Low mass ratio vortex-induced motion

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Abstract

The vortex-induced motion of floating structures is strongly influenced by their low mass ratios (i.e. the ratio of structural to displaced fluid mass). It has previously been demonstrated that the super-upper response branch encountered in two-degree of freedom systems at low mass ratio is characterised by much larger vibration amplitudes than those of the corresponding upper response branch. These low mass ratio super-upper response branch vibrations are clearly of immense importance in the design and analysis of offshore structures. It was the purpose of the current study to experimentally investigate the vortex-induced response of cylindrical structures at low mass ratio with particular emphasis on the relative magnitude of the super-upper response branch vibration amplitudes. The experimental apparatus utilised, consisted of a parallel linkage mechanism allowing translation motion of the cylindrical test section in both the inline and transverse directions. The configuration employed ensured identical mass ratios and natural frequencies in both directions. The mass ratio range covered was nominally 2.4 to 13.

Introduction

Vortex-Induced Vibration (VIV) is a flow-structure interaction phenomenon in which the structure is excited by forces induced by vortices shed alternately from the edges of a bluff object. The time varying non-uniform pressure distribution around the object resulting from the vortex shedding (causing a time varying lift force to be experienced by the object) creates structural vibrations both inline and transverse to the flow. Near the natural frequency of the structure, the vortex shedding frequency synchronises with the natural frequency and the vibration frequency. One of the primary mechanisms responsible for this synchronisation is the change in hydrodynamic mass, as demonstrated in the experiments of Vikestad [18]. The range of reduced velocity over which this synchronisation occurs is known as the lock-in range. Mostly, the ensuing vibrations are undesirable, resulting in increased fatigue loading and component design complexity to accommodate these motions. The transverse vibrations also result in higher dynamic relative to static drag coefficients.

With decreasing mass ratio an increase in the amplitude response is generally evident [17]. Also, the smaller the mass ratio, the larger the relative influence of the hydrodynamic mass on the vibration response of the structure. Since the hydrodynamic mass variation is largely responsible for synchronisation of the shedding and vibration frequencies, typically much wider lock-in regions are experienced at low mass ratio [17]. The limit of this trend is found at the critical mass ratio of around 0.54 [4], below which, there exists no de-coherence region and VIV occurs at all velocities above the initial lock-in.

Various definitions for the mass ratio are widely employed. In the present work, the mass ratio is defined by the relationship

$$m^* = \frac{m}{m_f}. \tag{1}$$

The denominator is the displaced fluid mass;

$$m_f = \rho L \pi \left(\frac{D^2}{4} \right). \tag{2}$$

The structural mass, m, includes enclosed fluid, but excludes the hydrodynamic mass. The term L is the submerged length of the cylinder. The mass ratio parameter is therefore the ratio of the oscillating structural mass to the displaced fluid mass. In the present study, the term low mass ratio refers to mass ratios of the order of one. These mass ratio magnitudes are common for marine structures.

The mass ratio parameter influences both the amplitude and frequency response of the cylinder. With higher mass ratios (e.g. a cylinder vibrating in air, with a mass ratio O(100)), changes in added mass are relatively insignificant due to the low density of the fluid. The natural frequency therefore, remains relatively unchanged throughout the lock-in range. When the fluid medium under consideration is much denser (e.g. a cylinder vibrating in water), distinct changes in the natural frequency are observed. The increasing natural frequency observed with increasing reduced velocity is directly attributable to the decreasing added mass throughout the lock-in range [16, 18].

The shedding of vortices from cylindrical bluff objects and the ensuing vibrations are well documented for single degree of freedom cases (see for example the review by Griffin [6]). Investigations tended to focus on the larger transverse vibrations and any interaction with the inline oscillations, which occur at twice the transverse vibration frequency, was ignored. Experimental investigations conducted by Williamson & Jauvtis [19] revealed the existence of a super-upper response branch when two degrees of freedom were considered. It was noted that these vibrations were extremely large and regular compared to those observed in the upper response branch. The regularity of these large-amplitude vibrations observed appear to indicate that the amplitude limitation due to disruption of the regular vortex shedding process (discussed in the text by Blevins [2]) is much more distinct in the two-degree of freedom than the single-degree of freedom upper response branch oscillations.

The super-upper response branch has also been observed in other investigations, such as the cantilevered cylinder experimental study by Pesce & Fujarra [12], the neutrally buoyant cylinder experiments by Hansen, Bryndum & Mayer [7] and the VIV work on non-linear compliant systems by Stappenbelt [16]. In the studies of Williamson & Jauvtis [19] and Stappenbelt [16], it was noted that the extremities of the super-upper response branch terminate at the single degree of freedom vortex-shedding mode boundaries. The vortex shedding mode observed during the super-upper response branch vibrations are an alternating pair of

triple vortices (referred to as the 2T mode) as opposed to the 2P (i.e. pairs of vortices) mode observed in the upper response branch. As in the single degree of freedom case, the initial response branch is still characterised by the classic von Karman 2S shedding mode in the two degree of freedom case.

