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Heat transfer from partially buried pipes

J.C. Morud[1] and A. Simonsen[1]
[1]SINTEF Materials and Chemistry, Trondheim, N-7465 Norway.
Corresponding author: John.Morud@sintef.no

Abstract

Analytical expressions for the heat loss from buried pipes have been known for a long time. In this paper, we derive approximate expressions for the heat loss from partially buried pipes. The approximations for the overall heat loss are accurate to within 10% of the rigorous solution of the heat conduction equation for the soil, and provide a smooth, continuous transition between the unburied and the completely buried case.

Introduction

Heat loss from buried pipelines is important in a number of applications, such as water distribution lines and on- and offshore oil and gas lines. Several authors in the past (ref [1] through [6]) have analyzed the case where the pipes are completely buried in the ground. However, the case of partially buried pipes is also important, and we have thus derived approximate analytical expressions for the partially buried case. The expressions provide a smooth continuous transition between the unburied and the completely buried case.

A brief outline of the paper: We first define the problem and list the known results for the completely buried case. We then derive approximate expressions for the partially buried case. Finally, we compare the results with numerical simulations.

Buried pipes occur in a number of applications, and it is difficult to be just to all previous authors. This paper is heavily influenced by the work by Bau and Sadhal (1982) who derived analytical expressions for completely buried pipes. Bau (1984) later extended the work to convective effects in the soil. This paper is also inspired by the work of Chung *et. al.* (1999). Of other related work may be mentioned Badr and Pop (1988), Balaix et. al. (1981) and Cucumo et. al. (2007).

Problem definition

For simplicity of discussion, we assume an offshore pipeline in the following. However, the results are rather generic, and may easily be applied in other applications.

Consider a partially buried pipe of radius R, as shown in Figure 1 where only the buried part is shown. The centre of the pipe is a distance H from the sea floor. The partially buried case is thus defined by $-R < H < R$.

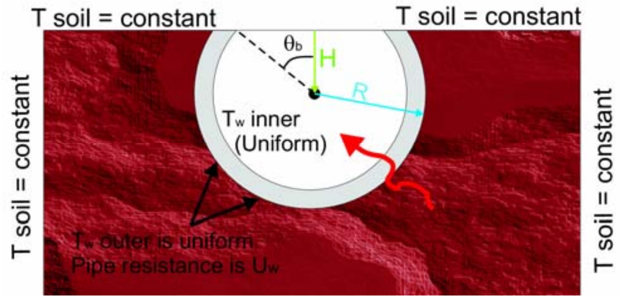

Figure 1. A partially buried pipe.

The pipe wall is assumed to have a heat transfer coefficient U_w [W/m^{2}K], and an unburied pipe is assumed to have an overall heat transfer coefficient of U_{sea} [W/m^{2}K] (Combination of U_w and convective heat transfer).

The heat transfer in the soil is assumed to be governed by heat conduction (Temperature T [K]):

$$\nabla^2 T = 0 \tag{1}$$

with an effective heat conduction k_{soil} [W/mK] in the ground.

The boundary condition is specified temperature at the sea floor and at the inner wall of the pipe. The soil is assumed to have infinite extent.

Results for the completely buried case

The results for completely buried pipes (H>R) have been known for a long time. From Bau and Saddhal 1982, we have the following approximate expressions for the overall heat loss (Combined heat transfer coefficient for pipe wall and soil, denoted "ground"):

$$\frac{U_{ground} R}{k_{soil}} = \frac{Bi}{\sqrt{1 + Bi^2 \alpha_0^2 + 2Bi\alpha_0 \coth(\alpha_0)}} \tag{2}$$

where

$$U_{ground} = \left(\frac{1}{U_w} + \frac{1}{U_{soil}} \right)^{-1} \tag{3}$$

is the overall (combined) heat transfer coefficient [W/m^{2}K],

$$\alpha_0 = \ln\left(\frac{H}{R} + \sqrt{\left(\frac{H}{R}\right)^2 - 1} \right) \tag{4}$$

is an auxiliary geometrical quantity and

$$Bi = \frac{U_w R}{k_{soil}} \tag{5}$$

is the Biot number of the pipe in the ground.

