Numerical Analysis of Flapping Wing Aerodynamics

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Abstract

Flapping-wing aerodynamics recently has generated a great deal of interest and increasing research effort because of the potential application in micro-air vehicles. The objective of this study is to critically review the recent progress of CFD analysis of flapping-wing aerodynamics. Critical parameters like flapping modes, frequency and amplitude for optimal thrust generation and propulsive efficiency are identified. Current gaps in this research area with suggestions for further research are discussed. A preliminary CFD study to analyse the effects of the reduced frequency \( k \), amplitude of oscillation \( h \) and the maximum non-dimensional flapping velocity \( kh \) on the thrust generation and efficiency of a NACA0012 airfoil undergoing pure plunge motion at a Reynolds number of 20,000 is performed and the insight gained is discussed. The results of the present study agree well with available experimental and computational data found in the literature, however at \( k = 2 \), \( h = 24 \) very high values of average thrust coefficient \( C_{T_{mean}} \) = (102) with very low propulsive efficiency \( \eta_P = 0.0006 \) are predicted.

Nomenclature

- \( A \) : maximum excursion of trailing edge, m
- \( C_{T_{mean}} \) : time averaged thrust coefficient
- \( C_L \) : coefficient of lift
- \( C_D \) : coefficient of drag
- \( C_{P_{mean}} \) : time averaged power coefficient
- \( c \) : chord, m
- \( f \) : frequency of oscillation, Hz
- \( h \) : non-dimensional plunge amplitude
- \( k \) : reduced frequency, \( \frac{2\pi f c}{U_o} \)
- \( St \) : Strouhal number, \( fA/U_o \) (\( kh/\pi \) for pure plunging)
- \( U_o \) : free stream velocity, m/s
- \( \alpha_{max} \) : maximum of angle of attack
- \( \eta_P \) : propulsive efficiency
- \( \phi \) : phase angle between pitching and plunging motion
- \( \theta_0 \) : non-dimensional pitch amplitude

Introduction

Inspiration from nature is a key element for research and scientific development. The field of flapping wing aerodynamics has been inspired by flying animals such as birds, bats, insects and efficient swimmers like fishes, which have extraordinary flying and swimming capabilities like forward flight, manoeuvre and hover. More recently the interest of researchers in this field has increased due to the possible application of flapping wing powered micro aerial vehicles (MAVs) and submerged vehicles. In the early twentieth century, Knoller [19] and Betz [3] were among the first to present the idea that during flapping motion, an oscillating wing makes an effective angle of attack which results in a normal force vector with both lift and thrust components. Katzynmayr [18] was the first to perform experiments to verify the Knoller-Betz effect in 1922. He placed a stationary airfoil into a sinusoidally oscillating wind stream and measured an average thrust force. The theory of Knoller-Betz did not account for the vorticity shed into the wake of the airfoil. During the flapping motion of the airfoil, the angle of attack of the airfoil changes continuously, resulting in lift production and also causing a change in circulation bound to the airfoil. According to Kelvin’s theorem, the total amount of circulation (vorticity) in the flow field must remain constant \( (D/\bar{D}t = 0) \). This means if the circulation bound to the airfoil varies, then there must be an equivalent circulation of opposite sign, shed form the airfoil into the wake [17]. In 1935, von Karman and Burgess [39] theoretically explained the generation of thrust and drag on the basis of the observed location and orientation of shed vortices. They modelled the wake of the flow past bluff bodies at low Reynolds numbers by an infinite row of alternating vortices, commonly known as von Karman Vortex Street. In this configuration of wake vortices, if the fluid is flowing from the left, then the upper row of vortices in the wake rotates clockwise and the lower row of vortices rotates counter clockwise, as shown in Figure 1. This causes a momentum deficit in the wake compared to the upstream flow and the body experiences drag; this configuration of wake is referred to as a ‘drag producing wake’. In contrast, the flow past a flapping flat plate or airfoil produces a wake in which the upper row of vortices rotates counter clockwise and the lower row of vortices rotates clockwise, as shown in Figure 2. These vortices induce a velocity or momentum surplus in the wake compared to the upstream flow and the airfoil experiences thrust; this configuration of wake is referred to as a ‘thrust producing wake’.

![Figure 1. Drag producing wake for a stationary NACA0012 airfoil (From Lai and Platzer [21])](image1)

![Figure 2. Thrust producing wake for a NACA0012 airfoil undergoing pure plunging motion, \( kh = 0.393 \) (From Lai and Platzer [21])](image2)

During the same period that von Kármán and Burgess theoretically explained the drag or thrust generation in flow past...
oscillating bodies, Theodorsen [35] successfully computed the aerodynamic forces and moments on an oscillating airfoil. Theodorsen’s formulation was based on the incompressible potential flow assumption and the Kutta condition at the trailing edge. Garrick[8] then used this approach for the derivation of thrust force and propulsive efficiency of harmonically oscillating foils as a function of the reduced frequency (k) and the maximum non-dimensional flapping velocity (kh). He showed that plunging airfoils generate thrust for all frequencies and that the thrust is proportional to the square of kh (with an additional dependence on k, for k < 4), while pitching airfoils generate thrust above a certain critical frequency (dependent on the pivot point location, e.g. for an airfoil pitching about a point ¼ c from the leading edge, this critical frequency will be k = 3.25).

