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Modelling of flow and tracer dispersion over complex urban terrain in the atmospheric boundary layer

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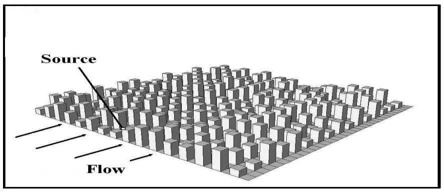

Figure 1: An example of one of the obstacle arrays.

Abstract

A new mathematical model of pollutant plume dispersion in an urban environment is presented. The model uses parameters that explicitly take into account turbulent flow close to the ground and the urban canopy parameters enabling an analytic calculation of the plume concentration profiles and concentration fluctuations. Model predictions are compared with some recent experimental data, showing a close match. The model developed can be used as an analytical tool for predicting pollutant plume behaviour in complex urban environments, or as a prototype and performance check for a new generation of dispersion models.

Introduction

There has been a growing interest in recent years in the modelling of hazards arising from the atmospheric dispersion of pollutant agents in the environment, and the threat that they pose to the population and military forces. This is a particularly challenging problem in an urban setting.

Dispersion of pollutant tracer in an atmospheric boundary layer (ABL) over a heterogeneous (urban) canopy is a complex process to be described by advanced methods of fluid dynamics, turbulence theory, diffusion and statistics. Using comprehensive modelling is computationally intensive and too time consuming when applied to operational problems when a reliable outcome has to be produced within a limited time frame. Plume characterisation requires the development of simplified analytical models of turbulent dispersion based on physical assumptions and "first principles" physics considerations. These models must still be simple enough to be easily treated numerically in an operationally viable way. Such models can also provide a theoretical foundation for "backtracking" problems, i.e. finding a pollutant source in a complex canopy under various meteorological conditions. The purpose of this paper is to summarise the recent research conducted by DSTO (HPPD) in the development of such models.

A NEW MODELLING FRAMEWORK

Mean Flow in a Complex Canopy

The flow model in a surface layer within and above the canopy should correctly describe the average (i.e. non-fluctuating) velocity field. The traditional model for ABL velocity profile is the celebrated log-law profile

$$U(z) = \frac{v_*}{\kappa} \ln\left(\frac{z-d}{z_0}\right), \tag{1}$$

where $U(z)$ is the horizontal velocity, z is the distance from the ground, v_* is the friction velocity, $\kappa = 0.4$ is Von Karman's constant, z_0 is the roughness height and d is the so-called displacement height. For the real ABL flow over the canopy both d and z_0 should be considered as fitting parameters.

It has been known for a long time (dating back to Prandtl, see [8]) that the ABL mean velocity profile can be fairly approximated by a power-law function:

$$U(z) = a v_* \left(\frac{z-d}{z_0}\right)^m, \tag{2}$$

where $U(z)$ is the horizontal velocity, a and m are constants (m is a main parameter of our model). For the ABL over a flat smooth surface the following relationship has been established between (1) and (2) [2]

$$a = \frac{\ln Re}{\sqrt{3}} + \frac{5}{2}, \quad m = \frac{3}{2\ln Re}, \tag{3}$$

where Re is the Reynolds number of the flow in ABL. Observed values of m in the atmosphere range from nearly 0 in very unstable conditions, representing perfect mixing and a uniform velocity profile, to nearly 1 in very stable conditions, approaching the Couette linear profile of laminar motion over a plane surface. For neutral conditions $m \approx 1/7$ [8]. The value of m also depends on surface roughness: roughness promotes mixing near the surface, which reduces the velocity gradient at small z and thus leads to larger variation in m. Based on the so-called distributed drag approach it has been recently shown (see Harman et al 2007) that the entire influence of the canopy on the ABL flow (2), (1) can be described by only one parameter that describes the ratio of the canopy surface area to the total area. For an array of identical cylinders (similar to Fig. 1) this parameter is approximately equal to

$$\varepsilon = \frac{2H}{r_0}\frac{1-\lambda_p}{\lambda_p}, \tag{4}$$

where all parameters in this formula are determined by the canopy morphology (H is the canopy hight, r_0 is the radius of the cylinders and λ_p is the packing density of canopy elements). The limiting values of ε correspond to sparse ($\varepsilon \gg 1$) and dense ($\varepsilon \ll 1$) canopies. We have developed a consistent theoretical framework that allows us to derive a "modified" velocity profile $U(z)$ (2) for a given value of ε , i.e. for a given canopy. Our approach is based on "smooth" matching of the two solutions of momentum balance (below and above the canopy) near the

