Effects of free-stream nonuniformity on boundary layer transition

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Abstract

Experiments are described in which fine wires are positioned upstream of the leading edge of a flat plate to generate well-defined FSN (Free Stream Nonuniformity). Large spanwise thickness variations in the downstream boundary layer are generated by the interaction of the laminar wakes with the leading edge. Regions of elevated background unsteadiness appear on either side of the peak layer thickness, which share many of the characteristics of Klebanoff modes, observed at elevated Free Stream Turbulence (FST) levels. However, for the low background disturbance level of the free stream ($u'/U < 0.05\%$), the layer remains laminar to the end of the test section ($R_e = 1.4 \times 10^6$) and there is no evidence of bursting or other phenomena associated with breakdown to turbulence. A vibrating ribbon apparatus is used to demonstrate that the deformation of the mean flow is responsible for substantial phase and amplitude distortion of Tollmien-Schlichting (TS) waves. Pseudo-flow visualization of hot-wire data shows that the breakdown of the distorted waves is more complex and occurs at a lower Reynolds number than the breakdown of the $K$-type secondary instability observed when the FSN is not present.

Introduction

Watmuff [11] reduced the background unsteadiness, $u'/U$, in a Blasius boundary layer by a factor of 30 by improving the flow quality of a small stand-alone wind tunnel. The effectiveness of the improvements was judged using contours of hot-wire data in spanwise planes through the layer. The contours demonstrated a form of boundary layer three-dimensionality in which locally concentrated regions of elevated background unsteadiness appeared to be correlated with small spanwise variations of the layer thickness. The characteristics of the unsteadiness (e.g. low frequency spectral content) in the concentrated regions were much the same as at other spanwise positions, where the $u'/U_1$ distribution was more uniform and the Blasius wall distance of the $u'/U_1$ maxima was $\eta = 2.3$. These characteristics have much in common with the Klebanoff modes observed by Klebanoff [10], Kendall [8, 9] and others [12] at elevated FST levels.

The most significant reductions in $u'/U_1$ were realised after painstaking improvements were made to the uniformity of the porosity of wind tunnel screens. During the course of the flow quality improvements, Watmuff found that even almost immeasurably small FSN variations (e.g. $\Delta U/U_1 = 0.05\%$) appeared to be associated with local concentrations of elevated unsteadiness. During the final stages, further improvements to the screen system produced only a relatively minor reduction in the FST level, but the additional decrease in the FSN led to a three-fold reduction of $u'/U_1$ within the layer.

It is almost certain that the small FSN variations are caused by minor imperfections in the weave of the screen cloth. For example, Watmuff measured the uniformity of the screens by scanning them between a fixed laser and photo detector pair. He found the peak $u'/U_1$ levels in the layer were reduced by a factor of 1.5 after the screens were re-ordered from least to most uniform in the flow direction, based on the laser scan results.

Insertion of FSN upstream of the leading edge

The extraordinary sensitivity to weak FSN encouraged the author to develop a means of introducing FSN for the purpose of deliberately embedding Klebanoff-mode-like features into the boundary layer. The technique consists of stretching a fine wire across the full-width of the test section at a position upstream of the leading edge. The wire is aligned perpendicular to the freestream and to the leading edge.

The motivation for the study was wholly based on the observations made during the painstaking flow quality improvements, i.e. that boundary layer phenomena resulting from FSN may be somehow related to phenomena resulting from elevated FST levels. Consequently, the study might provide some insight into physical processes that occur during bypass transition. At the time these experiments were conceived, the author was unaware of the numerical studies by Goldstein and colleagues [4,5,6] concerning the receptivity to spanwise vorticity in the free-stream. Their work is unique, in the sense that they are the only investigators who have attempted to account for Klebanoff modes as a result of the interaction of free-stream vorticity with the leading edge.

A range of wire diameters, at varying distances from the leading edge has been considered. At the outset, it was considered essential to introduce only a mean flow disturbance into the freestream, to avoid additional competing effects that might occur if additional unsteadiness was also introduced, e.g. vortex shedding from the wires could excite extraneous instabilities in the layer. The parameters for wires located in the test section are specified in Table 1, while the effect of weaker FSN produced by wires located upstream of the contraction are specified in Table 2.

