and the motion of the foil is introduced via a source term in the Navier Stokes equations for the plunge motion, and rotation of a circular zone around the foil for the pitching motion. This meshing approach allows second-order accurate time stepping within Fluent as discussed by Kinsey and Dumas [2]. The mesh is shown in figure 3.

![Figure 3. Close-ups of the mesh around the foil, showing pitching motion circular zone (top), and boundary layer mesh at the foil surface (bottom).](image)

The flywheel equation of motion is reduced to a system of first-order ordinary differential equations and discretised in time with a second-order Cranck-Nicolson scheme (where superscript \( n \) represents the time level):

\[ B = \left[ \begin{array}{c} \dot{\beta} \\ \beta \end{array} \right] \]  
so that \( B = H(B) \)  
\[ B^{n+1} = B^n + \frac{\Delta t}{2} (H^{n+1} + H^n) \]  
(7)

A Newton sub-iteration is performed (where superscript \( p \) represents the iteration level) with an approximate Jacobian:

\[ \left[ I - \frac{\Delta t}{2} \frac{\partial H}{\partial B} \right] (B^{p+1} - B^p) = B^n - B^p + \frac{\Delta t}{2} (H^p + H^n) \]  
with \( \frac{\partial H}{\partial B} = \left[ \begin{array}{c} f_\beta + M g_\beta \\ l_a g_\beta g_\beta \end{array} \right] \]  
(8)

Finally an under-relaxation factor \( \omega \) is introduced:

\[ B^{p+1} = B^p + \omega A^{-1} \left[ B^p - B^p + \frac{\Delta t}{2} (H^p + H^n) \right] \]  
with \( A = \left[ I - \frac{\Delta t}{2} \frac{\partial H}{\partial B} \right] \)  
(9)

Sub-iteration is performed within each time step using the user-defined-function capability of Fluent. At each iteration a new A matrix is calculated, from which new foil plunge and pitch positions are derived from the linkage functions \( f(\beta) \) and \( g(\beta) \). New aerodynamic lift \( L \) and moment \( M \) are then calculated. Residuals in the flywheel equation of motion are reduced by approximately eight orders of magnitude at each time step.

**Parameter Selection**

Even reduced to a single degree of freedom system, the flapping foil turbine has a very large number of parameters to be selected or optimised. These include the foil mass and inertia, flywheel inertia, plunge and pitch amplitudes, shape of the linkage functions and phase between pitch and plunge, foil pivot point location as a fraction of the chord \( x_{\text{ptv}} \), foil section, and rotational damper strength.

A NACA0012 airfoil section is used with a chord \( c = 0.15 \text{ m} \), and a free stream velocity of \( U_\infty = 1.0 \text{ m/s} \) in water (\( \rho = 998.2 \text{ kg/m}^3 \)) is assumed. The Reynolds number is set to \( Re = 1100 \) by adjusting the fluid viscosity, for comparison of results with the prescribed motion study of Kinsey and Dumas [2]. Later work
will address the effect of higher Reynolds numbers in transitional and turbulent regimes.

The plunge motion is given by \( y = f(\beta) = hc \sin(\beta) \), with the plunge amplitude fixed at \( h = 1.0 \) chords. The pitch motion is given as in equation (6), with phase \( \phi = 90° \), and optimum pitch amplitude \( \theta_0 \), pivot location \( \dot{x}_{piv} \) and stroke reversal fraction \( \Delta \dot{\beta}_R \) to be determined. For further reduction of the number of free parameters, the mass and inertia of the foil are ignored in comparison to the inertia of the flywheel.

The damper strength is non-dimensionalised by the idealised fluid damping on a flat plate rotating about its mid-chord, according to the linearised analysis of Theodorsen [9], by analogy with the approach used by Zhu [5-6] for a linear damper:

\[
c' = \frac{\epsilon_b}{16 \pi \rho c^3 U_{\infty} s} \text{ (10)}
\]

Similarly the flywheel inertia is non-dimensionalised by considering the ratio of kinetic energy in the flywheel and moving foil, compared to the kinetic energy of the ideal fluid added mass and inertia of a plunging and pitching plate (Brennen [10]):

\[
I' = \frac{1}{2} \left( I_b (\dot{\beta})^2 + m_a (\dot{y})^2 + I_{ac} (\dot{\theta})^2 \right) = \frac{l_b}{2/\pi \rho c^2 s [f_\beta ]^2} \text{ (11)}
\]

where again the foil mass and inertia are ignored in comparison to the flywheel, and the translational term dominates the rotational term in the denominator.

**Results**

**Verification**

A number of parameter combinations were run with coarse (61,000 cells), medium (116,000 cells, shown in figure 3), and fine (270,000 cells) grids. Time steps of \( \Delta t = 2.5 \times 10^{-3}, 1.25 \times 10^{-3} \) and \( 6.25 \times 10^{-4} \) seconds were tested, resulting in approximately 400, 800 and 1600 time steps per flapping cycle at a flapping frequency of 1 Hz (noting that frequency varied between 0 and 1.5 Hz across the range of parameters tested in this work). Minor differences in the time history of the foil and flywheel velocity were noted (not shown here due to space limitations). However the time averaged power production given by equation (2) showed little variation and the medium grid with \( \Delta t = 1.25 \times 10^{-3} \) seconds was chosen for remaining runs. This combination also provided excellent agreement with the time history of aerodynamic forces for the optimum prescribed-motion case of Kinsey and Dumas [2].

