Investigations of Eckert-Weise Energy Separation in the
Wakes of Turbine Blades and Circular Cylinders

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Abstract
A highly loaded turbine stage was evaluated experimentally in an annular cascade that matched engine Mach and Reynolds numbers. The turbine nozzle had a thick trailing edge to allow for cooling passages. In this ostensibly adiabatic arrangement, the central regions of the vane wakes exhibited a significant decrease in total temperature and their edges showed an increase.

To resolve these anomalous results, and obtain detailed information over the Mach number range, the mid-span section of the nozzle was tested in a large scale transonic planar cascade. At high subsonic speeds vortex shedding created energy redistribution in the wake, which was measured using new temperature probes having a bandwidth of over 80 kHz. Fluctuations in total temperature and total pressure, and hence entropy, were investigated. ‘Cold spots’, of decreased total temperature, were located close to the wake centre line and ‘hot spots’ at the wake edges. Good agreement was observed between measured and predicted values of total temperature at the cold and hot spots. The phenomenon of energy separation behind bluff bodies had been discovered by Eckert and Weise in 1943 and was confirmed by the results from the turbine cascade.

To observe the phenomenon under more generic boundary conditions, experiments were conducted at subsonic speeds on the vortex shedding behind a circular cylinder. From time-resolved measurements of wake total pressure and temperature the phenomenon of energy separation was again demonstrated. The measurements were compared with concurrent time-accurate numerical predictions. The numerical models again captured the non-uniform total temperature downstream of the cylinder.

Introduction
In 1943 Eckert and Weise [9] discovered that the time-averaged surface temperature at the trailing edge of a circular cylinder in cross flow dropped by as much as 20K compared with the upstream total temperature. The findings of Eckert and Weise were substantiated by the experiments of Ryan [15], who suggested that the phenomenon might be caused by vortex shedding. The results of Thomann [18] were the first to demonstrate that the low recovery factor at the rear of a cylinder propagates downstream. In effect these were the first measurements of energy separation in the wake of a circular cylinder. Little was subsequently achieved until the further work of Eckert [10] 1984, after which Kurosaka et al. [12] and Ng et al. [14] provided more experimental and computational evidence for the Eckert-Weise effect. This was now referred to as ‘energy separation’ when dealing with the relationship between wake temperature measurements and vortex shedding. Experimentally they showed that cold spots existed in the centre of the wake, appearing at the same frequency as the vortex shedding.

A number of theories have been proposed to explain the Eckert-Weise effect; the earliest was the potential flow theory of Ackeret [1], based on the energy equation and the potential of a free vortex. This theory predicted that the total temperature deficit was located between the two vortex rows and was proportional to the vortex shedding frequency and circulation. Thomann [18] showed that Ackeret’s potential flow theory agreed well with his own results despite its failure to consider boundary layer effects. Schultz-Grunow [17] considered turbulent heat-transport in the radial pressure field of a vortex. While the potential flow theories do approximate the redistribution of wake total temperature, their accuracy is inherently limited.

In an attempt to improve the energy separation theories Kurosaka et al. [12] presented three further possibilities. Central to the first is the energy equation in the form:

\[ c_{p} \frac{\partial T}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial \xi} \]  

(1)

In this theory, based on the Lagrangian frame, as a fluid particle moves around a convected vortex core, it moves through temporal pressure gradients. When the particle is on the downstream side of a vortex it moves through a negative temporal pressure gradient. From Eq. 1, it experiences a negative total temperature gradient. Conversely, as it moves around the upstream side of the vortex, it sees a positive pressure, and therefore total temperature, gradient. The particle’s total temperature is therefore at a maximum in moving from the upstream to the downstream face of the vortex and at a minimum when it passes from the downstream to the upstream face. Kurosaka et al. [12] offered an alternative explanation to the above theory by considering the components of the pressure field, \( \partial p/\partial \xi \), acting inwards towards the centre of the vortex against the centrifugal force caused by the particle rotation. They also presented two complementary theories based on the entrainment and ejection of fluid particles from the vortex street and on the kinematics of an idealised street of Rankine vortices. The theories proposed by Kurosaka showed that, in real vortex streets, total temperature separation does occur. The vortices are not all the same strength nor of constant strength and, where the street is not infinitely long, nor of constant width.