Since the explanation of thrust generation from oscillating airfoils in the early twentieth century, several analysis methods such as unsteady potential flow [15, 37] Navier Stokes computations [13, 25, 27, 30, 31, 36, 37, 40, 42-44] and experimental studies [2, 7, 10, 14, 20, 29] have been employed to investigate the flow field and to analyse the effect of flapping parameters on the thrust generation and propulsive efficiency from oscillating foils. These studies show that thrust generation depends on the particular combination of frequency, amplitude and phase difference between pitching and plunging motion. In the following sections the values of these parameters found in the literature for optimal thrust generation and propulsive efficiency will be discussed.

The purpose of this paper is to critically review the progress of CFD analysis of flapping wing aerodynamics. This study primarily covers 2D forward flight with a brief discussion of other important considerations in flapping wing flight such as three dimensional effects and hovering flight. A preliminary CFD study of the effects of the reduced frequency (k), amplitude of oscillation (h) and the maximum non-dimensional flapping velocity (kh) on the thrust generation and propulsive efficiency of a NACA0012 airfoil undergoing plunging motion is also presented. This study covers an extended range of flapping parameters (k = 0.5 – 24, h = 0.0125 – 48, kh = 0.05 – 48) which have not yet been reported in the literature.

Flapping wing aerodynamics
Several experimental and numerical investigations have been performed on flow over flapping wings to understand the unsteady mechanisms of aerodynamic forces generation and also to determine the effects of varying different flapping parameters such as flow Reynolds Number (Re), reduced frequency, plunge amplitude, mode of motion, and phase difference between pitching and plunging motion. We have gathered here some of the recent computational and experimental studies of 2D pure plunging, pitching and 2D combined pitching and plunging motion. Firstly, the studies of pure plunging and pitching motion of 2D airfoil sections are discussed, followed by the discussion of combined pitching and plunging studies. The interest here is to determine the values of flapping frequency and flapping amplitude best suited to generate maximum thrust with reasonable efficiency or maximum efficiency with a reasonable thrust.

Pure Plunge – 2D rigid foils
The case of 2D plunging motion has been considered by many researchers to provide insight into thrust generation and propulsive efficiency (inversion of the vortex street, leading edge separation) [22]. Following are some of the important studies in the literature on pure plunging for efficient propulsion.

Tuncer and Platzer [37] in 1996 computed the thrust force and propulsive efficiency using a Navier-Stokes (NS) code for the flow past a rigid NACA0012 airfoil undergoing pure plunging motion. The Reynolds number was 3 x 10^7. The value of k was varied from 0.2 < k < 3 and h was varied from 0.1 < h < 0.4 to find the optimal thrust and propulsive efficiency. They found that for a single plunging airfoil, maximum efficiency as high as 0.72 can be achieved for k = 0.2 and h = 0.4 but with a very low coefficient of thrust of 0.01. They also investigated the flapping/stationary airfoil combination in tandem configuration and found that if a stationary airfoil is placed downstream of the plunging airfoil separated by two chord lengths, more than 40% gain can be achieved in efficiency and 33% in thrust coefficient at k = 0.75 and h = 0.2.

Jones and Platzer[15] in 1997 reported their computational results using a 2D incompressible unsteady panel method (UPM) code for flow over different airfoil sections undergoing pure plunging motion and found that varying the thickness of the airfoil has a negligible effect on thrust generation and propulsive efficiency in the frequency and amplitude range considered, k = 0.01 – 10 and h = 0.1 – 0.4.

In 1998, Tuncer et al. [38] using a 2D compressible Navier-Stokes solver reported the thrust force coefficient and propulsive efficiency for a NACA0012 airfoil undergoing pure plunging motion at Re = 10^6. They stated that the maximum achievable thrust is a function of kh but it is limited by a critical value of kh = 0.35, above which dynamic stall occurs. They argued that one can either choose to select large amplitude and low frequency or vice versa under the critical value of kh. Furthermore, Tuncer et al. [38] argued that for optimal propulsive efficiency, it is advantageous to operate in the low frequency and large amplitude range. However, Platzer et al. [26] performed Navier-Stokes computations at Reynolds number 20,000 and reported that for a given kh, it is more advantageous to operate at a high k and a low h in order to minimize the adverse effect of the leading edge vortex. These two apparently contradictory statements have prompted us to examine flapping wing aerodynamics in more detail for a higher range of k and h and to answer the following questions.