Representative wake profiles are shown in figures 1 (a-c). Linearizing the convective terms reduces the momentum equation to a simple diffusion equation, leading to the following expression for the velocity defect of a laminar wake,

$$\frac{U_1 - U}{U_1} = C_D \left( \frac{R_e}{16 \pi} \right)^{1/2} \left( \frac{d}{x} \right)^{1/2} \exp \left( - \frac{U_1 x^2}{4 \nu} \right),$$

where $x$ is the streamwise distance from the wire, $z$ is the spanwise distance from the centreline, $R_e$ is the Reynolds number based on wire diameter, $d$, and free-stream velocity, $U_1$, and $C_D$ is the drag coefficient. The solid lines in figures 1 (a-c) have been calculated from (1). The agreement of the observations with (1) is excellent. A similar level of agreement is observed for the wakes generated by wires located further upstream of the leading edge.
PDF, with edge. Solid lines: in (a-c) from eq. (1), and in (d-f) by applying Gaussian wire centreline. All profiles located elevated levels are not the result of eddying motions. Instead, the cases (d) 2T wakes. All profiles located elevated background unsteadiness were also measured and the distribution is essentially the same as without the presence of the wires.

**Properties of base-flow boundary layer**

The spanwise uniformity of the base-flow boundary layer at the position $x=1.05$ m, i.e. $R (= R_x) = 837.5$, is shown in figures 3(a-d). The displacement thickness, $\delta_1$, is within ±1.5% of the theoretical value, except in the range $0 > z > -10$ mm, where $\delta_1$ increases to be about 3 % larger than the Blasius value. The shape factor, $H$, is uniform to within ±1% and the smaller variations are considered to be random errors introduced by hot-wire calibration drift or by small uncertainties in the wall distance. The contours in figure 3(d) demonstrate the low overall background unsteadiness level within the boundary layer, i.e. $u'/U_1 < 0.08 \%$. (Note that hot-wire signals are unfiltered and no allowance has been made for electronic noise).

**Effects of FSN on boundary layer**

The FSN introduced by the wires leads to a form of three-dimensionality and a pair of locally concentrated regions of elevated background unsteadiness in the layer as shown in figure 4(a-b). Contours of a similar but weaker form were observed before and at each stage during the flow quality improvements. The characteristics of $u'/U_1$ also have much in common with Klebanoff modes appearing at elevated FST levels [Klebanoff [10], Kendall [8,9] and others[12]]. For example, the two maxima in the background unsteadiness contours are located at the Blasius wall distance of $\eta \approx 2.3$. Furthermore, the elevated background unsteadiness occurs at low frequencies, as shown in figure 4(c). The frequency corresponding to the lower branch of the neutral stability curve at this streamwise position is $f=50$Hz. Most of the unsteadiness resulting from the FSN occurs at frequencies lower than those predicted from classical linear stability theory.

Figure 1. Properties of wakes generated by wires in test section. Mean velocity profiles: cases (a) 2T, (b) 4T, and (c) 5T. Unsteadiness profiles: cases (d) 2T, (e) 4T, and (f) 5T. (See Table 1). z-coordinate is relative to wire centreline. All profiles located $\Delta x = 63.5$ mm upstream of leading edge. Solid lines: in (a-c) from eq. (1), and in (d-f) by applying Gaussian PDF, with $\sigma_y = 15 \mu$m, to mean flow from eq. (1).

Figure 2. Mean velocity profiles of wakes generated by wires located upstream of contraction. Base flow, without wire, is solid (without symbol); cases 1U to 5U (see Table 2) generate successively stronger wakes. All profiles located $\Delta x = 63.5$ mm upstream of the leading edge. z-coordinate is relative to tunnel centreline.

Table 1. Parameters for wires located in test section.

<table>
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<tr>
<th>Case</th>
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<th>$R_x$</th>
<th>$C_p$</th>
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Table 2. Parameters for wires located upstream of contraction.