According to the authors, the prediction is accurate to within 2%. Moreover, the prediction can be shown to be a lower bound to the exact solution of the heat conduction equation for the soil.

Derivation of heat loss from partially buried pipes

In the following, engineering relations for the partially buried case are derived. The aim is simple-to-use engineering relations within engineering accuracy.

Consider the coordinate system indicated in Figure 2:

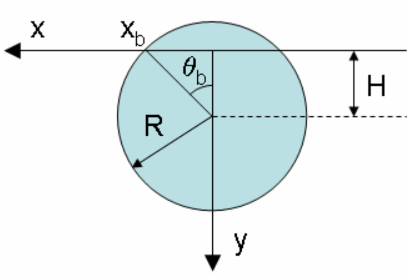

Figure 2. Definition of coordinates

Define the following complex quantities

$$z = x + iy$$
$$w = u + iv \tag{6}$$

where x,y are the physical coordinates and u,v are new coordinates defined by the following complex-variable (conformal) mapping.

$$w = Ln\left(\frac{z - x_b}{z + x_b}\right) \tag{7}$$

In the new u,v coordinate system the geometry becomes particularly simple, as shown in Figure 3:

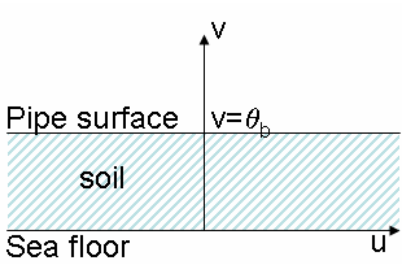

Figure 3. The mapped geometry in u,v-coordinates

For the asymptotic case of large Biot numbers, Bi>>1, the pipe surface temperature is approximately constant around the circumference, and the local heat flux, q [W/m^2], becomes:

$$q = -k_{soil}\left(\frac{\partial T}{\partial n}\right)_{z-plane} = -k_{soil}\left(\frac{\partial T}{\partial n}\right)_{w-plane}\left|\frac{dw}{dz}\right| \tag{8}$$

or

$$q = k_{soil}\frac{(T_w - T_{soil})}{\theta_b}\cdot\left|\frac{x_b}{Ry}\right| \tag{9}$$

The local heat transfer coefficient in the soil in the Bi>>1 case thus becomes

$$h_{soil} = \frac{k_{soil} x_b}{\theta_b R y} \tag{10}$$

Combined with the heat transfer coefficient of the pipe wall, U_w, we obtain a combined local heat transfer coefficient:

$$h_{ground} = \left(\frac{1}{h_{soil}} + \frac{1}{U_w}\right)^{-1} \tag{11}$$

Averaging this around the buried part of the circumference:

$$U_{ground} = \frac{1}{\pi - \theta_b}\int_{\theta_b}^{\pi} h_{ground}\, d\theta \tag{12}$$

The result of this operation is the average heat transfer coefficient, U_{ground}, for the buried part of the circumference, and may be summarized as:

$$\theta_b = \arccos\left(\frac{H}{R}\right) \tag{13}$$

$$C_1 = \sqrt{1 - \left(\frac{H}{R}\right)^2} \tag{14}$$

$$C_2 = \frac{H}{R} + \frac{C_1}{\theta_b \cdot Bi} \tag{15}$$

$$\frac{U_{ground} R}{k_{soil}} = \begin{cases} \dfrac{2}{\theta_b(\pi - \theta_b)}\dfrac{C_1}{\sqrt{C_2^2 - 1}}\left[\dfrac{\pi}{2} - \mathrm{atan}\left(\sqrt{\dfrac{C_2 + 1}{C_2 - 1}}\tan\left(\dfrac{\theta_b}{2}\right)\right)\right]; \; C_2 > 0 \\ \dfrac{1}{\theta_b}(\pi - \theta_b)\dfrac{C_1}{\sqrt{1 - C_2^2}} Ln\left(\dfrac{\tan\left(\dfrac{\theta_b}{2}\right) + \sqrt{\dfrac{1 - C_2}{1 + C_2}}}{\tan\left(\dfrac{\theta_b}{2}\right) - \sqrt{\dfrac{1 - C_2}{1 + C_2}}}\right); \; C_2 < 0 \end{cases} \tag{16}$$

Finally, the buried and unburied parts are combined into the desired overall heat transfer coefficient:

$$\boxed{U_{total} = \frac{\theta_b}{\pi}U_{sea} + \left(1 - \frac{\theta_b}{\pi}\right)U_{ground}} \tag{17}$$

The analysis above was made for the case Bi>>1, where the heat resistance is dominated by the soil. In this range, the analysis is sound.