- What are the values of plunging amplitude h, reduced frequency k and/or kh limiting the maximum thrust coefficient and optimal propulsive efficiency? What are the criteria that determine these values?
- For a given kh, what is the best combination of k and h for maximum thrust generation and optimal propulsive efficiency?
- How do physical phenomena like dynamic stall and leading edge separation affect the optimal thrust coefficient and propulsive efficiency?

In 2004, Young and Lai [44] numerically simulated the flow over a NACA0012 airfoil undergoing plunging motion at Re = 20,000. They studied the effect of leading edge flow separation on the force generation from a plunging airfoil at different k and h with kh = 0.6. They concluded that for thrust generation, constant kh cannot be a single controlling parameter, rather k and kh must be treated separately for k < 8.

Lewin and Hariri [22] examined numerically the flow over a plunging airfoil with k = 2 to 10, maximum heave velocity kh = 0.8 to 1.5 at Re = 2000 using Navier-Stokes solver. For the range of parameters covered in their study, they reported aperiodic and asymmetric solutions but with negligible effect on the propulsive efficiency. They examined in detail the effect of k on the thrust generation and propulsive efficiency and found that the leading edge vortex has significant effects on force generation and efficiency of airfoil. These included negative interference
between the leading and trailing edge vortices corresponding to low efficiency ($\eta_P < 0.7$) at $k = 3.333$ for all $kh$ values studied and also the separation of the leading edge vortex corresponding to a sudden drop in output power. They reported a rather low overall efficiency of 0.11 at $k = 5.333$ and $kh = 1.2$. The low efficiency ($\eta_P < 0.1$) at high frequencies ($k > 5.333$) for all $kh$ was due to interaction between the airfoil and the shed vortices inducing drag on the airfoil. At higher frequencies this effect becomes more prominent because of greater proximity of the vortices.

Pederzani et al. [25] in 2006 studied numerically the flow over rigid and flexible airfoil sections utilizing a 2D Navier Stokes incompressible viscous solver at $Re = 500$. They simulated the airfoil motion at different $k$ for two different values of $kh = 0.8$ and 1.0. They found that flexible airfoils are more efficient than rigid ones. Also, they studied the effect of density of the airfoil and found that a heavier airfoil generated thrust at rather lower input power than the lighter ones and therefore the heavier airfoils were more efficient.

In 2006, Sarkar and Venkatraman [30] numerically simulated the flow over a plunging airfoil using a 2D discrete vortex method. They investigated the effect of non-sinusoidal harmonic motion at three different values of $kh = 0.5, 1, 1.5$ and $k = 3$ to 8 at $Re = 10,000$. They found for constant $k$, asymmetric cases (downstroke and upstroke of different duration) give better thrust force than the pure sinusoidal case but the propulsive efficiency showed different trends at different $kh$ values (it remained approximately the same as for sinusoidal plunge motion at $kh = 1$ and around 50% increase in efficiency was observed at $kh = 0.5$). The other non-sinusoidal motions studied such as constant plunge rate and sinusoid with a gap were not advantageous in terms of thrust and propulsive efficiency compared to equivalent sinusoidal motions.