The studies of Jauvtis & Williamson [8, 9] arrived at the conclusion that for mass ratios above five or six, there was no significant change in the transverse oscillatory response from one to two degrees of freedom. This mass ratio boundary delineating the presence of the super-upper response branch was reported at a mass ratio of approximately 8.5 in the study of Stappenbelt and O'Neill [15]. These findings of course have potential bearing on the validity of the volumes of single degree of freedom VIV data employed in multiple degree of freedom system design situations. As suggested by Jauvtis & Williamson [9], offshore design codes should reflect the significant deviation from the single degree of freedom VIV data when dealing with the low mass ratios typically encountered in offshore structure design.

Methodology

The experimental investigation conducted, examined the single (transverse motion only) and two-degree of freedom VIV response of an elastically mounted rigid cylinder with low mass ratio under steady, uniform current conditions. The apparatus utilised consisted of a towing carriage and a parallel linkage mechanism capable of translation motion in the transverse and inline directions (the mechanism is illustrated in figures 1 and 2).
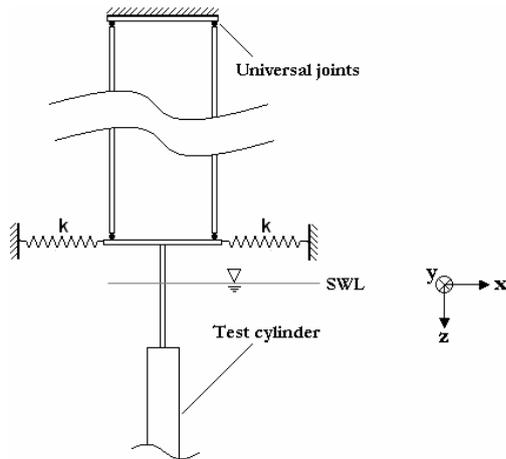
Figure 1 – Elevation view of the two degree of freedom experimental apparatus

The mechanism employed ensured identical mass ratios and natural frequencies in both directions. Inline motion was restricted for the transverse VIV only part of the experiment, through the use of diagonal cross braces between linkages. To avoid influencing the cylinder effective mass, the braces remained attached to the linkages for all experimentation. Experimental parameter values utilised in the present investigation are listed in table 1.

The apparatus for the experiment was lightly damped at around 0.6% of the critical damping. Damping ratios were effectively constant over the experimental amplitude response range. At very small vibration amplitudes however, the damping ratio increased marginally due to the effect of initial static frictional component in the universal joints employed. This low amplitude damping ratio increase did not extend to the vibration amplitudes experienced in the upper and super-upper response branches of interest in the present study.
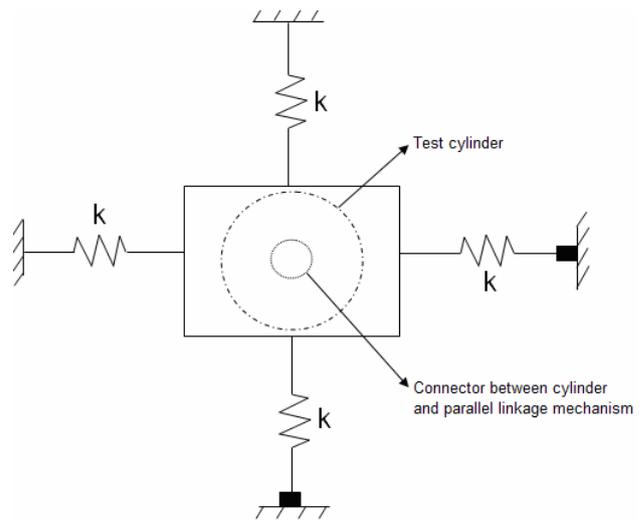
Figure 2 – Plan view of the two degree of freedom experimental apparatus

The mass ratio range covered was 2.36 to 12.96. This was achieved through the addition of lump masses to the plate attaching the cylinder test section to the parallel linkages. The test section support had an effective stiffness three orders of magnitude higher than that imposed by the linear coiled tension spring arrangement.

Table 1– Experimental parameters.

Parameter	Symbol	Value	Units
Cylinder diameter	D	0.0554	m
Cylinder length	L	0.4432	m
Normalised parallel linkage length	l/D	12.27	
Cylinder mass (incl. enclosed fluid)[1]	m	2.53	kg
System stiffness	k_1	453.0	N/m
Damping ratio	ζ	0.006	
Natural frequency in air[1]	ω_v	16.32	rad/s
Still-water natural frequency[1]	$\omega_{\omega v}$	10.75	rad/s
Mass ratio range	m^*	2.36-12.96	-

[1]Cylinder only (i.e. base case without lump masses attached).