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Figure 3. Spanwise uniformity of base-flow boundary layer for $R=837.51$ ($x = 1.05$ m) (a) displacement thickness, $\delta_1$, (b) shape factor, $H$, and contours of (c) mean velocity, $U/U_1$, (d) background unsteadiness, $u'/U_1$. Grid: ($N_x, N_z$) = (31,71), i.e. 71 hot-wire profiles, profile spacing, $\delta_x = 0.5$ mm (length of hot-wire filament).
The spanwise variation of $H$ in figures 5(a-b) is within the ±1% uncertainty limits of the Blasius value. A slight reduction in shape factor is evident for the largest diameter wire in figure 5(c). The insensitivity of $H$ is also evident when the FSN is introduced by wires upstream of the contraction as shown in figures 6(a-c). There is only about a 4% reduction in $H$ for the largest diameter wire in figure 6(c), despite the 70% increase in layer thickness. A reduction in $H$ is generally associated with decreased stability of boundary layers, e.g. adverse pressure gradient. Examination of individual profiles in the vicinity of the $H$ minima does not reveal an inflection point, even close to the test surface.

Klebanoff [10] suggested that the elevated layer unsteadiness at moderate FST levels could be the cause of relatively small amplitude modulation of the boundary layer thickness. The Blasius solution can be written, $U/U_1 = f^*(\eta) = f'(y/\delta_0)$, where $\delta_0 = (xU/U_1)^{1/2}$ is a measure of the boundary-layer thickness. The effect of a small perturbation, $\delta_p$, in thickness can be expressed as a Taylor series, i.e.

$$\frac{U}{U_1} = f\left(\frac{y}{\delta_0 + \delta_p}\right) = f'(\eta) - \frac{\delta_p}{\delta_0} f''(\eta) + \ldots$$

The difference between $U/U_1$ resulting from the perturbed thickness and the Blasius solution should be proportional to $\eta f''$, which leads to excellent agreement with Klebanoff's measurements, as shown in Goldstein & Wundrow [6]. This idea was presumably based on an earlier proposal by Bradshaw [2], which was subject to analytical treatment by Crow [3]. The more general analyses of Goldstein, Leib & Cowley [4], Goldstein & Wundrow [6], and Kendall [8, 9]. On the basis of...
the agreement with the measurements of Klebanoff, Kendall and others, Goldstein and his colleagues argue that there is a direct connection between the disturbances resulting from the FSN in their calculations and the Klebanoff modes observed at elevated FST levels. However their reasoning is unclear concerning the origin and form of the unsteadiness that is required for correspondence with the measurements.

It is clear that the perturbation in layer thickness, $\delta_p$, must vary with time in order for (2) to account for the observations. However, the author is not aware of any considerations that have led to a proposal concerning plausible forms of the $\delta_p$ variations.

The spanwise uniformity of the shape factor, $H$, shown in figures 5 and 6, indicates that the complete distribution of streamwise mean-velocity component in a spanwise plane can be determined by specifying the spanwise variation of the displacement thickness. Using this simple empirical model, the velocity at any $(y, z)$ position can be determined by interpolating $\delta_1(z)$ from a spline fit, for example, and then evaluating the local Blasius coordinate, using $\eta(y, z) = 1.7208y/\delta_1(z)$. The local streamwise velocity, $U(y,z)/U_1$ can then be determined for $\eta(y, z)$ from an interpolation of the Blasius solution.

The model described above has been used to explore the effects of small-amplitude modulations of the base flow, in an attempt to provide an explanation of the observations shown in figures 5 and 6. A smooth spanwise variation of $\delta_1$ is assumed for the “steady” base flow as shown in figure 7(a). The distribution is shown in dimensional form and it has been chosen to approximate the conditions corresponding to the results shown in figures 5 and 6, i.e. at the extremities of the span, $\delta_1 = 2.157 \text{ mm}$, which corresponds with the Blasius value for $x = 1.05 \text{ m}$, and the unit Reynolds number, $0.668 \times 10^6 \text{ m}^2\text{s}^{-1}$, used in the experiments. The spanwise variation of $\delta_1$ has an overshoot of 12% on the centreline and a small magnitude undershoot is imposed on either side, of approximately -3%, before recovery to the unperturbed Blasius value. These features are also evident in the measurements.

The mathematical form of the spanwise $\delta_1$ variation is not critical for the simulations, provided it is smooth. It is convenient to generate the $\delta_1$ distribution by summing two Gaussian distributions; one for the positive and the other for the negative contribution. Contours of the mean flow, $U/U_1$, and the spanwise derivative, $\partial(U/U_1)/\partial z$ are also shown for the base flow in figure 7(a). For the purposes of the simulation, the mean-velocity distribution will be considered to be two-dimensional, i.e. without streamwise derivatives.

