16th Australasian Fluid Mechanics Conference
Crown Plaza, Gold Coast, Australia
2-7 December 2007

Transient Natural Convection in a Differentially Heated Cavity with a Thin Fin of Different Lengths on a Sidewall

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Abstract

Direct numerical simulations of unsteady natural convection in a differentially heated cavity with a thin fin of different lengths on a sidewall at the Raleigh number of 3.8×10^9 are performed. It is found that the fin length significantly impacts on the transient thermal flow around the fin and heat transfer through the finned sidewall in the early stage of the transient flow development. The results also indicate that the oscillations of the thermal flow around the fin in the quasi-steady stage are very sensitive to the length of the fin.

Introduction

Enhancement and depression of heat transfer through a differentially heated cavity are of significance for industrial applications such as solar collectors and nuclear reactors. Considerable studies have been devoted to the problems of this aspect. Many techniques of enhancing or depressing heat transfer have been experimentally and numerically investigated. One technique which has attracted significant research attention is to place a fin on the sidewall in order to enhance or depress heat transfer. One of the earliest studies of natural convection induced by a fin on a vertical heated surface was reported in [2] in which transitional natural convection to turbulence is visualized using smoke. Studies of natural convection in a cavity with a fin on a heated or cooled sidewall have also been extensively reported in the literature. It has been demonstrated in [6] that there is no clear flow separation around a small square fin attached at a low position of the hot wall in the laminar flow regime.

Recently, the effect of the shape, size, material, and position of the fin on natural convection in the differentially heated cavity has been paid much attention (see [1,6,7,8]). In these studies, the fin is of a rectangular shape, and its thickness is usually considered small (so-called a thin fin). The fin length is one of the important controlling natural convection in the cavity. If the length of a poorly conducting fin is sufficiently large, multiple secondary circulations occur at the upper and lower corners of the fin respectively (refer to [3,4]). It is reported that heat transfer through the finned sidewall at low Rayleigh numbers is reduced as the fin length increases. This is because natural convection adjacent to the finned sidewall is depressed. However, a long fin results in secondary circulations of natural convection, and the secondary circulations, in some sense, increase convection adjacent to the opposite sidewall and thus enhance heat transfer through the opposite sidewall [3].

The aforementioned studies of the effect of the fin on flows and heat transfer focused on the steady laminar flow in the cavity at low Rayleigh numbers (e.g. $Ra < 10^7$). However, for unsteady natural convection at high Rayleigh numbers (e.g. $Ra > 10^7$), the conclusions drawn based on the steady laminar flows evidently need to be re-examined.

Recently, a direct numerical simulation of natural convection in a suddenly differentially heated cavity with a thin fin was performed in [9], in which the transition of natural convection from start-up to a steady state was described. It has been found that a thin fin, in the initial stage, blocks the upstream flow and forces it to detach from the hot finned sidewall. As a result, a lower intrusion front is formed under the fin. After the lower intrusion bypasses the fin, a starting plume arises and the thermal flow behind the plume front is drawn to the downstream sidewall. The transition of the thermal flow around the fin ultimately approaches a periodic flow. It has demonstrated that there are distinct variations of the flow structures and the wave properties between the cases at low Rayleigh numbers and those at high Rayleigh numbers.

This paper is relevant to that reported in [9], but the emphasis here is placed on the effect of the fin length on the transient natural convection and heat transfer since it is found from previous studies that the transient natural convection flow in the cavity is very sensitive to the fin length. Natural convection flows induced by two fins of different lengths are simulated and compared. The wave features of the thermal flows around the fin are characterized and discussed in this paper.

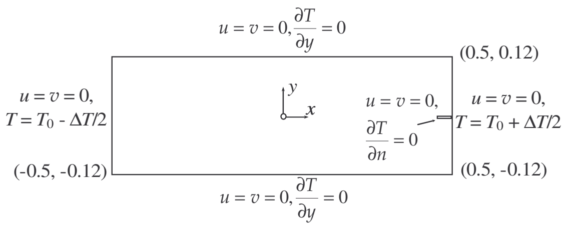

Figure 1 Computational domain and boundary conditions

Numerical procedures

Under consideration is a two-dimensional cavity of H (0.24-m) × L (1-m). An adiabatic fin of a thickness of 2-mm is attached at the mid height of the heated sidewall (refer to figure 1). Two fin lengths of 20-mm and 80-mm respectively are considered. Initially the working fluid (water in this case) is motionless and at a uniform temperature T_0. At time $t = 0$, the finned sidewall is suddenly heated to (T_0+ΔT/2) and the opposite sidewall cooled to (T_0-ΔT/2). The subsequent development of the flow and temperature fields in the cavity is governed by the two-dimensional Navier-Stokes and energy equations with the Boussinesq approximation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta(T - T_0), \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

where x and y are horizontal and vertical coordinates originated from the centre of the cavity, t the time, T the temperature, T_0 the initial temperature, p the pressure, u and v the velocity components in the x and y directions, g the acceleration due to gravity, β the coefficient of thermal expansion, ρ the density, κ the thermal diffusivity, and ν the kinematic viscosity.