<table>
<thead>
<tr>
<th>Author</th>
<th>Type of Study</th>
<th>Airfoil</th>
<th>$Re$</th>
<th>$k$</th>
<th>$h$</th>
<th>$kh$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tuncer and Platzer [37]</td>
<td>2D Compressible NS (M=0.3) and UPOT</td>
<td>NACA0012 3 x $10^6$</td>
<td>0.2 - 3</td>
<td>0.1 - 0.4</td>
<td>0.02 - 1.2</td>
<td>Max. $\eta_P = 0.72$, $C_{Tmean} = 0.01$</td>
<td></td>
</tr>
<tr>
<td>Jones and Platzer [15]</td>
<td>2D Incompressible Panel Code and BL algorithm</td>
<td>NACA0012, 0015, 0009, 0003</td>
<td>0.01 - 10</td>
<td>0.1 - 0.4</td>
<td>0.001 - 4</td>
<td>Max. $\eta_P = 0.519$, $C_{Tmean} = 2.13$ at $k = 4$, $h = 0.4$; Max. $C_{Tmean} = 2.5$, $\eta_P = 0.293$ at $k = 4$, $h = 0.4$</td>
<td></td>
</tr>
<tr>
<td>Tuncer and Platzer [36]</td>
<td>2D Compressible NS (M=0.3)</td>
<td>NACA0012 1 x $10^5$</td>
<td>0.8</td>
<td>0.4 - 0.7</td>
<td>0.32 - 0.56</td>
<td>Max. $\eta_P = 0.59$, $C_{Tmean} = 0.118$ at $k = 0.8$, $h = 0.4$.</td>
<td></td>
</tr>
<tr>
<td>Young and Lai [44]</td>
<td>2D Compressible NS (M=0.3) and UPOT</td>
<td>NACA0012 20000</td>
<td></td>
<td>0.6</td>
<td>0.0125 - 0.1</td>
<td>Max. $C_{Tmean} = 0.176$ $\eta_P = 0.55$ at $k = 0.8, h = 0.5$.</td>
<td></td>
</tr>
<tr>
<td>Lewin and Hariri. [22]</td>
<td>2D Incompressible Viscous NS</td>
<td>Elliptical 500</td>
<td>2-10</td>
<td>0.8-1.5</td>
<td></td>
<td>Max. $\eta_P = 0.11$ at $k = 5.333$, $kh = 1.2$</td>
<td></td>
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<tr>
<td>Pederzani and Hariri [25]</td>
<td>2D Incompressible Viscous NS</td>
<td>NACA0012 (rigid and flexible) 500</td>
<td>0.8, 1.0</td>
<td></td>
<td></td>
<td>Max. $\eta_P = 0.12$ at $k = 6, kh = 1$</td>
<td></td>
</tr>
<tr>
<td>Sarkar and Venkatraman [30]</td>
<td>2D Discret vortex method</td>
<td>NACA0012 10000</td>
<td>4, 6, 8</td>
<td>0.5, 1.0, 1.5</td>
<td>Max. $C_T = 1.2$ with $\eta_P = 0.25$ at $k = 4$, $kh = 1.5$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heathcote and Gursul [11]</td>
<td>Experimental, 2D and 3D</td>
<td>NACA0012 (rigid and flexible)</td>
<td>10000, 20000, 30000</td>
<td>0 - 14</td>
<td>0.175</td>
<td>Max. $\eta_P = 0.28$, $C_{Tmean} = 0.01$ at $k = 2$; Max. $C_{Tmean} = 2.5$, $\eta_P = 0.05$ at $k = 12.6$.</td>
<td></td>
</tr>
<tr>
<td>Platzer et al. [26]</td>
<td>2D Navier Stokes</td>
<td>NACA0012 20000</td>
<td>0.5, 2, 8</td>
<td>0.1, 0.175, 0.3</td>
<td>0.05 - 6</td>
<td>Max. $\eta_P = 0.28$, at $k = 2$, $h = 0.175$; Max. $C_{Tmean} = 1.8$ at $k = 5$, $h = 0.3$.</td>
<td></td>
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</table>

Table 1. Summary of recent computational and experimental studies of plunging airfoil

In 2007, Heathcote and Gursul [11] experimentally studied the flow over 2D airfoils undergoing plunging motion having different flexibilities. The Reynolds number based on chord length and incoming flow speed was 10000, 20000 and 30000. They studied the effect of varying $St = (kh)/n$ at constant $h = 0.175$. They reported that the thrust is maximum for the airfoil with intermediate flexibility which is consistent with their findings in [10] and also reported by Pederzani and Hariri [25]. The effect of varying $Re$ from 10000 to 30000 on the thrust force at $h = 0.175$ and $kh = 0 - 2.4$ ($St = 0 - 0.76$) was reported to be negligible. Heathcote et al. [10] in a similar study reported that the thrust coefficient increased with $kh$ and reached a maximum value at around $kh = 1.28 – 1.50$ and then decayed.
We have plotted here some of the reported results in Fig. 3 and Table 1 provides a summary of the studies described above. It is found that previous studies are focussed on a small range of parameters primarily between 0.1 < h < 1.4 and 0.2 < k < 8, except the most recent study by Platzer et al. [26]. It can be seen in Fig. 3 that both Navier-Stokes and panel method code predictions for $C_{\text{mean}}$ agree well with the measurements but there is a big discrepancy in the prediction of $\eta_P$ at low $kh$ values ($kh < 1$), because of the inability of the unsteady panel method code to predict leading edge separation in flow which occurs at high plunge amplitudes. Furthermore, it is observed that $C_{\text{mean}}$ increases with $kh$ without apparent limit. We suspect that there must be a limit to this increase in $C_{\text{mean}}$ with $kh$ and therefore we think it is worthwhile to extend the range of parameters to larger amplitudes and higher reduced frequencies, in order to explore the effect of phenomena like dynamic stall and leading edge flow separation on thrust force generation and propulsive efficiency.

### Pure Pitch – 2D rigid foils

Garrick’s analysis [8] showed that pure pitching motion is not very attractive for thrust generation as it produces positive thrust only at relatively high frequencies (approximately $k > 4$) and Koochesfahan [20] experimentally verified this result. More recently, Sarkar and Venkatraman [31] investigated numerically the flow over an airfoil undergoing pure pitching motion using a 2D discrete vortex method. They showed that flapping parameters such as mean angle of attack and pitch axis location influence thrust force generation. However, the values of thrust force produced for pure pitching were as low as $C_{\text{mean}} < 0.1$ for $k > 12$ and $\theta_p = 5^\circ$ and were not useful for practical propulsive vehicles. Since, the combined pitching and plunging mode is the most common flapping mode in all flying animals, it will be discussed in detail in the next section.