For the wakes, the elevated background unsteadiness in the free stream was successfully modelled by applying a Gaussian PDF to the spanwise position of the mean velocity profile. While the variation of $\delta_1$ depends only on the spanwise position, it is clear that gradients of the mean velocity in the layer will exist in both the $y$- and $z$-directions. Therefore the effects of varying both the strength and the spanwise position of the $\delta_1$ perturbation are considered. Variations in the strength of the $\delta_1$ perturbation might arise if the FSN was fixed in position but the magnitude of the velocity defect was to vary with time. Variations in the spanwise position of the $\delta_1$ perturbation could arise from corresponding variations in the spanwise position of the FSN with time.

The effect of varying magnitude is simulated by applying a Gaussian PDF to the strength of the $\delta_1$ perturbation, with a standard deviation given by, $\sigma_{\delta_1}/\delta_1 = 10\%$. The effect of the $\delta_1$ modulation on the mean flow is very small and the contours of $U/U_1$ in figure 7(b) are almost identical to the base flow. The rms unsteadiness distribution, $u'/U_1$ in figure 7(b), resulting from the $\delta_1$ modulation, shows that the maximum is located on the centreline. Larger values of $\sigma_{\delta_1}/\delta_1$ lead to much the same form of distribution for $u'/U_1$, i.e. the maximum corresponds with the mean value of the PDF, i.e. the centreline.

The effect of spanwise modulation is simulated by applying a Gaussian PDF to the spanwise position of the $\delta_1$ perturbation. The results for a standard deviation given by, $\sigma_z = 200 \mu m$ (approximately 10% $\delta_1$) are shown in figure 7(c). Again, the effect of the PDF on the mean flow is very small. The amplitude of $\sigma_z$ was chosen to generate approximately the same $u'/U_1$ maximum as for $\delta_1$ perturbation in figure 7(b). While the peak $u'/U_1$ levels may be similar, the distribution in figure 7(c) is markedly different. In this case, there are two concentrated regions of elevated amplitude on either side of the centreline, and the magnitude on the centreline is almost zero.

The overall similarity between the form and the levels of the $u'/U_1$ contours in figure 7(c) and the experimental observations in figures 5 and 6 support the conjecture that the elevated unsteadiness levels are the result of boundary layer thickness variations in the vicinity of the stationary probe, which arise from small-amplitude spanwise modulation of the layer thickness.

Cubic-spline fits to the observed spanwise $\delta_1$ variations are shown as solid lines in figures 5 and 6. Simulations have been conducted to determine the value of $\sigma_z$, which produces a simulated $u'/U_1$ distribution that most closely matches the observations, where an allowance has been made for the background component without the presence of wires of magnitude $u'/U_1 = 0.05\%$. For the spanwise $\delta_1$ variations resulting from wires located in the test section, a value of $\sigma_z = 90 \mu m$ has been found to closely match the $u'/U_1$ observations, while for the wires located upstream of the contraction, a value of $\sigma_z = 120 \mu m$ is required, i.e. about 30% larger. These results are shown in figures 5 and 6 and are labelled...
“simulation”. The values of $\sigma_1$ required to reproduce the $u'/U_1$ levels in the boundary layer is order 4% of the displacement thickness, which is remarkably small.

The $\sigma_1$ required to reproduce the $u'/U_1$ levels resulting from wires located in the test section shown in figure 5 is about six times larger than that required to reproduce the free-stream $u'/U_1$ wake profiles shown in figures 1(d-f). Therefore it is unlikely that the elevated unsteadiness in the boundary layer can be totally accounted for by spanwise oscillations of the mean flow pattern that merely “follow” the instantaneous spanwise position of the wake. The larger amplitude modulation required for the boundary layer simulation may indicate that some unaccounted form of instability mechanism is at play. For example, the mean flow pattern may be the result of a pair of counter-rotating streamwise vortices and the spanwise modulation of the mean flow could be associated with long-wavelength low-frequency small-amplitude spanwise modulation of the strength and/or the position of the vortex pair.