For the other extreme, Bi<<1, the assumption made for the soil (constant temperature at the pipe surface) is of course invalid. However, in this case, the heat resistance is dominated by the pipe wall, and the error made in the soil analysis has no influence on the final result. Thus, Equation 17 is applicable in the case Bi<<1 also.

To assess the approximate validity in the remaining case Bi~1, we have calculated the heat transfer numerically over this range of Biot numbers. As will be demonstrated in the following, the expression is valid to within 10% in the Bi~1 case. Thus, the formula for the overall heat transfer, Equation 17, may be used to engineering accuracy for the complete range of the Biot number, Bi.

Figure 4 shows the total heat transfer coefficient for the range 1<Bi<5, calculated with equation 17:

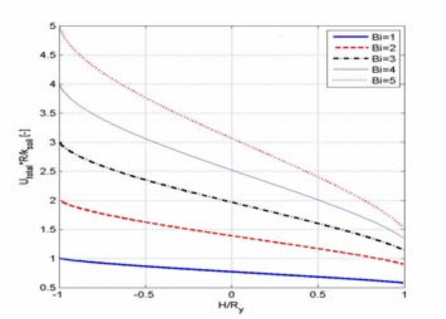

Figure 4. Shape factor as function of burial depth and Biot number.

Comparison against numerical (CFD) simulations

Comparisons of the accuracy of the approximate formula 17 was performed by one of the authors in an unpublished work [8], and it was found that the expression was always within 10% of the numerical calculation of the heat conduction equation.

In the following, we compare against an even more realistic simulation where there is two-dimensional heat conduction also inside the pipe wall. As will be demonstrated, the simple analytical model still yields results of engineering accuracy for the overall heat transfer coefficient U_{total} for this case, even though the approximation for the soil is weak in the barely buried case (i.e. when the pipe is on top of the sea floor).

Problem setup

The pipe is assumed to consist of three layers: steel; asphalt and concrete. Table 1 contains the data used. The data is taken from a typical pipe used in offshore gas lines. The resulting overall pipe wall coefficient is U_w=20.3W/m^2·K

Pipe wall properties			
Material	**k[W/m2.K]**	**Ri[m]**	**t[mm]**
Steel	50.00	0.4832	24.2
Concrete	0.74	0.5074	7.0
Asphalt	2.90	0.5144	100.0

Table 1. The wall properties of the simulated pipe.

The conductivities of soil and sea is taken as k_{soil}=2.97 and k_{sea}=0.60 W/m.K

The boundary conditions are shown in Figure 5 and the grid for the half-buried case in Figure 6. The CFD tool FLUENT was used for the simulations, and a k-ε turbulence model with standard parameters was used in all cases. Particular care has been made to have sufficient grid resolution at the outer surface of the pipe (y^+~1 for the first cell layer).

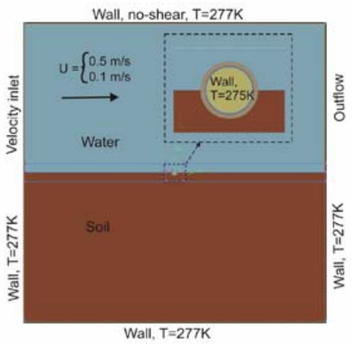

Figure 5. Geometry and boundary conditions for the CFD calculations

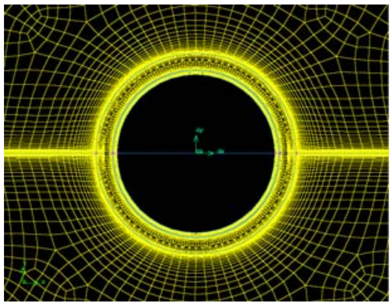

Figure 6. Grid for the half-buried case

CFD results

Figure 7 shows calculated temperature contours for the half-buried case.