### Combined Pitching and Plunging – 2D rigid foils

The combined pitching and plunging motion is closer to the real mode of flapping present in flying and swimming animals in nature, so it is necessary to analyse this mode of motion separately. Garrick [8] calculated the forces not only for pure plunge and pure pitching cases, but also for combined pitching and plunging motion. We have summarized here the recent numerical and experimental studies of flow over 2D combined plunging and pitching motion.

In 1998, Tuncer et al. [38] investigated the flow over a 2D NACA0012 airfoil undergoing combined pitching and plunging motion at $Re = 10^4$ using 2D compressible Navier-Stokes solver. They simulated the flow at $k = 0.3 \cdot 1, h = 1$, pitch amplitude $\theta_p = 10^\circ$ and the phase difference between the pitching and plunging motion $\varphi$ was varied from $30^\circ$ to $150^\circ$. They observed a peak in the propulsive efficiency at $\varphi = 90^\circ$ (pitching motion leads plunging motion by a phase difference of $90^\circ$) for all values of $k$. In a similar study in 2000, Tuncer and Platzer [36] tested the effect of dynamic stall on the propulsive efficiency and thrust force and reported that as soon as dynamic stall occurs, the propulsive efficiency starts to drop rapidly. They found a maximum value of thrust coefficient $C_{\text{mean}} = 1.3$ for $\eta_p = 0.15$ at $\varphi = 30^\circ$; Max $C_{\text{mean}} = 0.86$ at $k = 0.15, \varphi = 90^\circ$.

<table>
<thead>
<tr>
<th>Author</th>
<th>Type of Study</th>
<th>Ref. point</th>
<th>$Re$</th>
<th>$k$</th>
<th>$h$</th>
<th>$kh$ ($\alpha$)</th>
<th>$\theta_p$</th>
<th>$\phi$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tuncer et al. [38]</td>
<td>2D Compressible Navier-Stokes</td>
<td>1/2 c</td>
<td>10</td>
<td>0.3</td>
<td>1</td>
<td>0.3 - 1</td>
<td>10</td>
<td>30° - 150°</td>
<td>Max. $C_{\text{mean}} = 1.3$, $k = 0.15$, $\varphi = 30°$; Max $\eta_p = 0.86$ at $k = 0.15, \varphi = 90^\circ$</td>
</tr>
<tr>
<td>Tuncer and Platzer [36]</td>
<td>2D Compressible Navier-Stokes (M = 0.3)</td>
<td>1/2 c</td>
<td>10</td>
<td>0.3</td>
<td>1</td>
<td>0.3 - 1</td>
<td>10</td>
<td>30° - 90°</td>
<td>Max. $\eta_p = 0.86$, $C_{\text{mean}} = 0.072$ at $\varphi = 90^\circ$, $k = 0.3$; Max. $C_{\text{mean}} = 0.44$, $\eta_p = 0.25$ at $\varphi = 90^\circ$, $k = 1.0$.</td>
</tr>
<tr>
<td>Isogai et al. [13]</td>
<td>2D Compressible Navier-Stokes (M = 0.3)</td>
<td>1/2 c</td>
<td>10</td>
<td>0.3</td>
<td>1</td>
<td>0.3 - 2</td>
<td>10</td>
<td>7° - 20°</td>
<td>Max $\eta_p = 0.56$, $C_{\text{mean}} = 0.177$ at $\theta_p = 20^\circ, k = 1$; Max $C_{\text{mean}} = 0.44$ $\eta_p = 0.26$ at $\theta_p = 10^\circ$, $k = 1.3$.</td>
</tr>
<tr>
<td>Anderson et al. [2]</td>
<td>Experimental, 2D</td>
<td>1/3 c</td>
<td>4000</td>
<td>0.25</td>
<td>- 0.5</td>
<td>$0.07 - 0.94$ (0.05 - 0.6)</td>
<td>75° - 105°</td>
<td>Max. $\eta_p = 0.87$, $C_{\text{mean}} = 0.67$ at $kh = 0.94$, $\varphi = 75^\circ$, $\alpha_{\text{max}} = 20.2^\circ$</td>
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</tr>
<tr>
<td>Schouveiler et al. [32]</td>
<td>Experimental, 2D</td>
<td>1/3 c</td>
<td>4000</td>
<td>0.75</td>
<td>0.31 - 1.41 (0.1 - 0.45)</td>
<td>90°</td>
<td>Max. $C_{\text{mean}} = 1.05$, $\eta_p = 0.42$ at $kh = 1.41$, $\alpha_{\text{max}} = 30^\circ$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ramamurti and Sandberg [27]</td>
<td>2D Incompressible Navier-Stokes</td>
<td>1/4 c</td>
<td>1100</td>
<td>3.77</td>
<td>5.65</td>
<td>$3.77, 5.65$</td>
<td>90°</td>
<td>Max. $C_{\text{mean}} = 2.42$, $\eta_p = 0.24$ at $\varphi = 5.65$, $\alpha_{\text{max}} = 120^\circ$, Max. $\eta_p = 0.3$ at $\varphi = 90^\circ$ at $k = 5.65$.</td>
<td></td>
</tr>
<tr>
<td>Read et al. [29]</td>
<td>Experimental, 2D</td>
<td>1/3 c</td>
<td>4000</td>
<td>0.25</td>
<td>1.88</td>
<td>$0.094 - 0.69$ (0.06 - 0.44)</td>
<td>90°</td>
<td>Max. $\eta_p = 0.715$, $C_{\text{mean}} = 0.18$ at $\varphi = 90^\circ$, $kh = 5.02$, $C_{\text{mean}} = 2.41$, $\eta_p = 0.43$ at $\varphi = 100^\circ$, $kh = 1.88$.</td>
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</table>