Throughout the present study, the flow velocities are presented normalised (i.e. as reduced velocities, U_r) by the product of the cylinder diameter and the natural frequency of the structure in still-water. The natural frequencies in still water (table 2) of the various mass ratios examined were determined through free-decay tests.

Table 2 – Natural frequencies in still-water

m^*	$m^*\zeta$	f_n (Hz)	ω_n (rad/s)
2.36	0.014	1.711	10.75
3.68	0.022	1.502	9.44
5.19	0.031	1.359	8.54
6.54	0.039	1.261	7.92
7.91	0.047	1.153	7.25
8.76	0.053	1.151	7.23
10.63	0.064	1.084	6.81
12.96	0.078	1.025	6.44

To simulate steady, uniform current conditions the structure was towed through initially still water. The ratio of cylinder diameter to channel width was 1:25 (representing around 1% of the channel area). No significant variation in local current velocity

due to blockage effects was therefore experienced. All experimentation was conducted within the stable subcritical Reynolds number range and only smooth cylinders were considered.

The cylinder aspect ratio was selected to bring the test section closer to commonly employed cylindrical floating structure ratios of around 3 to 5. The experiment did not employ test section end-plates. The influence of the end-conditions on the cylinder forcing at this small aspect ratio are likely therefore to have resulted in lowered excitation frequencies. Strouhal number and vortex shedding frequency data are traditionally collected with very large aspect ratios (L/D>>100) and commonly, using the walls of the tank effectively as very large end-plates (maintaining two-dimensional flow at the cylinder ends). A number of studies have shown a decrease in the shedding frequency with decreasing aspect ratio. Examples include the low Reynolds number study of Lee & Budwig [10] and the higher Reynolds number study of Gowda [5].

Experimentation and analysis of previous work by Gerich & Eckelman [3] demonstrated regions near the cylinder ends where the shedding frequency was significantly lower (up to a reported 17% below the Strouhal frequency). Below an aspect ratio of approximately 15, the shedding frequency is dominated by these regions (i.e. the shedding is controlled by the end conditions). Further examination of the effect of the end-conditions (with Reynolds numbers up to 10^4) on vortex shedding by Norberg [11], also demonstrated a decreased shedding frequency with reduced end-plate size. In the limit of this trend is of course the present case of a cylinder without end-plates.

In the present study, the cylinder was non surface-piercing and the top cylinder end was at a distance 4D below the free surface. No compensating corrections were applied for the surface proximity or to account for the small hydrodynamic forces acting on the rectangular cylinder support structure. Of interest in the present experiment, were the cylinder displacement, acceleration and hydrodynamic forces. Both magnitude and spectral information was extracted from the data. The cylinder inline and transverse force measurements were inertially corrected to yield the fluctuating drag and lift forces respectively. Spectral force information was used to ascertain the vortex-shedding frequency also. For further details regarding the experimental arrangement, the reader is directed to the doctoral thesis by Stappenbelt [16].

The primary aim of the present study was to determine the point at which the variation between the single and two-degree of freedom transverse amplitude responses becomes significant. As applied in the studies by Jauvtis & Williamson [8, 9], the governing parameter was assumed to be the mass ratio. The point of prime experimental interest was therefore the upper mass ratio limit where the super-upper response branch was discernible on the transverse amplitude plot of both the single and two-degree of freedom data.

Results
Examples of the single and two-degree of freedom vortex-induced vibration time series are presented in figures 3 and 4. The cases illustrated are at a reduced velocity of seven, at the lowest mass ratio examined (m^*=2.36). These examples were selected to illustrate the difference in cylinder response in the super-upper and upper response branches.

A noteworthy feature of these time series is the relative regularity of the transverse motion maxima in the two-degree of freedom situation. The large amplitude and regularity of vibration of the super-upper response (figure 4) are evident when compared to the