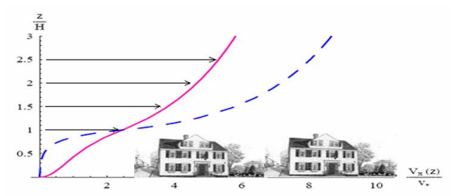

Figure 2: Average horizontal velocity profile. The pink/solid line is for a sparse canopy ($\varepsilon = 0.5$) and the blue/dashed line for a dense canopy ($\varepsilon = 2$).

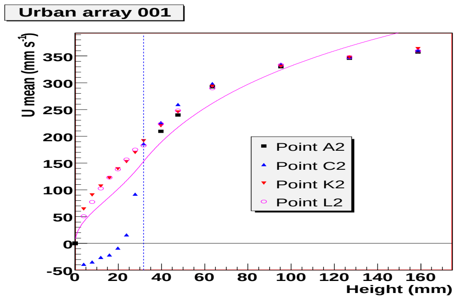

Figure 3: Measured velocity profiles for a simulated urban canopy at different positions relative to canopy objects.

canopy top. We have derived an algebraic system for functions $d(\varepsilon)$, $z_0(\varepsilon)$ to account for the effect of the canopy:

$$\frac{d}{H} = (1-\sigma), \quad \frac{z_0}{H} = \sigma e^{-1}, \quad \frac{1}{\kappa}\sqrt{\frac{\varepsilon\sigma}{2}} = \tanh\left(\sqrt{\frac{2\varepsilon\kappa^2}{\sigma}}\right). \quad (5)$$

The solution of the last equation in system (5) provides a value of $\sigma(\varepsilon)$ that should be substituted in the first two to obtain $d(\varepsilon)$, $z_0(\varepsilon)$ and hence a "canopy-modified" profile $U(z)$ given by (2) and (1). We found that for a large ε function $d(\varepsilon) \to 0$, $z_0(\varepsilon) \to 0$ as a power law (i.e. rather slowly) and $d(\varepsilon) = z_0(\varepsilon) = 0$ if $H = 0$. It should be emphasised that in the proposed framework, the entire morphological variety of canopies manifests itself only in different values of parameter ε.

Examples of velocity profiles calculated with (5) are presented in Fig. 2. In Fig 3 we present our experimental data from a water channel experiment ([5]). The urban canopy was modelled by an array of cubic obstacles that were packed in regular or random patterns (see Fig 1). The velocity measurements were conducted in various positions within a canopy cell (including wake areas). The solid line in Fig. 3 is our model prediction, which represents an average velocity profile for the whole cell. This is to be compared to the individual point measurements of velocity within each cell, which vary significantly from point to point. The point C_2 corresponds to the position directly behind the obstacle (wake area) with a clear visible reverse flow (negative velocity). Our simplified models attempt to capture the "averaged-over-cell" behaviour. For a variety of obstacle array configurations, we observed a reasonably good agreement between our model and the measured velocity profiles.

Mean Concentration Profile

For the derived velocity profile in and above the canopy we computed the mean concentration field from the advection-diffusion equation. For the power-law profile ((2) with $d = 0$) the mean concentration can be modelled by the well-known an-

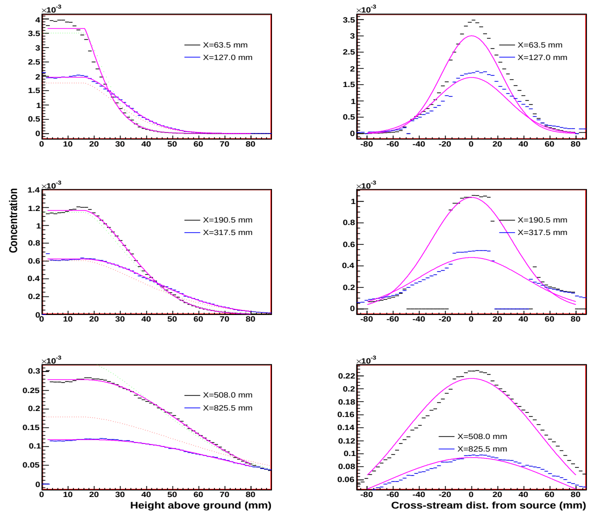

Figure 4: Concentration data fit in the "clipped" model for different downstream distances (C_z for the left column and C_y for the right column). The horizontal axis is the distance from the ground (left column) and cross-stream distance from the source position (right column) .