Table 2. Summary of recent computational and experimental studies of combined pitching and plunging NACA0012 airfoil.
\( C_{\text{mean}} \) was maximized at \( \phi = 120^\circ \) and for low pitch, \( \theta = 10^\circ \) and high plunge, \( h = 1 \), \( C_{\text{mean}} \) was maximized at \( \phi = 60^\circ \).

Anderson et al. [2] performed experiments to investigate the effects of Strouhal number on the thrust generation and propulsive efficiency of a rigid NACA0012 airfoil undergoing combined plunging and pitching motion at \( Re = 1100 \) and 40000. They reported \( C_{\text{mean}} \approx 1 \) with \( \eta_P \) as high as 0.87 at \( kh = 0.56 \) (\( St = 0.36 \)), \( h = 0.75 \), maximum angle of attack \( \alpha_{\text{max}} = 20.2^\circ \) and \( \varphi = 75^\circ \). At low angle of attack (\( \alpha = 5^\circ \)), \( \eta_P \) was less than 0.1. The phase angle between pitching and plunging \( \phi \) was identified as a critical parameter affecting the propulsive efficiency.

Guglielmini and Blondeaux [9] investigated the effect of different flapping parameters on the propulsive efficiency of a 2D elliptic foil undergoing combined pitching and heaving motion by means of numerical solution of the vorticity equation at \( Re = 1100 \). They found highest efficiency \( \eta_P = 0.45 \) for high pitching amplitudes between \( 30^\circ \) to \( 40^\circ \) and for \( kh = 0.94 \) -1.25 (\( St = 0.3 \) - 0.4). They also investigated the effect of varying \( \phi \) on the propulsive efficiency and found that maximum efficiency is obtained at \( \phi = 80^\circ \). They showed an increase in the peak of efficiency around 20% from \( Re = 1100 \) to \( Re = 3300 \). The pivot position of the pitching axis was also varied from -0.5c to 2c to find its effect on propulsive efficiency. The location for maximum efficiency was reported to be 1/3c from the leading edge.

Ramamurti and Sandberg [27] performed numerical simulation of the flow over a flapping NACA0012 airfoil using a finite element incompressible Navier-Stokes solver. The pivot point location for the airfoil was 1/4c and the flow \( Re = 1100 \). They found that the critical parameter which affects the thrust generation is \( kh \) rather than \( k \). They also found that maximum thrust is obtained when the pitching motion leads the plunging motion by 120° and the maximum propulsive efficiency occurs at \( \varphi = 90^\circ \).

Read et al. [29] and Schouveiler et al. [32] experimentally investigated the flow over a NACA0012 airfoil undergoing combined pitching and plunging motion at \( Re = 40,000 \) with the pitch axis location at 1/3c, \( h = 0.75 \), \( \varphi = 90^\circ \). The effect of varying pitch amplitude \( \theta = 10^\circ \) to \( 40^\circ \) and \( kh = 0.188 \) -1.38 was studied. Read et al. reported maximum propulsive efficiency \( \eta_P = 0.715 \) at \( \alpha_{\text{max}} = 15^\circ \) and \( kh = 0.502 \) with \( C_{\text{mean}} \approx 0.18 \). More significant results (simultaneously combining high thrust and efficiency) reported were \( \eta_P = 0.556 \), \( C_{\text{mean}} = 0.79 \) at \( \alpha_{\text{max}} = 20^\circ \), \( kh = 1.25 \) and \( \eta_P = 0.508 \), \( C_{\text{mean}} = 1.08 \) at \( \alpha_{\text{max}} = 25^\circ \), \( kh = 1.38 \). They found very high thrust coefficient \( C_{\text{mean}} = 2.43 \) with \( \eta_P = 0.49 \) at \( \varphi = 100^\circ \), \( kh = 1.88 \), \( \alpha_{\text{max}} = 35^\circ \) when higher-order heave motions were employed. They also reported that adding pitch bias angles to the pitching motion i.e., a mean pitching angle other than zero can produce a very strong side force which can be useful for manoeuvring.