upper branch response (figure 3). These are characteristic features of the super-upper response branch.
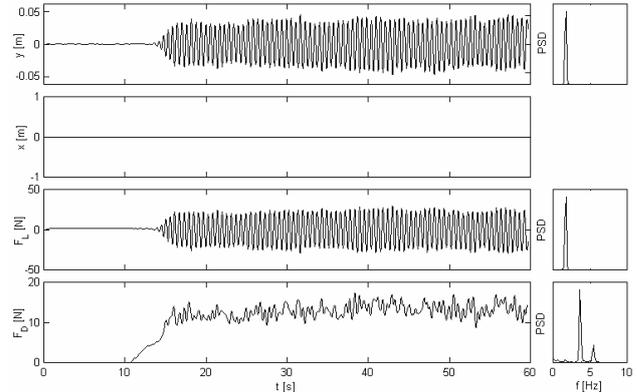
Figure 3 – Time series; m*=2.36, U_r=7, 1 dof.
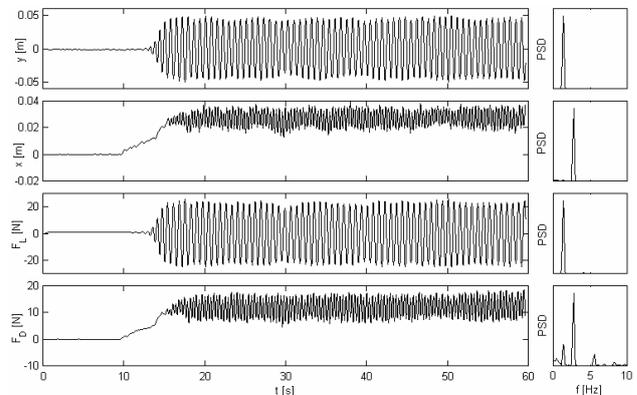
Figure 4 – Time series; m*=2.36, U_r=7, 2 dof.

The transverse vibration maximum amplitude response plots for each of the mass ratios covered are presented in figures 5a through to 5h. The maximum amplitude was defined as the mean of the top 10% of half peak to peak values. As expected, the lock-in range was wider with lower mass ratio. Distinct shifts of the lock-in point to higher reduced velocities and a shift in the lock-out point toward lower reduced velocity were observed at the higher mass ratios. There also exists a slight shift in the corresponding transverse amplitude maxima to higher reduced velocities with increasing mass ratio in the two-degree of freedom case. This trend was not present in the single-degree of freedom case. Also evident from figure 5 is the expected decrease in response amplitude with increasing mass ratio for both the single and two degree of freedom cases. Notably, the decrease was more pronounced for the two-degree of freedom case.

With the single and two-degree of freedom data presented on the same axes, the super-upper response branch is clearly discernible as a deviation between the plots. Note the distinct jump, in the low mass ratio cases, at the upper end of the super-upper response branch back to the lower response branch.

