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Hot-Wire Attenuation and Its Correction in Turbulence Measurements

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Abstract

A scheme is proposed for correcting the hot-wire attenuation in turbulence measurements in both frequency and time domains. The scheme is tested for simulated hot-wire signals under velocity impulse and sinusoidal perturbations of various frequencies. It is found that the frequency response of the hot-wire system and the calculated central moments, after applying the correction, can be greatly improved.

Introduction

In turbulence measurements using hot wires, it is generally assumed that the frequency response of the hot-wire system is similar to a low pass filter with constant response at low frequencies and a graduate roll off near the cut-off frequency. The hot-wire system is calibrated using either a 'static' calibration method (calibrate the system at various mean velocities, Brunn, 1995) or a 'dynamic' calibration method (Perry, 1982) with the hot wire being shaken at low frequency around *1 Hz*. The output voltage of the CTHWA is then related to the mean velocity by a polynomial of 3rd to 5th order depending on the calibrated velocity range.

However, the frequency response of the hot-wire system is not flat at frequencies below the cut-off frequency, rather it is attenuated. Specifically, the frequency response of the hot-wire system is flat at low frequencies, starts to decrease at a frequency $2\pi f_l \approx 40D/(2l)^2$, and gradually reaches a lower plateau at $f > 100f_l$ as according to Beljaars (1976). Here $D=k_w/\rho_w C_w$ is the thermal diffusivity of hot-wire materials and subscript w refers to the hot wire. Li (2004) found that the lower plateau is reached only at $f > 1000f_l$. Freymuth (1979) analysed the attenuation using an intuitive argument and found that for a $5\ \mu m$ platinum wire at $2l/d = 200$, the energy level at the second plateau is about 15% less than that at $f < f_l$. These conclusions have been supported by the numerical results of Morris and Foss (2003) and the analytical results of Li (2004). The frequency where attenuation begins is quite low ($f_l = 160\ Hz$ for a $5\ \mu m$ platinum wire at a length to diameter ratio of *200*) and its effect extends to frequencies (Li, 2004) beyond the cut-off frequency of the hot-wire system. Unfortunately, this frequency range covers most of the turbulence measurements today. The effects of this attenuation on measured mean velocity and turbulent intensities may be small, but can be large on the estimation of turbulent energy dissipation, vorticity and structure functions of high orders since contributions to these turbulent quantities are mainly from small scale turbulence or turbulence at high frequencies. So far, there has been no study to the possible consequences of this attenuation on turbulence measurements. The reason for this is that there has been no experimental method available in determining this attenuation.

Recently, in studying the dynamic response of the hot-wire system under various perturbations, Li (2006) found that the frequency response of the hot-wire system to a square wave perturbation at the offset of the second stage amplifier in the feedback circuit of the CTHWA is close to that of the analytical solution of Li (2004). Some differences exist between the two, such as the cut-off frequency determined from the frequency response of the hot-wire system to a square wave perturbation at the offset is in general 10-15% higher than the true one. Nevertheless, the frequency response of the hot-wire system to a square wave perturbation at the offset showed the attenuation before the cut-off frequency and follows that of analytical solution closely. This provides, for the first time, the possibility of deriving a practical method to determine the hot-wire attenuation and thus offers an opportunity to correct its effects in turbulence measurements.

In this paper, a scheme is proposed for correcting this hot-wire attenuation in turbulence measurements. This involves designing a digital filter in both the frequency and time domains. The digital filter is applied to the dynamic response of the hot-wire system to the velocity impulse perturbations and sinusoidal velocity perturbations at various frequencies. The results show that the attenuation effects of the hot wire in turbulence measurements can almost be completely corrected at frequencies below the cut-off frequency. Weiss, Knauss and Wagner (2001) proposed a similar method to determine the frequency response of the hot-wire system. Their focus was in the high frequency region near the cut-off frequency and they showed that the correction can be used to improve the signal to noise ratio. The focus of this paper is from the low to high frequency region and the hot-wire attenuation effect.