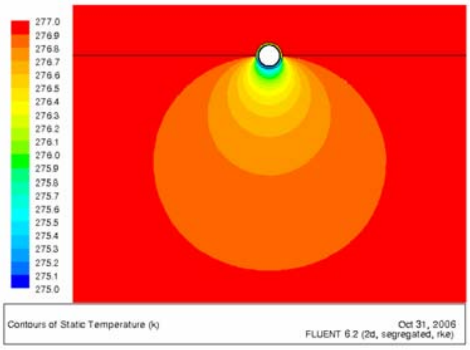

Figure 7. Temperature contours for a half-buried pipe

Figures 8 through 10 show closeup temperature contours for the cases H/R=-0.99, 0 and 0.99 respectively.

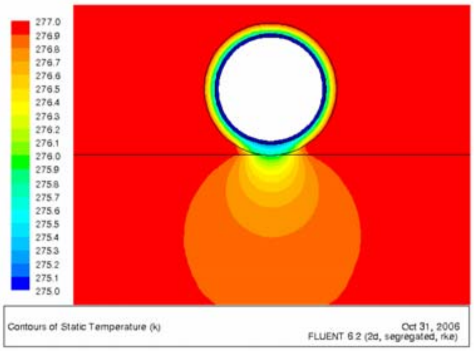

Figure 8. Temperature contours for a barely-buried pipe (H/R=-0.99)

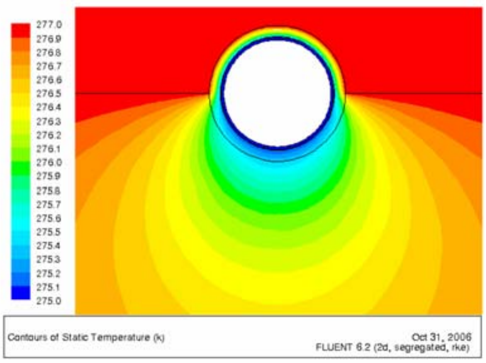

Figure 9. Temperature contours for a half-buried pipe (H/R=0)

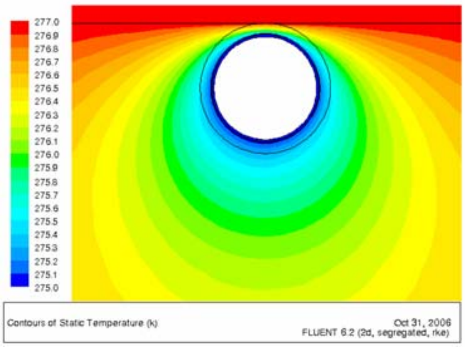

Figure 10. Temperature contours for a almost-buried pipe (H/R=+0.99)

Comparison of CFD and analytical results

Figure 11 and 12 show comparisons between the CFD- and the corresponding analytical results (Equation 17) as function of burial depth. For the analytical calculations, the convective heat transfer resistance in the sea is simply neglected.

Figure 11 compares the heat transfer coefficient, U_{ground}, of the buried part of the partially buried pipe. As can be seen, the error is not large as long as the pipe is sufficiently buried, but the error in the buried part is significant when the pipe is just touching the ground. A scrutiny of the temperature contours (Figure 8) reveals that heat transfer in the circumferential direction of the pipe wall is significant in this situation, and this effect was neglected in the analytical derivation.

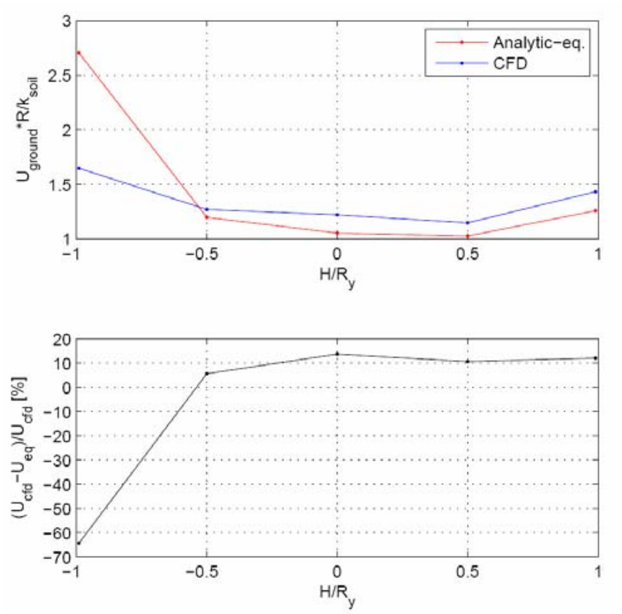

Figure 11. Comparison of the heat transfer coefficient of the buried part of the pipe circumference.