The difference in maximum amplitudes of the single and two-degree of freedom responses decrease with increasing mass ratio. There appears to be a gradual alignment of the two-degree of freedom vibrations and the transverse motion only case with increasing mass ratio. At m^*=10.63 there is general agreement of the two cases (less than 2.5% variation between the peaks of the response plots). At m^*=12.96, this agreement is within 1.5%. The initial and lower response branches at all mass ratios are essentially identical for the single and two degree of freedom cases as is the de-coherence region post lock-in.
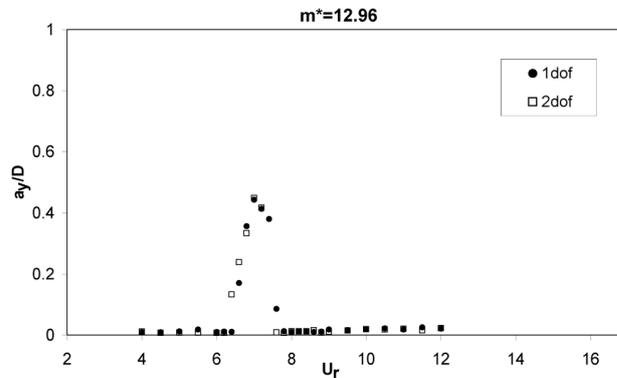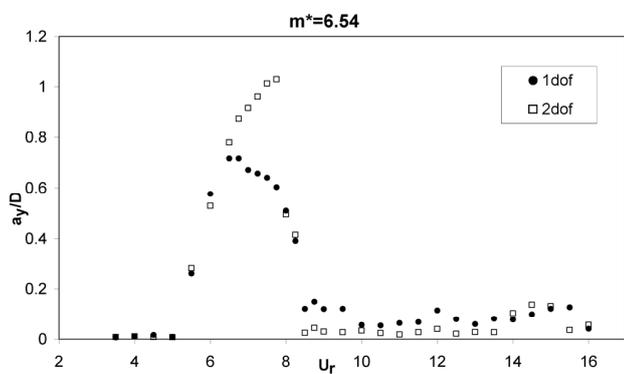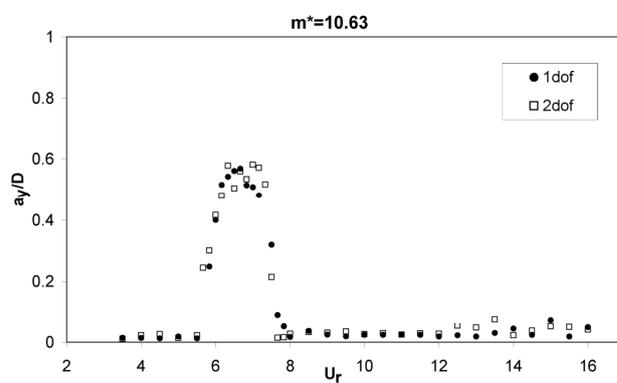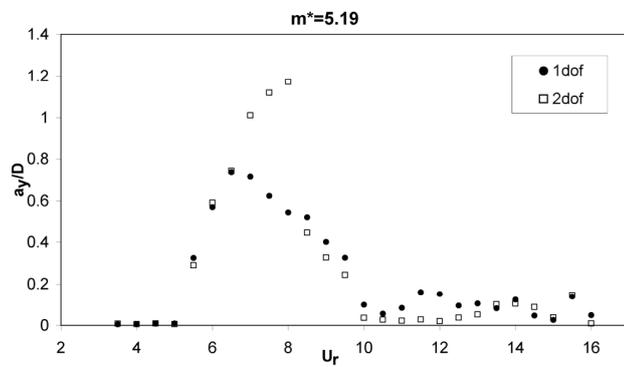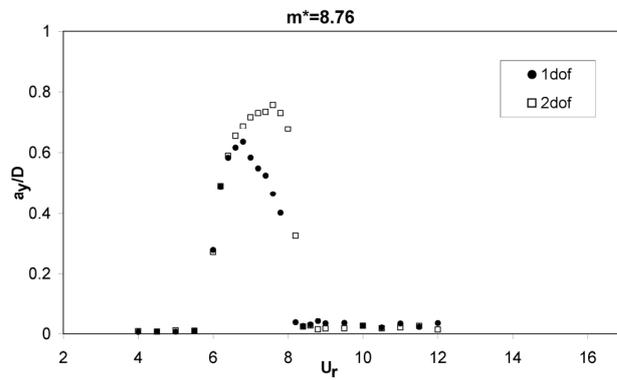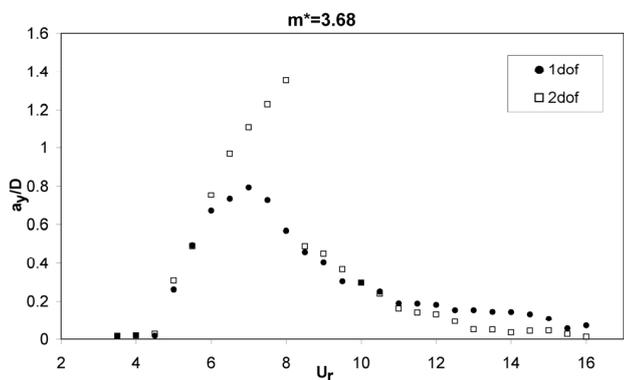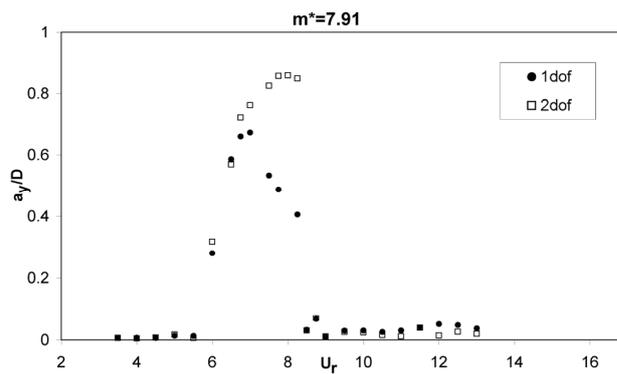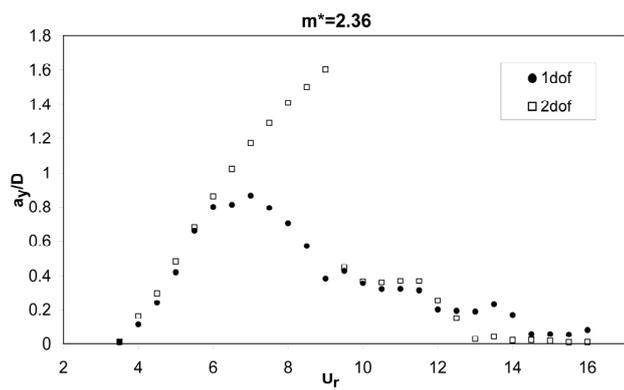
Figure 5 – Transverse response amplitude single and two-degree of freedom; a) m*=2.36, b) m*=3.68, c) m*=5.19, d) m*=6.54, e) m*=7.91, f) m*=8.76, g) 10.63 and h) m*=12.96.