The non-uniform total temperature distribution observed in the cylinder wake also occurs downstream of turbine blades with thick trailing edges. The non-uniform total temperature and pressure distributions downstream of turbine blades with thick trailing edges were studied by Carscallen et al. [6].
Time-accurate numerical models have been developed to reproduce the physics of the compressible wake flow, which is dominated by von Karman vortex shedding. To reproduce the large-scale instabilities, the short-time Reynolds Averaged Navier Stokes (RANS) equations are solved over the flow domain. In addition, results of concurrent time-accurate experimental work, performed at the National Research Council of Canada (NRC), are documented. The integration of the experimental and numerical approaches provides a powerful tool for developing an improved understanding of this phenomenon.

### Experimental Arrangements

The 2-D profile used in the cascade tunnel was the mid span section of the turbine nozzle vanes. The nozzle blading, as tested in the cascade tunnel, was scaled to 4.3 times engine size and is shown in figure 1. The nozzle turning angle was 76° and the isentropic exit Mach number was 1.16 at the design pressure ratio of 2.3. The inlet metal angle was selected to give -10° incidence. The nozzle was designed to accomplish most of the flow turning in the upstream portion of the flow channel and the vane trailing edge was thick to allow for internal cooling passages. The planar cascade had six nozzle vanes and five nozzle passages. It was operated as an in-flow facility in which 3.83 kg/s of laboratory air was drawn through the cascade by a 2.0 MW exhauster. Distinguishing features of this transonic nozzle planar cascade were the realistic aspect ratio and the thickness of the inlet end-wall boundary layer. Adjustable boundary layer bleeds were installed on both sidewalls, adjacent to and upstream of the first and sixth nozzle vanes, to remove sidewall boundary layers and corner vortices. An adjustable tailboard was perforated, with a 23% open area ratio, and this eliminated shockwave reflection.

The 1.5 m trisonic blow down wind tunnel at NRC was used for the experiments on circular cylinders. A 381 mm long cylinder of 37.5 mm diameter was mounted on the wind tunnel balance upstream of the wake traverse position (figure 2). The tip of the total temperature and pressure probes was maintained at a distance of 6D downstream of the cylinder trailing edge for the duration of the experiments. A high frequency response total temperature probe, developed at Oxford University [5], and a high frequency, 1.6 mm diameter total pressure transducer were used. These probes were used to measure time-accurate total temperature and pressure fluctuations in the cylinder wake. The probes were traversed across the wake in increments of 3 mm from a distance of 132 mm below to 120 mm above the wake centreline. A separate high frequency, 1.6 mm diameter total pressure transducer was mounted flush with the cylinder surface. This probe was used as a phase reference in the wake traverse experiments, during which the probe was set at an azimuth of 120° from the upstream stagnation point. In addition, the surface mounted total pressure probe was used to measure the surface pressure at 5° intervals around the cylinder circumference in the azimuth range of -55° to +55°. For the surface pressure measurements the total pressure wake probe downstream of the cylinder was the phase reference.

### Numerical Modelling

Time accurate computational modelling of the wake flow behind the turbine cascade blades has been performed by Currie [8], Brooksbank [4], Bennett [3] and Mahallati [11].

The flow solver implemented by Currie [8] used a quasi-three-dimensional form of the Reynolds-Averaged Navier-Stokes (RANS) equations over an unstructured grid. A second order accurate MUSCL implementation of Roe’s flux difference splitting scheme (16) was used. Turbulence was modelled with a zonal $k-\epsilon$ SST [13] formulation, using the $k-\epsilon$ model near walls and the $k-c$ model elsewhere.

Brooksbank [4] used an inviscid code and obtained results that agreed well with the experimental results, adding detail, particularly to the understanding of the near wake flow. For validation purposes Brooksbank compared the surface isentropic Mach number distribution calculated by his code with the experimental data. The results showed that the inviscid code accurately matched the surface isentropic Mach number distribution providing evidence of the code’s validity.

In Bennett’s work [3] a numerical model solved the short-time RANS equations. Turbulence closure was obtained using the $k-\epsilon$ model. Second order formal accuracy was achieved spatially also using MUSCL interpolation with the Roe scheme. An adaptive multigrid refinement algorithm was used, locally refining the mesh in regions of large density gradient.