In Figure 4, the variation of \( C_{\text{mean}} \) and \( \eta_P \) with \( \varphi \) is plotted from the data available in some of the previous CFD analyses of flows over a combined pitching and plunging airfoil. In addition to the \( k \) and \( h \), \( \varphi \) is also identified as a critical parameter on which the efficiency and thrust are dependent. Most of the studies discussed here concluded that \( \varphi \) should be close to 90° for peak efficiency. Table 2 provides a summary of the studies discussed here.

### Preliminary Calculations

#### Kinematics

For the preliminary computations, a NACA0012 airfoil section undergoing pure plunging motion is considered. The plunging motion is defined by following equation:

\[
y(t) / c = h \sin(2\pi ft)
\]

with \( h = 0.0125 \) to 48 and \( k = 0.5 \) to 24, at \( Re = 20,000 \). The amplitude and frequency range covered in this paper has not been explored previously and will provide an insight into the effects of leading edge flow separation and dynamic stall on thrust force generation and propulsive efficiency associated with high amplitude motion of airfoil.

![Figure 4. Variation of \( C_{\text{mean}} \) and \( \eta_P \) with \( \varphi \) of 2D NACA0012 airfoil undergoing combined plunging and pitching motion.](image-url)

**Forces**

The output for different combinations of \( h \) and \( k \) include the mean (i.e., time averaged) thrust coefficient \( C_{T,\text{mean}} \), mean input power coefficient \( C_{P,\text{mean}} \) and propulsive efficiency, \( \eta_P \):

\[
C_{T,\text{mean}} = \frac{1}{T} \int_0^T C_D(t) \, dt
\]

\[
C_{P,\text{mean}} = \frac{1}{U_\infty} \int_0^T \left[ C_L(t) \dot{y}(t) \right] \, dt
\]

\[
\eta_P = \frac{C_{T,\text{mean}}}{C_{P,\text{mean}}}
\]

**Solver**

The unsteady flow field around a NACA0012 airfoil undergoing pure plunging motion was simulated using the commercially available CFD package Fluent version 6.2, with an unsteady incompressible solver and second-order upwind spatial discretization. The plunging motion of the airfoil was modelled by using the ‘dynamic mesh’ feature and the whole grid and airfoil was moved as a rigid body. The use of the dynamic mesh feature limited the unsteady formulation to first order in time. The flow field is assumed to be laminar and it is shown in the validation of results that the laminar assumption provides good agreement with experiments for this Reynolds number range.

**Grid**

The structured grid was developed in C-topology as shown in Figure 5. The upstream inlet velocity boundary was at 9c, downstream pressure outlet was at 11c, and across the flow the boundaries were 10c from the airfoil. The number of grid points was 901 x 101 (streamwise x normal) with 401 points distributed on the airfoil surface. The first grid point was located at a normal distance of 0.004c with a wall Yplus value of order 1 and approximately 15 grid points normal to the flow direction in the boundary layer.

**Grid and Timestep refinement**

To assess the independence of the numerical solution with grid refinement, a grid independence study was carried out at
$Re = 20,000$, $k = 2.0$, and $h = 2.0$. The grids tested were $251 \times 101$ (101 points on the airfoil surface), $451 \times 101$ (201 points on the airfoil surface), $1801 \times 101$ (801 points on the airfoil surface), $451 \times 201$ and $901 \times 201$ points, all with timestep $\Delta t = 0.05$ sec. It was found that the $901 \times 101$ grid was sufficiently refined and that the lift and thrust coefficients were grid independent as shown in Figure 6. Furthermore, a timestep refinement study was also performed for $\Delta t = 0.1$, 0.05 and 0.01 sec and found that $\Delta t = 0.05$ sec was sufficiently refined.

For the validation of Navier Stokes computations, the results were compared with the measured values of Heathcote et al. [11]. In Figure 7, the time averaged coefficient of thrust and propulsive efficiency are plotted against $kh$. It is observed that our results are in good agreement with the experimental results. However, there is discrepancy between our results and Garrick’s linear analysis results for $kh < 0.4$. This is because at very low $kh$, the thrust produced by the airfoil is insufficient to overcome the viscous drag and therefore gives negative efficiency for the Navier-Stokes calculations whereas Garrick’s analysis assumed a non-viscous flow. At $kh > 0.5$ the effect of the leading edge vortex separation causes Navier-Stokes computations to under-predict the efficiency compared to Garrick’s results. Similar results are reported in the literature [26] as discussed in previous sections.