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The inline response amplitude (figure 6) follows the same trend as the transverse response amplitude with respect to maximum vibration amplitude (i.e. decreased maxima with increasing mass ratio). The range of the amplitude response region for the inline vibrations also widens with decreasing mass ratio. The inline response peaks align best with the two degree of freedom maxima rather than the single degree of freedom maxima as was observed in the work of Stappenbelt [16] and the review by Sarpkaya [14].
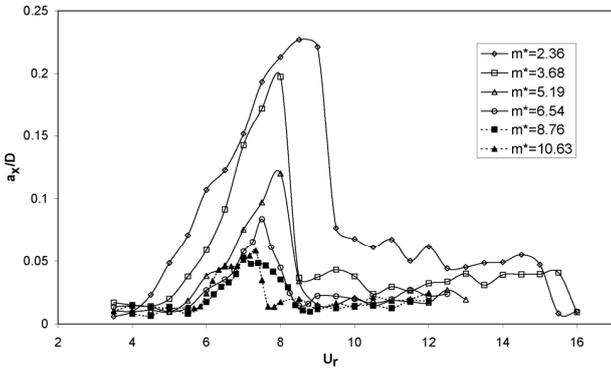
Figure 6 – Inline response amplitude

Examination of the vortex-shedding and vibration frequencies at each mass ratio reinforce the prior observations regarding the range of the amplitude response region. Figures 7 and 8 illustrate the m^*=2.36 case and figures 9 and 10 the m^*=10.63 case. The lock-in region is defined by the coincident vortex shedding and cylinder transverse vibration frequencies.

Synchronisation of the vortex-shedding and vibration frequencies begins at lower reduced velocity and extends to higher reduced velocity for the lower mass ratio cases (see table 3). No significant difference is noted in the lock-in region for the single and two-degree of freedom cases. The lock-in range remained essentially unaltered from one to two degrees of freedom regardless of whether the super-upper or upper response branch was present. This is not unexpected in light of the previously discussed observations regarding the vortex-shedding mode boundaries of the super-upper response branch. It is also consistent with the observation that the initial and lower response branches are essentially identical for the single and two degree of freedom cases since the extremities of these branches delineate the lock-in and lock-out points.

Table 3 – Lock-in values

m*	dof	Initial lock-in (U_r)	Lock-in range (ΔU_r)
2.36	1	4.0	8.5
3.68	1	5.0	7.0
5.19	1	5.5	5.0
6.54	1	5.5	4.5
7.91	1	5.5	3.5
8.76	1	5.5	2.7
10.63	1	5.8	2.0
12.96	1	6.4	1.4
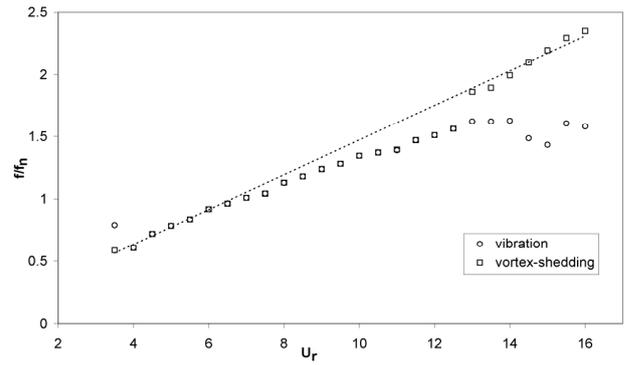
Figure 7 – Vibration and vortex shedding frequencies; m*=2.36, 1 dof.
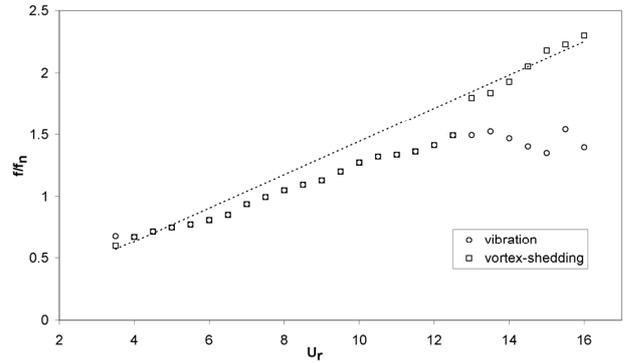
Figure 8 – Vibration and vortex shedding frequencies; m*=2.36, 2 dof.
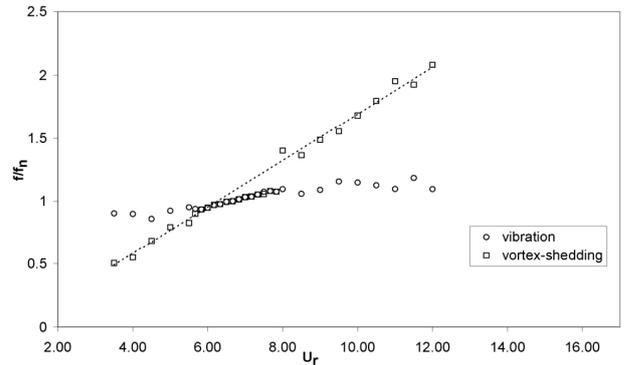
Figure 9 – Vibration and vortex shedding frequencies; m*=10.63, 1 dof.
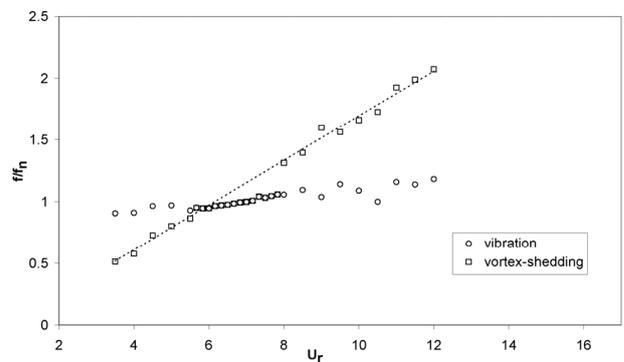
Figure 10 – Vibration and vortex shedding frequencies; m*=10.63, 2 dof.