A measure of the integrated effect of the FSN on the development of the boundary-layer mean flow is the growth of $\delta_1$ relative to the Blasius value. Determination of the spanwise development of the boundary-layer mean flow is the growth of $\delta_1$ calculated from the spanwise profiles is shown for all six cases in figures 8(a-f). The $z$-coordinate shown in all of the figures is relative to the peak $\delta_1$ at each streamwise position. A small spanwise shift in the peak $\delta_1$ is evident in the results, corresponding to about 7.5mm over the streamwise range used for the measurements, i.e. $\delta x = 1.5$ m.

An alternative and more approximate method was used to estimate the growth of the $\delta_1$ variations, which only requires the quantity $U/U_1$ to be determined along a single spanwise profile at fixed wall distance, at each streamwise position, for each case. The Blasius wall distance, $\eta(z)$, is interpolated from each $U/U_1$ measurement at each data point, by interpolation of the known Blasius solution. The displacement thickness variation is then calculated using, $\delta_1(z) = 1.7208 \eta_*/\eta(z)$, where $\eta_*$ is the distance of the hot-wire filament from the test surface. The streamwise development of the spanwise variation of $\delta_1$ calculated from the spanwise profiles is shown for all six cases in figures 8(a-f). The $z$-coordinate shown in all of the figures is relative to the peak $\delta_1$ at each streamwise position. A small spanwise shift in the peak $\delta_1$ is evident in the results, corresponding to about 7.5mm over the streamwise range used for the measurements, i.e. $\delta x = 1.5$ m.

The basis of the method for the determination of the $\delta_1$ variation from the spanwise profiles is the assumption of spanwise uniformity of the shape factor, $H$, which has not been determined at these positions. Nevertheless, comparison of cursory test results obtained at $x=1.05$m indicated that $\delta_1$ estimated from the spanwise profiles, compared surprisingly well with the $\delta_1$ obtained by integration of the mean-velocity profiles shown in figures 5 and 6. Overall, the estimated $\delta_1$ estimates were found to be within about ±5% of the values obtained by integration of the mean-velocity profiles.

It is possible that the phenomena responsible for the spanwise thickness variations may exist only in the vicinity of the leading edge. In this case, the elevated thickness relative to the undisturbed layer might be expected to decrease with streamwise distance, owing to viscous diffusion, for example. On the other hand, growth of the spanwise thickness variations with streamwise distance would indicate that the phenomena responsible for the spanwise thickness variations persist in the downstream boundary layer. The development with $R$ of the maximum and minimum $\delta_1$ across the layer is shown in figures 9(a-b). For all cases where the wires are located in the test section, shown in figure 9(a), the effect of the FSN on the boundary-layer mean flow is such that the growth rates of the $\delta_1$ minima and maxima are much the same as that of the undisturbed Blasius boundary layer. This observation still suggests that the phenomena responsible for the spanwise thickness variations is present in the downstream boundary layer, but that some equilibrium condition may be at play to balance a decrease of the thickness extrema with streamwise distance, due to viscous diffusion, for example. In contrast, the FSN generated by the wires located upstream of the contraction, shown in figure 9(b), has a much stronger effect. While the growth rates of the $\delta_1$ minima appear to be much the same as for the undisturbed Blasius boundary layer, it is evident that the growth rates of the $\delta_1$ maxima are much larger. The growth rate for the cases 1U and 4U appear to be reasonably constant while that for case 5U appears to diminish with $R$. It is unknown whether the apparent reduction in the growth rate for this case is associated with errors introduced by a significant reduction of the shape factor with $R$. Evidently, for the FSN generated by the wires located upstream of the contraction, the phenomena responsible for the spanwise thickness variations clearly maintain their presence in the downstream boundary layer.

Another feature of Klebanoff modes generated by elevated FST levels is that the growth of $u'/U_1$ with streamwise distance appears to be algebraic rather than exponential. Kendall [8,9] reports that the growth varies as $x^3$ for FST generated by a turbulence grid. Westin et al. [12] present measurements from
their own investigations and by others concerned with elevated FST levels that tend to support Kendall's observations. However, the growth rate appears to vary from facility to facility, i.e. there is disagreement concerning the factor of proportionality. Bertolotti & Kendall [1] suggest that one possible reason for the discrepancies is that the Klebanoff modes are associated with the exceedingly low frequency components and that some form of ac-coupling of the hot-wire signal may have been employed. They point out that details concerning the signal processing are often unreported in the literature. Also, the low frequency components require much longer sampling periods than usual for accurate resolution. The author suggests that an alternative explanation for the variance of the factor of proportionality may be that each facility incorporates a different leading edge. Differences in the leading edge geometry combined with the facility dependent FST characteristics are probably a more likely source of the variance in the factor of proportionality.