Corrections to the hot-wire attenuation effects

Hot-wire attenuation has been analysed by Freymuth (1979) using an intuitive argument. This is confirmed numerically by Morris and Foss (2003) and analytically by Li (2004). Figure 1 shows the central moments of the sinusoidal perturbation at the hot wire. For detailed CTHWA circuit and other details on numerical simulation, refer to Li (2006). The perturbation magnitude is 25% of the mean velocity. The results show that the attenuation effect of the hot wire increases with increasing orders. This effect, as pointed by Freymuth (1979) and Bremhorst and Gilmore (1978), is due to heat waves along the finite hot-wire length with a non-uniform mean temperature profile. Li (2004) pointed out that this attenuation effect could not be corrected either by the static calibration method or by the dynamic calibration method of Perry (1982), because the dynamic calibration involves only 'shaking' the hot wire at around *1 Hz*. Beljaars (1976) suggested that the attenuation effect starts at $2\pi f_l = \omega_l \approx 10/\tau$ and this has been confirmed in Li (2004). For the hot wire used for the results in figures 1, $f_l \approx 419\ Hz$. This is well above the 'shaking' frequency of *1 Hz* during dynamic calibration. Li (2004) also pointed out that in practice it is very difficult to calibrate the hot wire to take into account this attenuation effect of the hot wire and it can only be remedied by a correction scheme during data analysis.

Figure 2 shows the frequency response of a hot-wire system to various perturbations. As pointed out in Li (2006), the dynamic response of the hot-wire system, the energy spectral density of the dynamic response signal, the second order centre moments from sinusoidal perturbation at the hot wire all give the same results. The frequency response from the square wave perturbation follows the true frequency response of the system from low to some frequency before the cut off frequency. But difference exists between the true frequency response and that determined from the square wave test.

The results in figures 2 point to the possibility of correcting the attenuation effect of the hot wires after the data are collected (by a data acquisition system, say). In performing corrections, the objective here is to achieve the frequency response of the hot-wire system to be similar to that of a low pass Butterworth filter

$$H_B(f) = \frac{1}{1+(f/f_c)^{\sigma}} \tag{1}$$

where constant σ is to be determined so that the frequency response will be as close as possible to the frequency response of the hot-wire system near the cut-off frequency. A different low pass filter such as a second order system with optimal damping ratio can also be used. Here we correct only the attenuation effect of the hot wire and it is assumed that the Butterworth filter has the same cut-off frequency as that of the hot-wire system.

The process of this attenuation correction is as follows. First, a digital filter in both the frequency and time domains was designed based on the frequency response of the hot-wire system. Then, the coefficients in the time domain from the designed digital filter were used to convolute with the raw experimental data to obtain results with attenuation being corrected.

The designing of the digital filter begins with tuning the hot wire by using a step perturbation at the offset to achieve an optimal response following a procedure as suggested in Li (2006). The dynamic response of the hot-wire system will be sampled using a data acquisition system and the spectral density will be calculated using FFT. The cut-off frequency f_c of the hot-wire system at *–3dB* can then be determined from the energy spectral density which has been normalized to be zero dB at low frequency. From these, a second low pass filter is constructed as

$$\begin{aligned} F(f) &= \frac{H_B(f)}{E(f)} \frac{1}{1+(0.5f/f_c)^{\sigma}} \qquad f < f_c \\ F(f) &= \frac{1}{1+(0.5f/f_c)^{\sigma}} \qquad f \ge f_c \end{aligned} \tag{2}$$

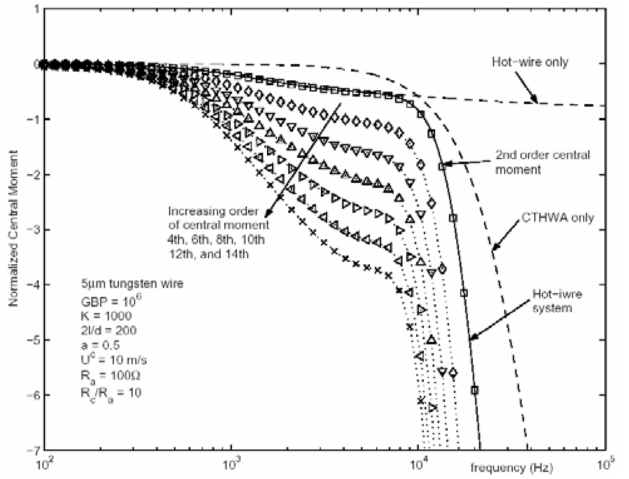

Figure 1 The normalized central moments of different orders at $\Delta U = 0.25U^c$.