**Further Parameter Selection**

The effect of stroke reversal fraction \( \Delta \dot{\beta}_R \) was evaluated for several different flywheel inertias and damper strengths as shown in figure 4. The non-sinusoidal pitching engendered by \( \Delta \dot{\beta}_R = 0.2 \) showed a clear advantage, as has been reported by our group for several different flywheel inertias and damper strengths as shown in figure 5. There is a trend of efficiency asymptoting with flywheel inertia, except at the higher pitch amplitude for very low inertias. Based on these results an inertia \( I = 5.67 \) was selected for further runs, although future work will examine performance more fully at lower inertias.

The optimal pitch amplitude and non-dimensional damper strength \( c' = 1.209 \) as shown in figure 5. There is a trend of efficiency asymptoting with flywheel inertia, except at the higher pitch amplitude for very low inertias. Based on these results an inertia \( I = 5.67 \) was selected for further runs, although future work will examine performance more fully at lower inertias.

The effect of flywheel inertia was evaluated for two different pitch amplitudes and non-dimensional damper strengths as shown in figure 5. There is a trend of efficiency asymptoting with flywheel inertia, except at the higher pitch amplitude for very low inertias. Based on these results an inertia \( I = 5.67 \) was selected for further runs, although future work will examine performance more fully at lower inertias.

![Figure 4](image1.png)  **Figure 4. Efficiency versus stroke reversal fraction, for \( \theta_0 = 45° \), \( \dot{x}_{piv} = 0.5 \).**

![Figure 5](image2.png)  **Figure 5. Efficiency versus non-dimensional flywheel inertia for two pitch amplitudes.**

Finally pivot point location as a fraction of the chord, \( \dot{x}_{piv} \), was evaluated for \( \theta_0 = 75° \), \( I = 5.67 \) and \( c' = 1.209 \) as shown in figure 6. Being clearly advantageous, \( \dot{x}_{piv} = 0.5 \) was selected for all further runs.

![Figure 6](image3.png)  **Figure 6. Efficiency versus foil pivot point location.**

**Optimum Pitch Amplitude and Damper Strength**

The effect of pitch amplitude and damper strength were further evaluated in the form of a contour map of efficiency as shown in figure 7. All runs used \( \Delta \dot{\beta}_R = 0.2, I = 5.67 \) and \( \dot{x}_{piv} = 0.5 \).

![Figure 7](image4.png)  **Figure 7. Contours of turbine efficiency vs non-dimensional damper strength \( c' \) and foil pitch angle \( \theta_0 \). Black dots represent runs conducted.**

There is a relatively broad ridge of high efficiency (up to 30%, commensurate with the 34% reported by Kinsey and Dumas [2] for prescribed motion), although there is a rapid drop-off in performance for higher amplitudes and damper strengths. The high pitch amplitudes for optimum performance are an indication of the importance of leading edge separation in power generation. One would also expect an optimum damper strength, as higher values lead to higher power extraction in equation (2), but also slow the flywheel lowering \( \beta \).
Three cases (A: $\theta_0 = 65^\circ$, $c' = 3.0$, B: $\theta_0 = 65^\circ$, $c' = 3.5$ and C: $\theta_0 = 45^\circ$, $c' = 3.0$) are examined in figure 8. The first is the optimum case from figure 7, and the other two are off-optimum. Figure 8 shows the rotation rate of the flywheel and the torque on the flywheel due to aerodynamic forces for one cycle of motion. Also shown are the vorticity field at three different times in the pitch reversal at the top of the upstroke, and during the constant-pitch phase during the downstroke. The optimum Case A shows a large leading edge vortex (LEV) behind the foil pivot point, which results in favourable torque on the flywheel. Case B, with slightly more damping, reduces the flywheel rate significantly and thus slows the foil. This results in a very strong trailing edge vortex which keeps the LEV ahead of the foil pivot point for longer during pitch reversal and creates a much more pronounced reduction in flywheel torque. During foil translation it also moves the LEV further from the foil, resulting in less lift and thus less flywheel torque. Case C is more similar to Case A, except that the lower pitch angle results in a smaller LEV and lower aerodynamic forces.

### Conclusions

The results of this work indicate that the efficiencies reported in previous prescribed-motion studies are achievable for fully flow-driven motion. Non-sinusoidal motions are found to produce higher efficiencies than sinusoidal motions. Further work remains to examine the role of flywheel inertia, as well as foil mass and inertia, more closely. Further gains might be made by altering the phase of the pitch relative to the plunge $\phi$ and by extending the plunge amplitude $h$. Also the stability and role of the leading edge vortex remains to be studied for higher Reynolds numbers in the transitional and turbulent regimes.

### References


<table>
<thead>
<tr>
<th>Flywheel Rate $\beta$ (rad/s)</th>
<th>Flywheel Torque (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A: $\theta_0 = 65^\circ$, $c' = 3.0$</td>
<td>Case B: $\theta_0 = 65^\circ$, $c' = 3.5$</td>
</tr>
</tbody>
</table>

Figure 8. Flywheel rotation rate $\dot{\beta}$ and aerodynamic torque on the flywheel $T_{aero} = L_f \dot{\phi} + M_g \dot{\phi}$ for one cycle of motion (left two columns), and vorticity magnitude at different times in the cycle (right three columns). Location in the cycle is indicated by black vertical lines in the left two columns, and represents three times during the stroke reversal at the top of the upstroke (top three rows), and during the constant-pitch translation in the downstroke (bottom row). Note that the flapping frequency is different in each case, so location in the cycle is referenced to flywheel angle rather than time.