The growth with Reynolds number of $u'/U_1$ resulting from FSN introduced by wires in the present work has been determined from hot-wire measurements obtained along a series of spanwise profiles, at a wall distance closely corresponding to the maxima in the background unsteadiness. Separate measurement grids were designed for each case, on the basis of cursory observations, allowing the wall distance of the measurement grids to grow with $x^+$ was found to generate grids that pass closely through the locations of the $u'/U_1$ maxima. The growth of the background unsteadiness in the layer resulting from the FSN introduced by the wires located in the test section is shown in figure 10(a) and the growth resulting from FSN introduced by wires located upstream of the contraction is shown in figure 10(b). The growth for both peaks in the background unsteadiness (i.e. at the two spanwise positions) is shown in both figures.

The growth of the peak background unsteadiness levels associated with FSN introduced by the wires located in the test section is weak. For the smallest diameter wire (case 2T), the amplitude remains almost constant, with the second peak in the distribution showing a slight decay. For the larger diameter wires, the growth is positive, but still very weak. The growth associated with FSN generated by wires located upstream of the contraction is larger and weakly nonlinear, with an increasing growth rate for larger $R$. The growth rate for the FSN generated by the largest diameter wire is approximately three times larger than that for the smallest diameter wire. The effect on the layer of the FSN originating from wires upstream of the contraction appears to more closely follow the behaviour observed for Klebanoff modes at elevated FST levels.

**Interaction between phenomena resulting from FSN and Tollmien-Schlichting waves**

The boundary layer disturbances resulting from the FSN have always been observed to remain stable, provided that disturbances in the form of Tollmien-Schlichting (TS) waves are not deliberately introduced into the layer. Stability has been observed, even at higher free-stream velocities, such as $U_1=15$ m/s. Only very occasional bursting has been observed at these higher free-stream velocities, and only towards the end test plate, $x=2.1$ m, for $U_1=15$ m/s, where the Reynolds number based on streamwise distance is given by $R_e = 2.1 \times 10^6$, i.e. $R = 1450$.

![Figure 10](image-url)
The vibrating ribbon technique is used to explore the interactions between TS waves and the phenomena resulting from the FSN. The operating point is given by \( F = 2\pi f v / U_1^2 = 60 \times 10^6 \) and \( R = 524 \), where \( f \) is the frequency. The hot-wire signals are averaged on the basis of the phase of the ribbon vibration using 64 phase intervals. It is assumed that each velocity measurement can be described using, \( U = \bar{U} + u_\phi + u_\phi' \), where \( \bar{U} \) is the temporal mean velocity, \( u_\phi \) is the phase-averaged fluctuation about the mean and \( u_\phi' \) is the random fluctuation. For consistency with the transition literature, the term, \( u \), will be used to represent wave amplitude, defined as the rms of \( u_\phi \) over all phase intervals. The quantity \( u'^2 \) represents the background unsteadiness at constant phase. The term \( u' \) is defined as the rms over all phases.

In the base flow, the growth of rms wave amplitude, \( u/U_1 \), closely follows the predicted growth with \( R \) obtained using the Parabolized Stability Equations (kindly provided by F. Bertolotti). Extensive spanwise measurements also demonstrate that the waves are highly 2D for \( u/U_1 < 1.0 \% \). The onset of \( K \)-type secondary instability occurs for larger \( u/U_1 \) (Herbert [7]) followed by the formation of \( \Lambda \)-shaped vortex loops, increased randomness and ultimately the demise to fully turbulent flow.

Introduction of the FSN leads to considerable phase and amplitude distortion of the TS waves. However, for small wave amplitudes, e.g. \( u/U_1 < 1.0 \% \), the distortion is benign in the sense that the streamwise development of contours of \( u/U_1 \), in a spanwise plane maintain much the same shape and form, as shown in figures 11(a-e). Also, contours of the random background fluctuations appear to be unaffected by the presence of the wave motions. For these measurements, the ribbon amplitude is adjusted to ensure that the rms wave amplitude, \( 0.1\% < u/U_1 < 0.5\% \) at each streamwise position. The spanwise grid spacing, \( \delta_x = 0.5 \text{mm} \), which is equal to the length of the hot-wire filament. The disturbance from the FSN displaces the wave amplitude contours away from the test surface and this region is also associated with a larger wave amplitude. The largest \( R \), shown figure 11(e), is close to Branch II.