alytical solution (see [8])

$$C(x,y,z) = C_y(x,y)C_z(x,z). \quad (6)$$

where C_y is the horizontal (lateral) and C_z is the vertical concentration profile. The function C_y can be presented in the standard Gaussian form and for the vertical profile C_z we have the stretched exponential solution:

$$C_z = C_0 \exp\left(-B\zeta^{\alpha}\right). \quad (7)$$

with

$$C_0(x) = \frac{Q}{v_0 z_0}\frac{\alpha}{\Gamma(\beta)}B^{\beta}, \quad (8)$$

$$\zeta = \frac{z}{z_0}, \; B(x) = \frac{x_0}{x}, \; \frac{x_0}{z_0} = \frac{d}{\alpha^2}, \; \beta = \frac{1+m}{\alpha}, \quad (9)$$

where $\Gamma(\cdot)$ is the Gamma function, $\alpha = 1+2m$, $u_0 = av_*$, $d = const$ and Q is the rate at which the source releases the pollutant (for details see [8]).

It is evident that the solution (7) is a valid representation for the concentration profile above the canopy top (i.e. for $z \gg d$). In order to have a consistent profile for all z it should be matched with the pollutant concentration modelled within the canopy (i.e. for $z \leq d$). Two models of the concentration profile within the canopy were validated. The first model was a "clipped" profile, when we simply assumed a constant value of concentration for $z \leq d$. The justification for such a model is the strong process of turbulent mixing that occurs within the canopy that should "smooth out" all concentration gradients. The data fit to the "clipped" model for different downstream positions is presented in Fig 4. The left column is the vertical concentration profile and the right column is the lateral structure of the plume with a Gaussian fit.

The second evaluated concentration model was based on allowing the variation of α with height to provide the best data fit i.e. $\alpha = \alpha(z)$. The rationale behind this framework was the known limiting values of α: $\alpha = 1+2m$ for $z \gg d$, and $\alpha = 2$ for $z \leq d$ (Gaussian diffusion in stagnation areas near canopy floor). As a reasonable approximation we proposed

$$\alpha = (1+2m)(1+\phi\exp(-z/d)), \quad (10)$$

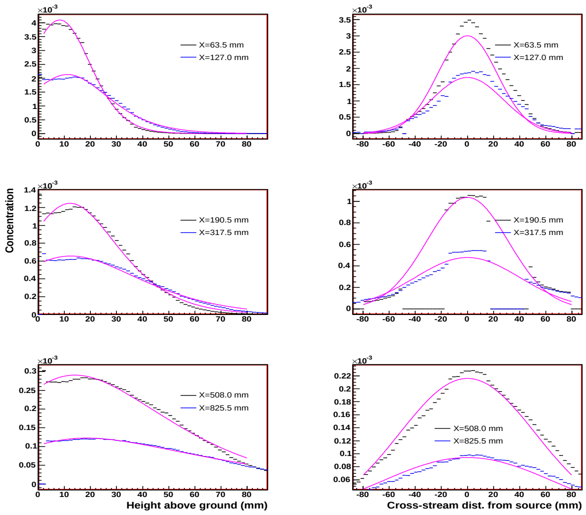

Figure 5: Concentration data fit in "variable α" model. All notations are as in Fig 4.

where $\phi = (1-2m)/(1+2m)$ is the function of the velocity profile parameter m and canopy parameter ε (since $d = d(\varepsilon)$). It is worth noting that C_z (7) depends on α in rather a convoluted way (not only through an exponential power) causing a complex deformation of the concentration profile even with minor change in $\alpha(z)$.

The data fit to the "variable α" model is presented in Fig. 5 (left column). As in Fig. 4, the right column is the lateral structure of the plume with a Gaussian fit. In general we observed that both models are in good agreement with the experimental data. The data fits are nearly indistinguishable downstream of the source. Closer to the source the "variable α" model seems to be a better representation of the vertical plume structure. The better performance of the "variable α" model can be attributed to the more adequate description of the process of turbulent mixing in the canopy layer (i.e. mixing is changing within the canopy with height). The "clipped" model corresponds to constant mixing in the canopy. For dense canopies, with the stagnation flows near the ground, changes with height are not so important ([6]) and both models produce very similar results.