Two-dimensional time-accurate numerical simulations of the midspan flow were performed by Mahallati [11] over the speed range, using a commercial CFD code. The Roe flux difference-splitting scheme was again used for discretization of spatial derivatives. Temporal terms were treated with the second-order implicit method. The $k-\epsilon$ turbulence model was used for closure. Convergence was accelerated using multi-grid techniques. The mesh was refined such that the minimum value of $y+$ was less...
than unity. No wall functions were therefore needed and the turbulence model was integrated all the way to the wall. As an example of this approach an instantaneous numerical schlieren of the turbine blade flow is given in figure 3.

Time accurate computational modelling of the wake flow behind the circular cylinder was performed by Bennett [3] using a similar approach to that used for the turbine cascade. A non-uniform, structured mesh of rectangular topology formed the baseline mesh of the multigrid hierarchy. This mesh was body-fitted around the cylinder and proportional grid stretching was employed normal to the surface to obtain good boundary layer resolution. An initial uniform inlet condition was imposed throughout the computational domain. The numerical model was time-marched from the initial distribution to a quasi-stationary vortex shedding regime using an inviscid numerical method. The quasi-stationary inviscid flow prediction was then used to prime the turbulent computation. Free stream values for the short-time averaged levels of \( k \) and \( \omega \) were defined throughout the computational domain. The turbulent flow field was then progressed to a quasi-stationary vortex shedding prediction. This method for the computation of the turbulent flow field is computationally efficient.

**Results**

An initial objective of the turbine cascade testing was to establish whether the blunt trailing edge was causing vortex shedding. Schlieren optics, with a sub-microsecond argon jet flash light source, were used for this. It was established that von Karman vortex shedding was occurring at all Mach numbers. An example from a discharge Mach number of 1.16 is given in figure 4. At these higher Mach numbers there were also time periods during which vortex shedding occurred in ‘exotic’ configurations [7].

Total temperature and pressure probes were traversed in a plane normal to the wake. Time-resolved total temperature contours downstream of the trailing edge are given in figure 5. These show the cold vortical structures on the wake centre line and the higher temperature hot spots at the wake edges. To check whether time-averaged total temperatures on the wake centreline are in agreement with previous thermocouple measurements the results were compared and are given in figure 6. As seen in the figure the agreement is good. This indicates that the temperature fluctuations induced by vortex shedding are entirely sufficient to account for the steady state measurements of Eckert-Weise energy separation observed in the wake.

Having both instantaneous total temperature measurements and total pressure measurements it was possible, using the entropy relation:

\[
s_2 - s_1 = c_p \ln(T_2/T_1) - R \ln(P_2/P_1),
\]

(2)
to plot contours of time-resolved entropy increase downstream of the trailing edge and these are given in figure 7. Vortex structures in the wake are again resolved.

In order to investigate Eckert-Weise energy separation on a more generic model, measurements were made on the circular cylinder in cross-flow. The results shown in figure 8 gives experimental [2] and computational [3] time-resolved total temperature contours 225 mm (6D) downstream of the cylinder trailing edge.
Experimental results:

![Figure 8](image1.png)

Numerical results:

![Figure 9](image2.png)

Figure 8. Time resolved total temperature ratio contours six diameters downstream of the cylinder trailing edge.

Figure 9. Time-averaged total temperature ratio six diameters downstream of the cylinder trailing edge.

Figure 8 gives the time-averaged total temperature ratio 225 mm downstream of the cylinder trailing edge. This shows the characteristic trend in energy separation with a significant decrease in total temperature ratio on the wake centre line and attendant regions of hotter than average fluid at the wake edges.

Conclusions

Strong vortex shedding occurred from the trailing edge of turbine blades at all Mach numbers. Thermocouple measurements had indicated wake energy separation, peaking at a discharge Mach number of 0.95. Wide bandwidth time-resolved total temperature measurements demonstrated that the energy separation was entirely a result of the vortex shedding which created cold vortical structures in the wake core and hot spots at the edge of the wake. Time-accurate computational work using a number of different codes substantiated the periodic nature and vortical structure of the temperature fluctuations.

Vortex shedding from the circular cylinder was strongest and most coherent at a Mach number of 0.6. Contours of time-resolved measurements of wake total temperature ratio showed the characteristic cold vortical wake core with hot spots at the edge of the wake. These measurements of total temperature ratio and entropy increase were compared with computational predictions giving reasonable qualitative agreement.

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References