**Results**

The Navier-Stokes computations were performed for an extended range of $k$ and $h$. The variation of $C_{T\text{mean}}$ and $\eta_P$ with $kh$ for constant $k$ is plotted in Figure 8. It is found that $C_{T\text{mean}}$ increases with increasing $kh$ and follows the same trend as predicted by Garrick’s linear analysis despite the formation of strong leading edge vortices at high $kh$. However the $C_{T\text{mean}}$ values are very much less than those predicted by Garrick’s linear theory because of viscous effects and separated flows. We expected $C_{T\text{mean}}$ to stop increasing at very high $kh$ values but the results of the present Navier-Stokes computations do not show any reduction in $C_{T\text{mean}}$ with increase in $kh$. It was suspected that due to the laminar assumption, the prediction of thrust might be in error at very high $kh$. As $kh$ represents the maximum non-dimensional plunge velocity of the airfoil, at high values of $kh$, the actual Reynolds number observed by the airfoil must be much higher than the flow Reynolds number based on the free stream flow. Therefore for a particular case of $k = 8$, $h = 0.75$, the one-equation turbulence model (Spalart-Allmaras) was employed but the $C_{T\text{mean}}$ value is still as high as predicted by the laminar flow assumption. We consider this phenomenon should be explored in more detail with other turbulence models, refined meshes and time steps. The propulsive efficiency in Figure 8 shows that the peak in efficiency occurs only at low $kh$ values and it asymptotes to zero for higher values of $kh$. Although the plunging of the airfoil at high $kh$ generates high thrust, the power requirement to plunge the airfoil increases much more rapidly than the increase in thrust, giving a low efficiency.

In Figure 9, the variation of $C_{T\text{mean}}$ and $\eta_P$ with $kh$ is plotted. It is found that for the flow Reynolds number considered, it is beneficial to operate in the high $k$ and low $h$ combination rather than the high $h$ and low $k$ combination because a high thrust coefficient can be achieved at high frequency and leading edge separation can be avoided at low amplitudes which can benefit in better propulsive efficiency. However, if very high $h$ is used as shown in Figure 10, $k = 0.5$, $h = 12$, strong leading edge separation will be encountered, resulting in very low propulsive efficiency.

**Other key Parameters in Flapping wing aerodynamics**

We have explored the physical phenomena involved in thrust generation by flapping airfoils but still much effort is required for the efficient design of MAVs and submerged vehicles using flapping wings. The other important parameters which will influence flapping wing aerodynamics are discussed below:

- The effect of 3D flow over finite span flapping wings. Several attempts have been made to study these effects [1, 4-6, 12, 16, 23, 24, 28, 33, 34, 41]. Most of the studies have reported the successful modelling of realistic natural flapping motions for a single flight condition, and discussed
the role of interconnected vortex loops in generating a large amount of lift produced, especially for hovering in insects. Spentzos et al. [33] reported that aspect ratio has very little effect on the evolution of the dynamic stall vortex on a plunging rectangular wing but the interaction of leading and trailing edge vortices with wing tip vortices varies for different aspect ratio wings. Much detailed study for an extended range of flapping parameters for forward flight still needs to be explored, incuding:

- The effect of pitch axis location on the thrust generation and propulsive efficiency of flapping wings.
- The optimization of different flapping wing parameters to maximize thrust generation or propulsive efficiency using genetic algorithms.
- The effect of chordwise and spanwise flexibility on the thrust generation and propulsive efficiency of flapping wings.

Figure 10. Vorticity contours at $k = 0.5$ and $h = 12$

Conclusions
A review of recent computational and experimental studies in the context of 2D flapping wing forward flight is presented. For a 2D rigid airfoil undergoing pure plunge motion, maximum propulsive efficiency $\eta_P = 0.72$ at $k = 0.2$, $h = 0.4$ with $C_T\text{mean} = 0.01$ [37] and maximum thrust coefficient $C_T\text{max} = 2.5$ at $k = 4$, $h = 0.4$ with $\eta_P = 0.293$ [15] are reported in the literature. For a 2D rigid airfoil undergoing combined pitching and plunging motion maximum propulsive efficiency $\eta_P = 0.87$ at $kh = 0.94$, $\phi = 75^\circ$, $a_{max} = 20.2^\circ$ with $C_T\text{mean} = 0.67$ [2] and maximum thrust coefficient $C_T\text{mean} = 2.42$ at $k = 5.65$, $h = 1$, $\phi = 120^\circ$ with $\eta_P = 0.24$ [27] are reported. The results of a preliminary CFD study to determine the effects of extended range of parameters, $k = 0$ to 24, $h = 0.05$ to 48 and $kh = 0.05$ to 48 on the thrust generation and efficiency of a NACA0012 airfoil undergoing pure plunging motion at $Re = 20,000$ are also presented. The results for thrust coefficient and propulsive efficiency agree well with previous computations for $kh$ values reported in literature but for very high $h$ and $kh$ values, a thrust coefficient as high as $C_T\text{mean} = 102$ at $k = 2$, $h = 24$ is found with a corresponding efficiency of 0.006. In comparison to previous studies, increasing $h$ provides higher thrust coefficient but such a high value of $C_T\text{mean}$ demands to be explored in further detail with a more refined solution.

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