Table 4 lists the Strouhal frequencies for all cases covered with the corresponding index of fit. As expected, the Strouhal frequencies are significantly lower than the commonly reported values of around 0.2 to 0.21 [17] due to the previously discussed effects related to the low aspect ratio of the cylinder and the lack of cylinder end-plates.

It must be noted at this point that since the cylinder is not stationary at any towing velocity over the experimental range, the shedding frequencies pre and post lock-in allow for the determination of estimates of the Strouhal frequency only. From table 4, it may be noted that the Strouhal estimates for each mass ratio are not identical. This would appear to indicate that the vibration and shedding frequencies outside the lock-in range still influence one another. The observed trend was one of lowered Strouhal frequency with decreasing mass ratio.

Table 4 – Strouhal frequencies

m*	dof	St	R²
2.36	1	0.140	0.995
3.68	1	0.145	0.997
5.19	1	0.155	0.994
6.54	1	0.166	0.998
7.91	1	0.176	0.996
8.76	1	0.180	0.996
10.63	1	0.186	0.995
12.96	1	0.198	0.991

Prior experimentation [2, 13] purports the existence of a linear relationship between the mean drag and the transverse oscillation amplitude of the form

$$\overline{C}_d = \overline{C}_{d0}\left[1 + \kappa\left(\frac{a_y}{D}\right)\right]. \tag{3}$$

The experimentally determined curve fit parameter, κ, has been reported at around 2 [13] and 2.1 [2] for single degree of freedom cases. The curve fit parameters for the present study are presented in table 5. As noted in the study by Stappenbelt [16], the two-degree of freedom gradient is lower than that observed in the single degree of freedom case. The stationary cylinder drag coefficients match the aspect ratio adjusted fixed-cylinder drag data of Achenbach [1] reasonably well.

Table 5 – Mean drag coefficient as a function of transverse displacement amplitude for m*=2.36.