Concentration Fluctuations

The approach outlined above models the development of a "mean plume" within a complex environment. This is the time averaged behaviour of a real dispersing plume, or equivalently, the average pattern that would be seen if an identical release of material was performed many times. Model analysis of pollutant release events also requires the development of "concentration realisation" models that give a statistically sound representation of possible instantaneous patterns of plume dispersion. This is important to enable the investigation of uncertainty or risk in hazard assessments, as well as to provide realistic synthetic environments for operational analysis studies.

It is well-known that tracer fluctuations are very intermittent, so a correct description of intermittency is an important step in building realistic models of tracer fluctuations (see [5], [10]). Intermittency (i.e. the fraction of time when concentration has zero/non-zero values) manifests itself as a "singular" term in the PDF (Probability Density Function) of tracer concentration (see [5])

$$f(C) = \gamma\psi(C) + (1-\gamma)\delta(C), \tag{11}$$

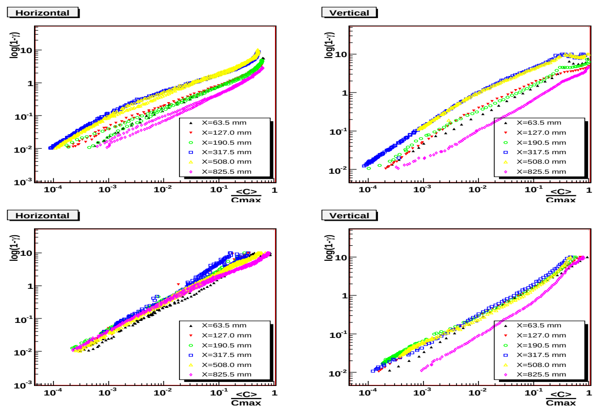

Figure 6: Examples of intermittency profile as a function of inverse mean concentration: slice across the centre of the plume, different downstream positions (top row - no-obstacle canopy, bottom row - $1H$ regular obstacle array). Straight lines correspond to Eq.(13).

where $\delta(\cdot)$ is the delta function, γ is the intermittency parameter ($\gamma = 1$ corresponds to the no-intermittent distribution), $\psi(C)$ is any "regular" concentration PDF (see [5]).

A simple model of intermittency can be developed based on the following physics-based consideration. Let us assume that tracer is advected in the form of consolidated structures (blobs, sheets [4]). This is a realistic assumption since the system is far from equilibrium, so the tracer distribution is far from being perfectly mixed. The existence of discrete structures of tracer suggest that Poisson statistics may provide a useful framework for describing tracer fluctuation behaviour, with "an individual event" being attributed to the "tracer blob" passing a particular point of the flow. Then for the probability of the k occurrences of such an event we can apply the celebrated formula of the Poisson distribution

$$\mathrm{Prob}(k,\lambda) = \frac{\exp(-\lambda)\lambda^k}{k!}, \tag{12}$$

where λ is the average number of occurrences. Then for the "no-occurrences" event we simply get $\mathrm{Prob}(0,\lambda) = \exp(-\lambda)$ irrespectively of any statistical properties of "blobs distribution". This evidently results in the following formula

$$1-\gamma \sim \exp\left(-\beta\frac{C}{C_{max}}\right), \tag{13}$$

where we have used the obvious relationships: $\gamma = \mathrm{Prob}(0,\lambda)$, $\lambda = \beta C/C_{max}$, $\beta = -\ln(1-\gamma_{max})$. Thus we may expect that intermittency factor γ exponentially decreases with C .

The formula (13) was validated with our data from the water channel experiment. The results are presented in Fig.6, where $\ln(1-\gamma)$ is plotted against C_{max}/C in the Log-Log scale, so straight lines correspond to Eq.(13). We can see that the relationship (13) holds for wide range of parameters (concentrations, downstream positions and types of obstacle arrays).

Our next step was to developed a simple model for concentration intensity $i = \overline{C^2}/\overline{C}^2 - 1$. The important result was found in [5] where it was shown that for Gifford's isotropic meandering plume model

$$i+1 \sim \left(\frac{C_{max}}{C}\right)^{\zeta}, \tag{14}$$

with $0 \le \zeta \le 1/2$.