Natural convection flows in a differentially heated cavity are characterozed by three main dimensionless parameters: the Rayleigh number (Ra), the Prandtl number (Pr) and the aspect ratio (A), defined as follows,

$$Ra = \frac{g\beta\Delta TH^3}{\nu\kappa}, \tag{5}$$

$$Pr = \frac{\nu}{\kappa}, \tag{6}$$

$$A = \frac{H}{L}, \tag{7}$$

All computational boundaries are non-slip, and the fin and the top and bottom walls are adiabatic. Figure 1 shows the above boundary conditions.

The governing equations are implicitly solved using a finite-volume SIMPLE scheme. All the second derivative and linear first derivative terms are approximated by a second-order center-differenced scheme. The advection terms are discretized by a second upwind scheme. The time integration scheme is a second-order backward difference method. The discretized equations are iterated with under-relaxation factors.

Considering the features of natural convection flows in a differentially heated cavity (also see [5,9]), we constructed a non-uniform grid system with finer grids concentrated in the region of all the wall boundaries (a zone of 50-mm adjacent to a wall) in which the grid expands at constant rates from the wall to the interior edge of this region. Similarly, the vicinity of the fin is finely meshed in order to accurately capture the features of the thermal flow around the fin.

Grid dependence tests have been conducted on two grid systems for different fins. Since the oscillations of the thermal flow around the fin are important features of the flow (see [9]), a set of parameters including the average, amplitude, and frequency of temperature waves as well as the total heat transfer rate of the hot finned sidewall are examined with different meshes. The results are listed in table 1. It is clear in Table 1 that the frequency of the travelling waves in the vertical thermal boundary layer, the average temperature at the monitoring point, and the heat transfer rate of the finned sidewall are not sensitive to the present grid resolutions. Although the calculated amplitude changes slightly between different grids, these variations are unlikely to change the overall features of the unsteady natural convection flows induced by the fin. Therefore, in consideration of the computational time, the relatively rough grid is adopted in this study.

Table 1 Parameters in grid dependence tests (5900 s ~ 6000 s)

Fin's cross section (mm^2)	Grid ($H \times L$)	Temperatures at (0.498 m, 0.09 m)			H_{fin} (W)
		Average (K)	Amplitude (K)	Frequency (Hz)	
20×2	211×467	299.520	2.3091	0.1563	729.40
	259×596	299.472	2.4154	0.1563	735.85
40×2	211×538	299.449	1.2856	0.1172	746.39
	259×743	299.309	1.3825	0.1172	742.87

H_{fin} is the heat rate through the finned sidewall

Time-step-dependence tests have also been conducted with a time-step of 0.1 s and 0.05 s respectively. For the two cases with different time-steps, there is no discernible difference in the transient thermal flows. Therefore, the time-step of 0.1 s, similar to that adopted in [9], is considered to be sufficiently small for capturing the major flow features, and is adopted in the paper.

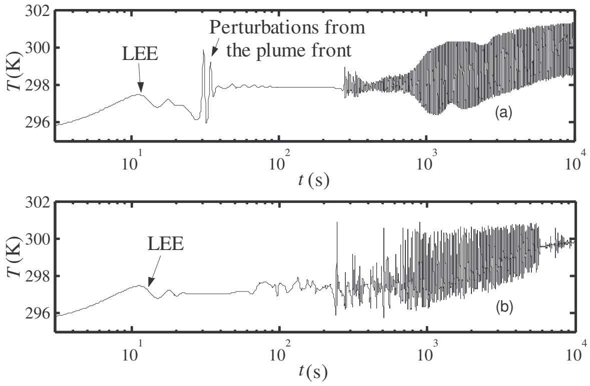

Figure 2 Time series of the temperatures at the point (0.498 m, 0.09m). (a) For a smaller fin. (b) For a larger fin.

Results and discussion

Wave features of temperatures

The numerical results presented in what follows is for the case with $Ra = 3.8 \times 10^9$ and $Pr = 6.7$. To illustrate the evolution of the thermal boundary layer, figure 2 plots time series of the temperatures at the point (0.498 m, 0.09 m) for the cases with the smaller fin of 0.02×0.002 m^2 (figure 2a) and the larger fin of 0.08×0.002 m^2 (figure 2b). For the smaller fin, the LEE in the earlier stage is clear (also see [5]), and the transition of the thermal boundary layer flow finally approaches a periodic flow, as seen in figure 2(a). Similarly, the LEE may be observed in the earlier stage of the flow with the larger fin (figure 2b). However, the amplitude of the travelling waves in the downstream thermal boundary layer significantly reduces after 5800 s, and it eventually approaches a constant but reduced value.