Degrees of freedom	κ	$\overline{C}_{d0}$	R²
1	1.80	0.75	0.92
2	1.34	0.83	0.97

The super-upper response branch and the corresponding mass ratio limit are also discernible from the drag coefficient plots. Figures 11 to 14 illustrate the gradual coalescing of the drag coefficient plots for the single and two-degree of freedom cases with increasing mass ratio as observed with the transverse response amplitude plots.
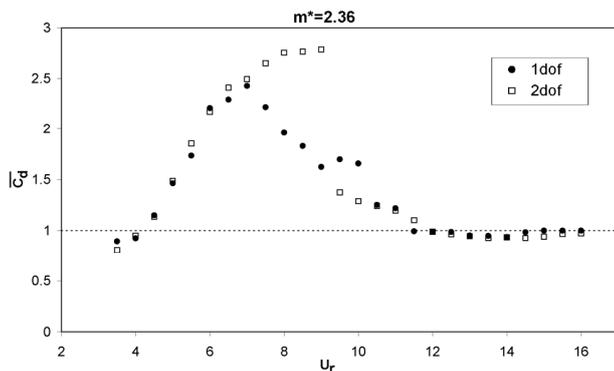
Figure 11 – Mean drag coefficients; m*=2.36.
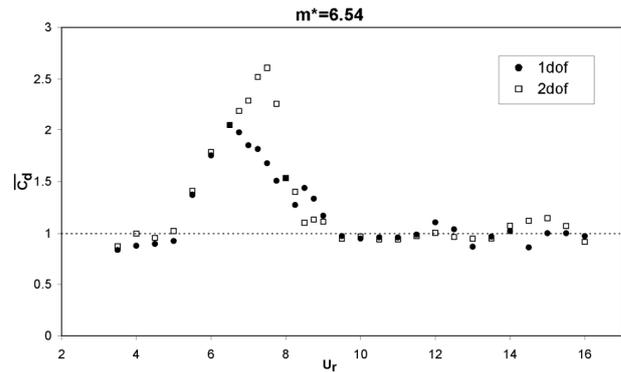
Figure 12 – Mean drag coefficients; m*=6.54.
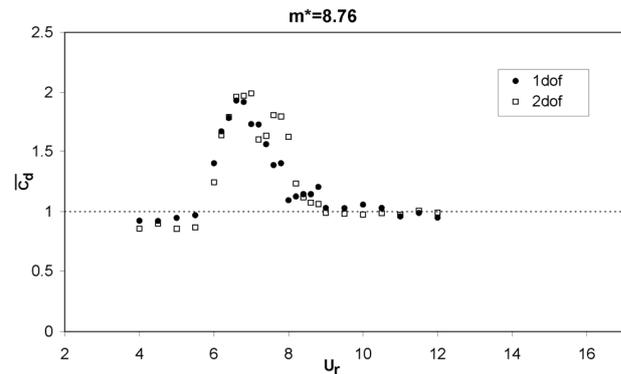
Figure 13 – Mean drag coefficients; m*=8.76.
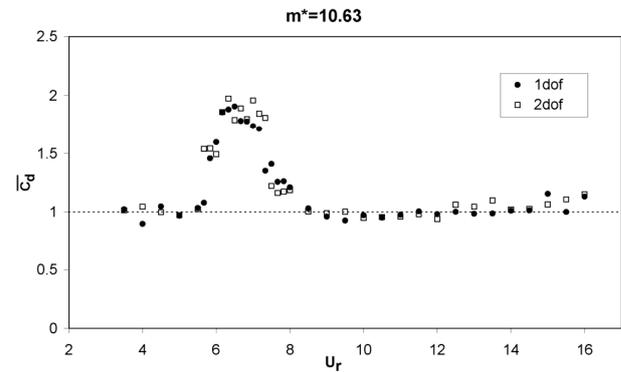
Figure 14 – Mean drag coefficients; m*=10.63.

Conclusions

Consistent with prior studies, a widening of the lock-in region and increased transverse response amplitudes are observed with decreasing mass ratio. The increasing influence of the hydrodynamic mass at low mass ratio is largely responsible for the changes in the lock-in region. The decrease in transverse amplitude response magnitude with increasing mass ratio was more pronounced with two degrees of freedom.

The present study provides evidence consistent with the existence of the super-upper response branch in two-degree of freedom low mass ratio systems. The super-upper response branch was clearly observable in the two degree of freedom cases up to a mass ratio of 8.76. At the mass ratio of 10.63 the single and two-degree of freedom transverse amplitude response plots peaks align to within 2.5%. The alignment does not present as an abrupt change, but rather a gradual coalescing of the single and two-degree of freedom system responses with increasing mass ratio as illustrated in figure 15.
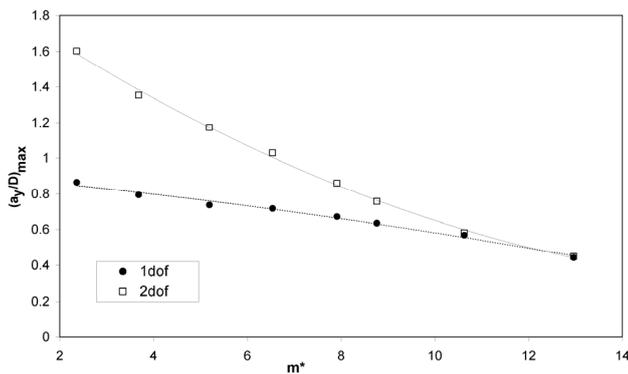
Figure 15 – Transverse response amplitude maxima.

The disparity between the present results and the prior work by Jauvtis & Williamson [9] regarding the upper mass ratio limit implies that the mass ratio may not be the sole determining parameter in the prediction of the occurrence of the super-upper response of a cylinder undergoing VIV. The difference in the results reported by Jauvtis & Williamson [9] and the present study may in part be due to the order of magnitude variation in structural damping ratio. The present study, with the higher damping ratio displayed the alignment between single and two-degree of freedom cases at a higher mass ratio.

The authors are in agreement with the suggestion by Jauvtis & Williamson [9] that offshore design codes should reflect the significant deviation from the single degree of freedom VIV data when dealing with the low mass ratios typically encountered in offshore structure design. However, the mass ratio limit of the super-upper response branch reported in these studies does not appear to be a suitable conservative cut-off point (for the use of empirical relationships based upon single degree of freedom VIV data) in light of the present results. Further investigation is required before any conclusions may be reached regarding the boundaries of the occurrence of the super-upper response branch and the governing parameters so that these may be amalgamated into existing design methods.

Acknowledgments

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