In designing the digital filter, the cut-off frequency of the Butterworth filter is assumed to be the same as that of the hot-wire system but the cut-off frequency of the correction filter ($2f_c$) has been chosen such that the correction filter does not affect the turbulence signal sensed by the hot wire. A different cut-off frequency can be chosen for the correction filter as long as it is larger than $2f_c$. For the circuit given in Li (2006), constant σ = *4.5* was determined from the frequency response of the hot-wire system to square wave perturbation at the offset. This $\sigma = 4.5$ is close but different from that of a second order dynamic system as suggested by Perry (1982) ($\sigma = 4$) and quite different from that of a third order dynamic system as suggested by Freymuth (1977a) ($\sigma = 6$). The non-sharp vertical edge of the correction filter at $f > 2f_c$ is necessary in order to avoid the damped 'ringing' (Press et al, 1992) of the digital filter in the time domain.

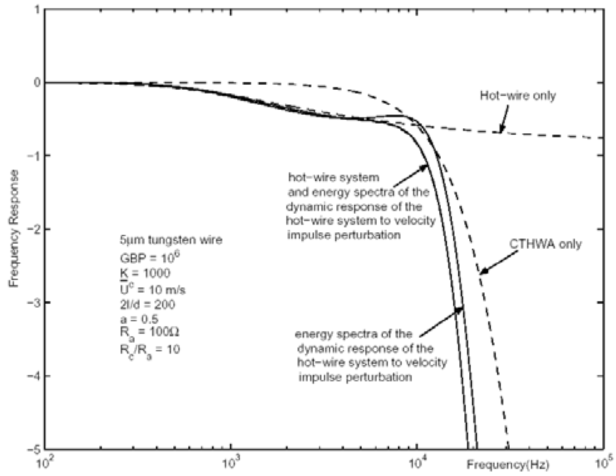

Figure 2 The normalized energy spectral densities from the dynamic response of the hot-wire system under different perturbations. Also shown are the frequency response of the hot-wire system as from Li (2004)

The hot wire could then be calibrated (using either the static or the dynamic calibration method) and used to take measurements in a turbulent flow. For single wires, if only the correction to the turbulence spectral densities is of interest, the filter function (2) can be used to multiply the energy spectral densities of turbulence in the frequency domain and the correction is easily achieved. If turbulent quantities involving velocity differences such as calculating energy dissipation, structure function or vorticity are needed from normal hot wire measurements, digital filter in time domain is required to correct the experimental data (digital filter should be applied to data from each wire in case multi wires are used). For turbulent measurements of velocity components across the mean flow direction using multi-wires (say crossed wires), digital filters in the time domain are required to correct the experimental data from each wire. The reason for applying the digital filter to experimental data from each hot wire is to take into account the differences of different wires.

The design of the digital filter in the time domain is as following. The discretized filter function (2) in the frequency domain was extended from f_M to f_{2M} first such that

$$\begin{aligned} \hat{F}(f_m) &= F(f_m) \qquad m \le M \\ \hat{F}(f_m) &= F(f_{2M+1-m}) \qquad M < m \le 2M \end{aligned} \tag{3}$$

Here f_M ($\ge 2f_c$) is the maximum bandwidth of the correction filter. This will ensure that the correction filter function in the time domain is real. Inverse FFT was then applied to the square root of the extended filter function (3) in the frequency domain. The results were a filter function in time domain arranged in wrap-around order. This can be re-arranged such that

$$\begin{aligned} c_{K+k} &= \Psi(t_k) \qquad 1 \le k \le K+1 \\ c_k &= \Psi(t_{2M-K+k}) \qquad 0 \le k \le K \end{aligned} \tag{4}$$

where $\Psi(t_k)$ is the inverse FFT of the square root of $\hat{F}(f_m)$ and $T_f = 2t_K$ is the duration of the digital filter. Figure 4 shows the filter function in the time domain produced from the digital filter function in the frequency domain as that shown in figure 3, which was based on the dynamic response of the hot-wire system to square wave perturbations at the offset. The sampling frequency was $1/\Delta t_1$ with $\Delta\tau_1 = 64\times10^{-7}$ *sec*. In practice, the sampling frequency does not need to be this high. Using Nyquist criteria, a sampling frequency of $4f_c$ will be sufficient. In figure 10, $N_f = 2K = 200$ coefficients of the digital filter are shown, this gives a duration of the digital filter in the time domain as $T_f = 1.28$ *ms*. The coefficients in (4) have also been normalized such that

$$\sum_{k=0}^{N_f} c_k = 1 \qquad (5)$$

and this would ensure that the digital filter does not change the energy of the hot wire signals and the calibration constants of the hot wire can be applied to determine the instantaneous velocity. This finishes the design of the digital filter in the time domain.