For larger wave amplitudes the onset of breakdown of organised wave motions occurs at a lower \( R \) and is of a more complex form than the \( K \)-type instability observed in the base flow. Contour surfaces and contour lines of the wave motions, \( u_\phi/U_1 \) and random fluctuations, \( u''/U_1 \), are shown in figures 12(a-b). Note that both \( u_\phi/U_1 \) and \( u''/U_1 \) are phase-averaged quantities. The contours have been constructed by using phase as the third (streamwise) coordinate. The spanwise variations of \( u_\phi/U_1 \) in the base flow in figure 12(a) demonstrate the onset of \( K \)-type instability (Herbert [7]), and it is evident that a small level of randomness is associated with the negative wave amplitudes resulting from cycle-to-cycle variability. The complex distortion of the waves resulting from the FSN is evident in figure 12(b). Note that the \( u''/U_1 \) levels are larger than for the base flow and that the contours have a well-defined structure.

Orthographic views of results corresponding to larger initial wave amplitude are shown in figures 13(a-b). Note that both the \( u_\phi/U_1 \) and \( u''/U_1 \) surfaces in figure 13(a) clearly show the \( \Lambda \)-shape, which provides additional evidence that the negative waves evolve into vortex loops. With the presence of FSN, the \( u_\phi/U_1 \) surfaces are highly intertwined, and a pair of additional elevated random motions are present.

Part of the reason for complexity is that is misleading since contour perturbations at these larger amplitudes. For example, the positive contour surface visible in figure 13(a) is misleading since contour surfaces of the total phase-averaged velocity clearly show only a \( \Lambda \)-shaped deflection associated with the negative level, as shown in figure 14(b). However, the complex sinuous motions resulting from the presence of the FSN are still evident in figure 14(a).

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**Figure 11.** Interaction of small-amplitude waves with FSN from \( d = 50.8 \mu \text{m} \) wire. Case 47. Contours of rms wave amplitude, \( u/U_1 \), in spanwise plane, with increasing \( R \) (streamwise distance). Ribbon amplitude adjusted to ensure \( 0.1 < u/U_1 < 0.5 \% \). (a) \( R = 659.0, u_{\text{rms}} = 0.44 \% \), (b) \( R = 707.8, u_{\text{rms}} = 0.30 \% \), (c) \( R = 837.5, u_{\text{rms}} = 0.22 \% \), (d) \( R = 913.8, u_{\text{rms}} = 0.11 \% \), (e) \( R = 1017.5, u_{\text{rms}} = 0.20 \% \), (legend). (u')

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**Figure 12.** Plan view of pseudo-flow contour surfaces of \( u_\phi/U_1 \) and contour lines of \( u_\phi/U_1 \) and \( u''/U_1 \) for \( R = 837.5 \). (a) \( K \)-type instability in base flow. (b) Same ribbon amplitude, but with FSN, Case 57. Dark surfaces: \( u_\phi/U_1 = +2.5 \% \). Light surfaces: \( u_\phi/U_1 = -2.5 \% \). Contour lines in plane \( y = 1.2 \text{mm} \). For \( u_\phi/U_1 \), Contour Range, CR = \( \pm 6.25 \% \), Contour Increment, CI = 0.5 \%, black lines are positive- and grey lines negative-levels. For \( u''/U_1 \), CR = 0.45 \%, CI = 0.05 \%.
Conclusions

The wave distortion appears to be only of relatively minor significance until the wave amplitudes reach a level for the onset of secondary instability when the FSN is not present. The onset of random behaviour, characteristic of turbulent flow, occurs at lower \( R \) and breakdown of the organized wave motion is more complex than when the FSN is not present.

The motivation behind these studies is to gain some insight into transition at elevated FST levels, which has remained a mystery since the thirties. A simplistic proposition is that FST may be viewed as unsteady FSN and that transition at lower \( R \) is caused by interactions of a similar nature to those observed in the present study. A better understanding of the dependency on the leading-edge geometry and on the length-scales of the FST might help explain the disparities between controlled experiments noted by Westin et al. [12].

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References