Fortunately, the overall heat transfer coefficient for the pipe, U_{total}, is dominated by the unburied part of the pipe wall in this barely-buried situation, and the error in the overall heat transfer coefficient is small. Figure 12 compares results for the overall heat transfer coefficient. The error is now of the order of 10% and less for the overall heat transfer coefficient for the whole range of burial depths, H/R.

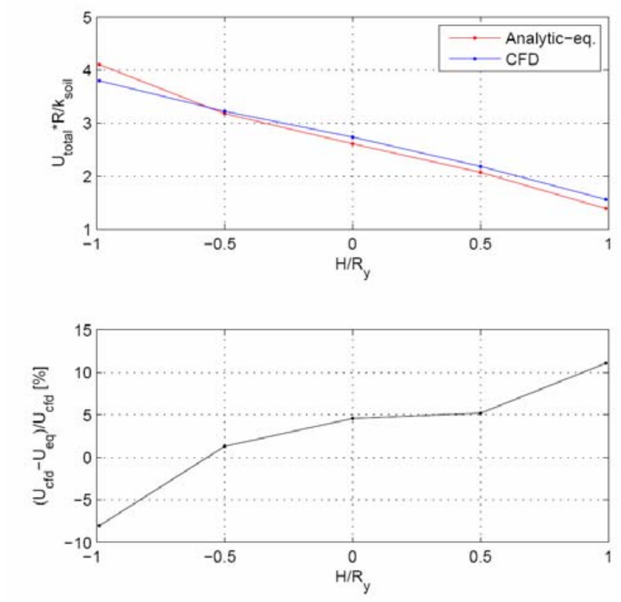

Figure 12. Comparison of the total heat transfer coefficient of the partially buried pipe.

Discussion

The approximation for the overall heat transfer coefficient of a half-buried pipe, Equation 17, is accurate to within ~10% of the exact solution of the heat conduction equations of the soil. This is sufficient for many engineering applications.

Further improvement of the analytic results may be achieved with a more sophisticated model. This is clearly indicated by the temperature contours in Figures 8 through 10. Effects that may be important are:

- Removal of the simplifying assumpion of constant temperature at the outer pipe wall, valid for Bi>>1. Analytical results (simple engineering formulas) may still be possibe by truncating a series expansion.
- Introduction of heat conduction in the circumferential direction in the pipe wall, which can be significant in the barely-buried case.
- Introduction of convection in the soil. See e.g. the paper by Bau (1982) for the completely-buried case.
- Transient effects. The steady state solution of the heat equation for the soil is influenced by the soil at a large distance from the pipe. However, the typical time scales of the soil at this length scale may be quite large, and in many cases much larger of the time scales of the pipe operation. In such cases transient effects should be accounted for.

Circumferential temperature variations was also neglected by Bau and Sadhal[1]. It is believed that a two dimensional analytic solution of the governing equations is achievable both for the partly buried as well as the fully buried scenarios.

Even though the overall heat loss from the pipe is predicted with sufficient accuracy, there may be a need for a more accurate prediction of the variation of the heat flux around the pipe circumference. Examples include the local prediction of corrosion rates in gas pipelines, where the local condensation rate of water is important, and the prediction of local hydrate formation in pipelines, where the local heat flux may govern the local formation rate of hydrates.

Conclusion

Approximate relations for the overall heat loss from a partially buried pipe have been developed. The prediction of the overall loss is within ~10%, which is sufficient in many applications.

There is still room for improvements, in particular with respect to the variation of the heat flux around the circumference. Effects that may be included are heat flux in the circumferential direction, convective effects in soil, transient effects as well as the removal of simplifying assumptions in the analysis.

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