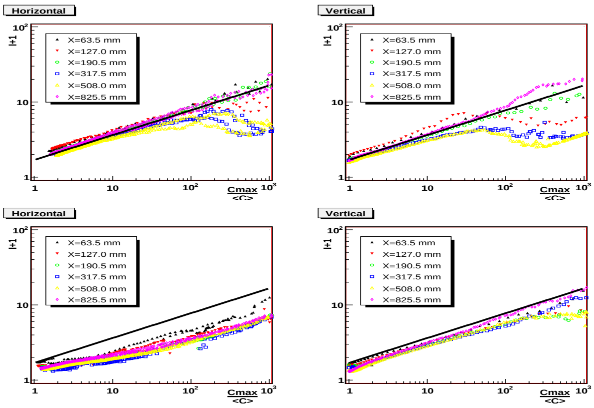

Figure 7: Examples of concentration intensity profile as a function of inverse mean concentration: slice across the centre of the plume, different downstream positions (top row - no-obstacle canopy, bottom row - $1H$ regular obstacle array. Solid line is Eq. (14) with $\zeta = 1/3$.

A similar formula can be derived in another simplified case - tracer dispersion in the surface layer when any dependency on cross-stream direction can be neglected (line source). In that case all concentration moments can be described by the same universal profile [7] , so $\overline{C^2} \sim \overline{C}$. Then for i we arrive at the same Eq.(14), but now with $\zeta = 1$. These results provide a solid foundation for an assumption that the functional relationship of (14) should hold in a more general case (i.e. when the plume is strongly anisotropic and 3D effects cannot be neglected). We also expect the "shape" parameter to have a rather universal value.

The above assumption was validated with our experimental data and the results are presented in Fig. 7. The value of $i+1$ is plotted against C_{max}/C in the Log-Log scale, so straight lines should correspond to (13). Again, we can see that the relationship (13) holds over a wide range of the model parameters. The solid line corresponds to the "fitting" value $\zeta = 1/3$.

A more advanced model of plume concentration fluctuations has been developed based on the so-called "fluctuating plume" approach, where overall fluctuations are represented as a combined effect of slowly oscillating plume meander and fast in-plume fluctuations. Thus, for the conditional PDF of concentration in the absolute frame f the following general representation was adopted [5]:

$$f(C,\, \mathbf{x}) = \int f_r(C,\, \mathbf{x} - \mathbf{X}_c) f_c(\mathbf{X}_c) d\mathbf{X}_c, \qquad (15)$$

where f_c is the PDF of centroid meander, f_r is the concentration PDF in the relative frame (associated with plume centroid), $\mathbf{X}_c(t)$ is the position of centroid. Development of realistic models for f_r and f_c requires the application of rather complicated statistical methods and are described in detail in our other publications (see [5], [10], [9]). It is well-known that the PDF for horizontal meander is always close to Gaussian [5]. Based on Large Deviation Theory we have proposed a model where the PDF for vertical plume meander z_c can be described by a Gamma distribution [9]

$$f_{cz}(Z_c) \sim Z_c^{a-1} \exp\left(-\frac{Z_c}{b}\right), \qquad (16)$$

where, based on the properties of Gamma distribution [1], parameters a and b can be expressed in terms of the the first two momentums of Z_c: $ab = \overline{Z_c}$, $ab^2 = \overline{Z_c}^2 - \overline{Z_c}^2$. For plumes in the ABL the estimate $\overline{Z_c}^2 \approx 2\overline{Z_c}^2$ holds over a wide range of parameters [8], so $a-1$ in (16) seems to be rather small.

We have proposed a new model for in-plume fluctuations in a recent paper [3], where we related the concentration fluctuation intensity i with the ABL flow above the canopy. The important conclusion is that the statistical properties of the plume in the canopy can be parameterised with the flow parameters (m, ε), so a proposed two-parameter model (2), (4) also provides a consistent framework for concentration fluctuation modelling.

CONCLUSIONS

Physics based models of a plume in an urban canopy allow a simplified (but still adequate) analytical description of pollution transport in a complex environment which is particularly useful for hazard management applications. The proposed theoretical framework has been validated against our water channel experimental data and has provided a close match. The proposed framework can help to validate and justify some more empirically based and heuristic assumptions of some operational dispersion models. Our modelling framework can thus be used as a valuable performance check of such models, or be extended to an operational model prototype, able to be linked to larger modelling systems.

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