Figure 3 shows that, after applying the digital filter, the dynamic response of the hot-wire system has been delayed by $T_f/2$. This is because the digital filter as according to equation (4) is causal (Press et al, 1992) and the output at time step n depends on inputs at that particular time step or earlier. In the present results, the numerical simulation starts with impulse being applied at $t = 0$, and a causal digital filter is appropriate. In practice, the number of data points sampled can be more than required and an acausal digital filter can be applied such that the output can depend on earlier and later inputs. In this case, K points from the head and K points from the tail of the output data can be discharged. As according to Press et al, (1992), acausal filters generally give superior performance than causal filters.

Figure 4 shows the frequency response of the hot-wire system to a velocity impulse after applying the digital filter, and this is compared with that without applying the digital filter. The results clearly show that the digital filter has greatly improved the frequency response of the hot-wire system by completely eliminating the attenuation effect. Between *1 kHz* and *3.5 kHz*, the corrected response is slightly above that of the zero dB line. This is because the response of the hot-wire system to a square wave perturbation at the offset (figure 3) is slightly below that from the velocity impulse perturbation in the corresponding frequency regions, and slight over-correction has been applied. Nevertheless, the improvement of the digital filter on the frequency response of the hot-wire system is significant. By design, both responses have the same cut-off frequency $f_c = 15.6$ *kHz* and figure 4 shows that both responses completely collapse together at $f > 11kHz$.

The corrected response shown in figure 4 was obtained by applying a digital filter of $T_f = 1.28$ *ms* duration ($N_f = 200$ at $\Delta t_1 = 64\times10^{-7}$ *sec*). It was found that this long duration is necessary because figure 4 shows that the attenuation of the hot wire to a velocity impulse starts at *419 Hz*. Figure 5 shows the effect of using digital filter duration of T_f, $0.5T_f$ and $0.25T_f$. It can be seen from the figure that the results obtained using digital filter duration less than T_f does not result in sufficient correction. As mentioned above, the attenuation of the hot wire to velocity impulse starts at $f_l = 419$ *Hz*, this gives $T_l = 1/f_l = 2.38$ *ms*. This shows that $T_f \approx 0.5T_l$ is needed to fully correct the attenuation effect of the hot wire in designing the digital filter in the time domain.

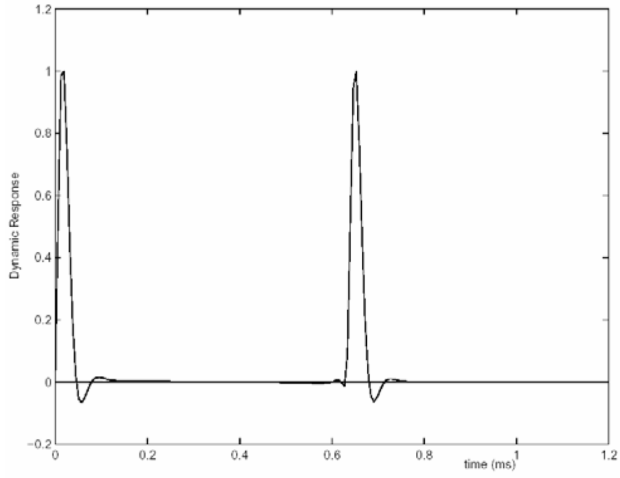

Figure 3 Dynamic response of the hot-wire system to a velocity impulse at the hot wire and that after applying the digital filter

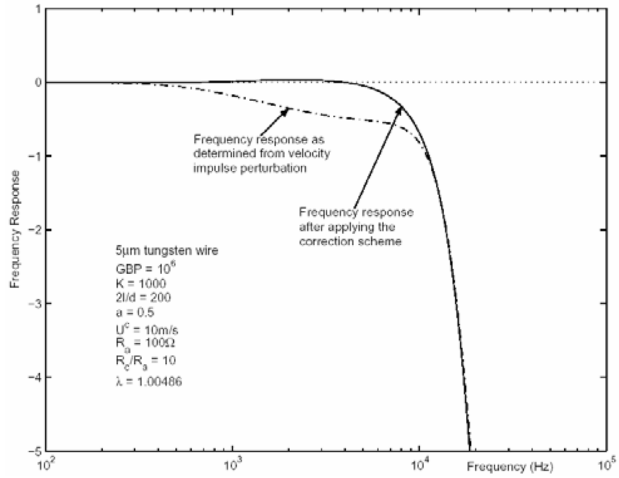

Figure 4 Frequency response of the hot-wire system under the velocity impulse, with and without applying the digital filter with $\Delta U = 0.25U^c$.