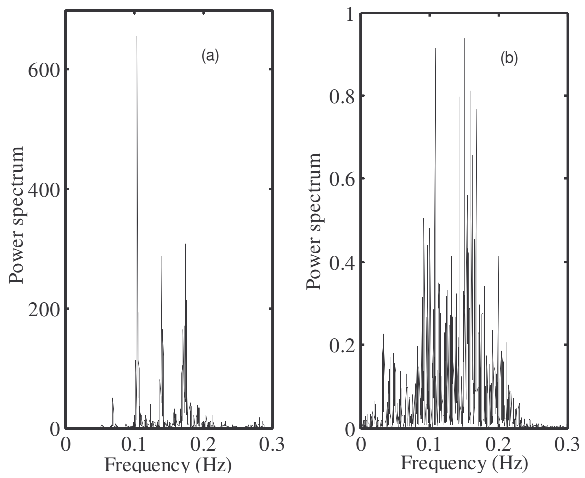

Figure 3 Spectra of the temperature series at the point (0.498 m, 0.09m). (a) For the smaller fin. (b) For the larger fin.

Figure 3 shows the spectra of the temperature series in figure 2. For the case of the smaller fin, figure 3(a) indicates that the temperature wave has a noticeable dominant frequency of approximately $f = 0.1$ Hz with some additional higher frequency modes. However, for the larger fin, the spectrum in figure 3(b) indicates that, in addition to a frequency of approximately $f = 0.1$ Hz, higher frequency modes are significant. This implies that, although the dominant frequency does not vary much between the two cases with different fin lengths (also see [9]), multi-modes of oscillations are present, particularly in the case with the larger fin, as seen in figure 3(b).

Flow features in transition to the quasi-steady state

As indicated in figure 3, the fin length has a significant impact on the wave features of the temperature series. The variations of the wave features result from the changes of the thermal boundary layer flow. To illustrate transient flow features of the thermal boundary layer, figure 4 shows the plume fronts bypassing the thin fin in the early stage. For the smaller fin, since the plume front is closer to the hot wall after it bypasses the thin fin, the reattachment of the plume front to the downstream wall is clear, as seen in figure 4(a). The perturbations from the reattachment of the plume front may be observed in the temperature time series, as seen in figure 2(a). However, for the larger fin, figure 4(b) displays that the development of the plume front is different from that with the smaller fin. With the larger fin, the plume front directly moves upwards rather than reattaches to the hot sidewall, and thus no perturbation from the plume front is observed (figure 2a). The development of the plume front and wave features of the thermal boundary layer in the early stage are similar to the case of the thin fin of 0.04×0.002 m^2, as indicated in [9]. Figure 4 also indicates that, due to the difference of the length between the two fins, the time it takes for the thermal flow to bypass the fin is evidently different.

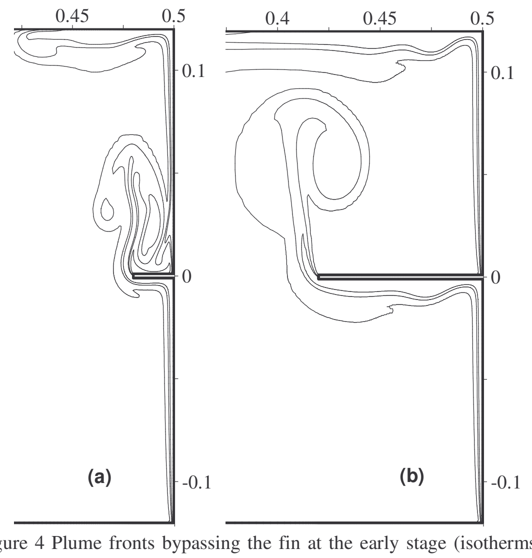

Figure 4 Plume fronts bypassing the fin at the early stage (isotherms of 296.56, 296 and 297 K). (a) For the smaller fin at t = 24 s. (b) For the larger fin at t = 48 s.

Figure 5 shows the thermal flows in the vicinity of the hot sidewall in the quasi-steady stage. The isotherms in figure 5(a) indicate that, although the fin of 0.02×0.002 m^2 is small in comparison with the other fin, separation of the thermal flow around the fin is clear. The thermal flow around the fin further triggers an instability of the downstream thermal boundary layer, as seen in figure 2(a). For the thin fin of 0.08×0.002 m^2, since the flow separating from the thin fin is further from the hot sidewall, the thermal flow bypassing the fin has less impact on the downstream thermal boundary layer flow as seen in figure 2(b). As the position of the separation moves further from the hot sidewall with the passage of time, travelling waves in the downstream thermal boundary layer induced by the thermal flow around the fin becomes weaker. This is consistent with the temperature results shown in figure 2; that is, the amplitude of temperature waves significantly reduces after approximately 5800 s.

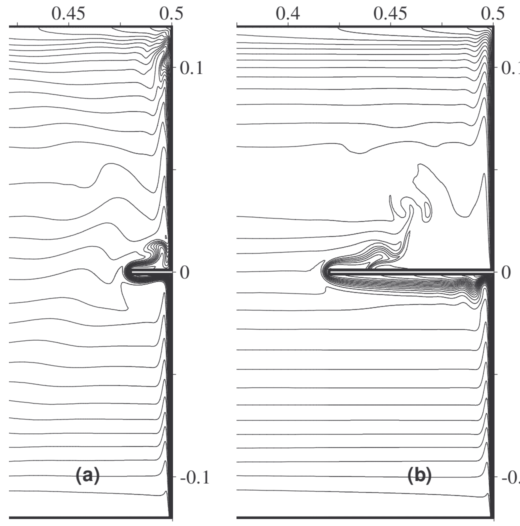

Figure 5 Thermal flows in the vicinity of the hot sidewall at t =6000 s (isotherms: from 287.94 to 303.16 K with an interval of 0.39 K). (a) For the smaller fin. (b) For the larger fin.

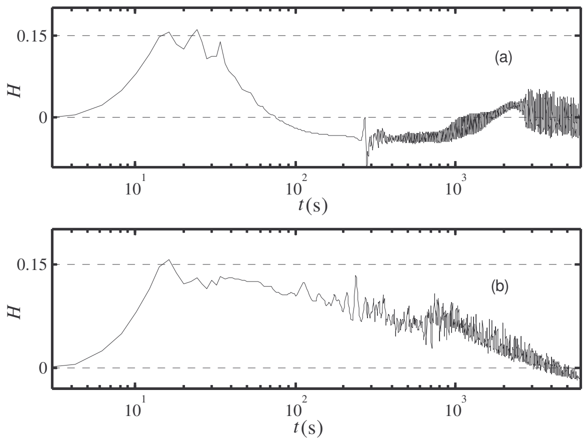

Figure 6 Normalized difference of heat rate of the hot wall between with and without a thin fin against time. (a) For the smaller fin. (b) For the larger fin.

Heat transfer

It is worth noting that the fin length also has a direct impact on heat transfer through the finned sidewall. Figure 6 shows the normalized difference of the heat rate of the finned sidewall against time, which is defined by,

$$H = (H_{fin} - H_{nofin}) / H_{nofin}. \tag{8}$$

where H is the normalized difference of the heat rate of the finned sidewall, and H_{nofin} the heat rate without a fin. For the smaller fin, although heat transfer is significantly enhanced in the earlier stage by up to 16%, the normalized heat rate variation rapidly reduces to below zero, and then oscillates about zero, as seen in figure 6(a). Figure 6(b) indicates that, for the larger fin, the maximum enhancement of heat transfer is also up to 16%. The normalized difference of the heat rate decreases slowly with time, and it becomes negative approximately after 4000 s. Qualitative comparisons with the results obtained with a thin fin of 0.04×0.002 m^2 reported in [9] indicate that heat transfer through the finned sidewall is dependent on the fin length. It is anticipated that there may be an optimal fin length with which a maximum enhancement of heat transfer through the finned sidewall can be achieved. In order to determine the optimal fin length, a great number of calculations will be required. That is out of the scope of this paper and thus is not discussed.

Conclusions

In this paper, natural convection flows in a suddenly differentially heated cavity with a fin of different lengths placed on the sidewall are numerically simulated. Comparisons of the cases with different fin lengths indicate that the flow and heat transfer properties are dependent on the fin length, particularly in the early stage. The fin, in some sense, blocks the upstream thermal boundary layer flow and forces it to detach from the hot sidewall.

For the case with the smaller fin, due to the entrainment of the downstream thermal boundary layer, the plume front bypassing the fin reattaches to the downstream thermal boundary layer. At the quasi-steady stage, a periodic flow is formed. It is found that the fin significantly changes not only the flow structures of the downstream thermal boundary layer but also heat transfer through the finned sidewall.

For the case with the larger fin, the plume front does not reattach to the downstream thermal boundary layer immediately after bypassing the thin fin. Instead, it moves directly upwards and strikes the intrusion under the ceiling. In this case, the larger fin significantly enhances the heat transfer through the finned sidewall in the early stage.

Comparisons of the present numerical results obtained with a smaller fin and a larger fin with that reported in [9] with a medium fin indicate that heat transfer through the finned sidewall is dependent on the fin length, and an optimal fin length may exist, which will result in a maximum enhancement of heat transfer through the sidewall..

Acknowledgment

The authors gratefully appreciate the financial support of the Australian Research Council.

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