To further test the correction scheme, the designed filter was applied to the simulated hot-wire outputs to sinusoidal velocity perturbations of various frequencies at the hot wire with $\Delta U = 0.25U^c$. A total number of time steps $N = 10^5$ were simulated for each frequency and all with $\Delta t = 10^{-7}$. To apply the digital filter, the data were re-sampled at $\Delta t_1 = 64\times10^{-7}$. After applying the digital filter to each sinusoidal wave output, the central moments of even orders up to 14th were calculated. Figure 6 shows the normalized results. The normalization was achieved by dividing each central moment by its theoretical value. Comparing this with that of figure 1, it can be seen that the central moments of higher orders have been greatly improved. The central moments stay almost constant till $f = 5$ *kHz*. Beyond this frequency, the results in figure 6 show that the corrected frequency response of the hot-wire system starts to roll off from the zero dB line. Figure 6 also shows that between 1 *kHz* $< f < 4$ *kHz*, the high order central moments have been slightly over corrected. As stated before, this is because of the slightly over corrected frequency response as that shown in figure 6. However, figure 6 shows that the over correction is small.

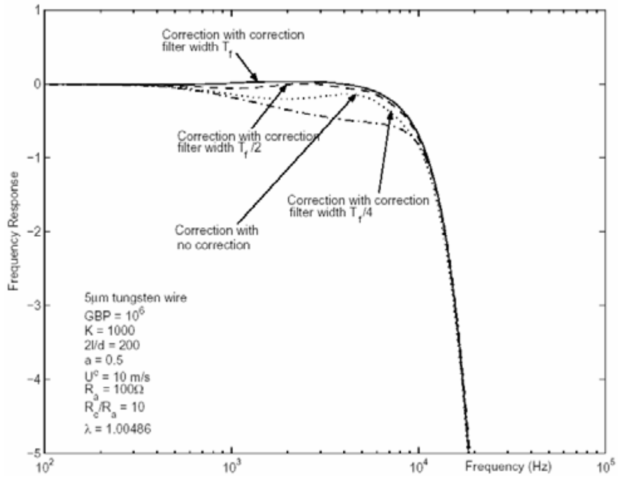

Figure 5 Effect of filter width in time domain on the corrected frequency response with $\Delta U = 0.25U^c$.

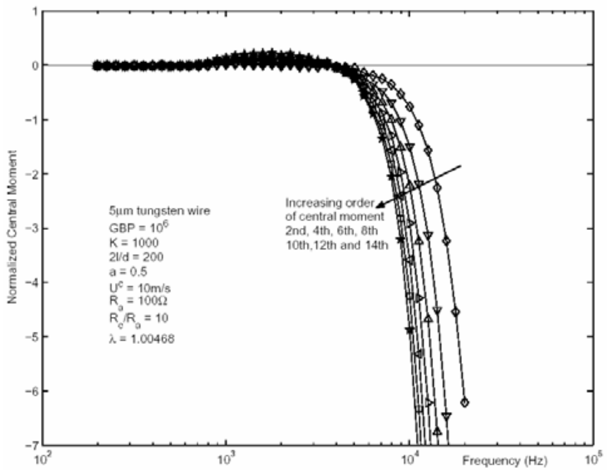

Figure 6 Corrected central moments of even orders of sinusoid wave perturbation with $\Delta U = 0.25U^c$.

During turbulent measurements, if the signal to noise ratio of the hot-wire system is high enough, the digital filter can be designed such that the corrected frequency response of the hot-wire system can have a cut-off frequency higher than that given in figures 3 and 6. The procedure will be similar to that presented above as that in Weiss et al. (2001).

Conclusions

The effects of the hot-wire attenuation on the high order central moments have been shown and it is shown that the higher the order, the servere the attenuation effects. A correction scheme based on the frequency response of the hot-wire system has been proposed and applied to the simulated data. It is found that the correction works well before the rolling-off frequency. It is also found that the width of the digital filter in the time domain needs to be large. This is because the hot-wire attenuation starts at fairly low frequency. In future work, experimental techniques will be developed to apply the proposed scheme to experiments results.

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