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# The Influence of Probe Resolution on Measurements of Fluctuating Scalar And Its Dissipation Rate

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## Abstract

This paper investigates the effect of probe resolution on measurements of the scalar fluctuations and the scalar dissipation rate. A simple spectral method is employed to estimate the influence of the spatial resolution on both the measured mean squares of a fluctuating scalar and its streamwise derivative by filtering the signals at different cut-off frequencies. Temperature differential above ambient acts as a passive scalar for the present investigation and is measured in the far field of a slightly heated circular jet. It is found that accurate measurements of the scalar dissipation rate require the smallest length scale of the scalar fluctuation (defined as the Batchelor scale) to be resolved. However, the resolution requirement for accurate measurements of the statistics of the scalar fluctuations is much less stringent.

## Introduction

Statistic behaviour of a passive conserved scalar in turbulent flows has been investigated extensively during the past decade or so. As a consequence, our understanding of turbulent mixing processes has improved dramatically. While the mixing of two or more fluid components can occur at various scales, that at the molecular level and the finest scales of turbulence plays a dominant role in those practical applications where chemical reactions take place. It is well known that the fine-scale mixing is commonly characterised by the statistic behaviour of the scalar gradient.

To measure accurately a fluctuating scalar and particularly the scalar dissipation rate, the measurement probe must have sufficient spatial and temporal resolution to ensure that all spatial fluctuations in the scalar gradient occur on scales larger than the effective measurement volume and the product of the measurement time and local velocity. That is, the probe is required to have a spatial resolution scale ($\lambda_r$) of order of the finest length scale of scalar fluctuations, defined by Batchelor [4] as

$$\lambda_B \equiv \left( \frac{\nu D^2}{<\varepsilon>} \right)^{1/4}, \tag{1}$$

which is commonly called 'Batchelor scale'. In definition (1), ε is the turbulent energy dissipation rate, ν is the kinematic viscosity, $D$ is the scalar (molecular) diffusivity, and angular brackets denote time averaging (this applies hereafter). In order to estimate the relative resolution $\lambda_r/\lambda_B$, correctly obtaining $\lambda_B$ becomes important. Most attempts to estimate $\lambda_B$ for measurements of the scalar gradient are based on analogies with those of the velocity gradient. The finest scale on which velocity fluctuations occur is the Kolmogorov scale defined as

$$\lambda_K \equiv \left( \frac{\nu^3}{<\varepsilon>} \right)^{1/4}. \tag{2}$$

Combining equations (1) and (2) yields the relationship between the Batchelor and the Kolmogorov scales:

$$\lambda_B = \lambda_K Sc^{-\frac{1}{2}}, \tag{3}$$

where $Sc$ is the molecular Schmidt number defined by $Sc \equiv \nu/D$. An important implication of this relationship is that the Batchelor scale is one and a half order of magnitude smaller for measurements in liquid-phase flows ($Sc$ ~ 1000) than for measurements in gaseous flows ($Sc$ ~ 1). This renders complete resolution of the Batchelor scale extremely difficult, and almost impossible with planar imaging techniques, in high-$Sc$ flows.

For high-Reynolds-number flows, the amount of the turbulence energy dissipated at the smallest eddies, <ε>, is considered to be equal to the supply rate of the turbulence energy generated by the large eddies, which is of order $U^3/L$ (Tennekes and Lumley [22]); that is:

$$<\varepsilon> = C\frac{U^3}{L}, \tag{4}$$

where $U$, $L$ are the local characteristic velocity and length scales and $C$ is a constant determined by experiments. Substitution of (4) into (2) allows the Kolmogorov scale to be estimated by

$$\lambda_K / L = C_1 Re_L^{-\frac{3}{4}} \tag{5}$$

where $Re_L$ is a Reynolds number defined by $Re_L = UL/\nu$ and $C_1$ is an empirical constant equal to $C^{-1/4}$. This constant may change for different turbulent flows. It follows that the Batchelor scale can be expressed as

$$\lambda_B / L = C_1 Re_L^{-\frac{3}{4}} Sc^{-\frac{1}{2}}. \tag{6}$$

Since a direct measurement of total dissipation rate <ε> has not been possible, the finest scales are frequently estimated using (5) and (6). These scales are also often estimated by measurements of the streamwise component of <ε> invoking Taylor's hypothesis as well as using the assumption of local isotropy.

It is important to note that the proportionality $C_1$ of Eqs. (5) and (6) does not vary with the choice of the local characteristic scale $L$ in any flow. However, to estimate the required spatial resolution scale $\lambda_r$ for scalar measurements in jet flows, different values for $C_1$, ranging between 5 and 25, have been considered plausible in a number of previous studies [5,6,7,9,10,17,20,21]. We notice that those studies did not use the Batchelor scale $\lambda_B$ defined by (1) as the smallest length scale of scalar mixing; instead, they used a so-called 'smallest strain-limited diffusion length scale', often denoted by $\lambda_D$. For example, Miller and Dimotakis [17] utilised the relation $\lambda_D = \lambda_\nu \mathrm{Sc}^{-1/2}$, where $\lambda_\nu$ is the spatial scale, called the 'inner viscous scale', from which the action of viscosity becomes important. They deduced from the velocity spectra that $\lambda_\nu \approx 25\lambda_K$, thus taking $\lambda_D = 25\lambda_K \mathrm{Sc}^{-1/2}$ as "the smallest expected scalar diffusion scale". (More recently, Dimotakis [8] has defined $\lambda_\nu \approx 50\lambda_K$ where the turbulence spectrum departs from the –5/3 power-law.) Buch and Dahm [5] and Dahm and Dimotakis [7] utilised the relation $\lambda_D/L = 25\mathrm{Re}_L^{-3/4}\mathrm{Sc}^{-1/2}$, which is the same as that of [17] if $\lambda_K = \mathrm{Re}_L^{-3/4}L$. Later, the average value of $C_1$ was found [6,20] to be 11.2. Of course, the resolution of $\lambda_D$ is significantly more attainable than that of $\lambda_B$ in liquids, providing strong incentive for choosing the less-stringent criterion should it be valid. Several previous studies [5,6,7,9,10,17,20,21] claim that this less-stringent criterion is valid, i.e. that the scalar measurements are fully resolved, if the spatial resolution scale $\lambda_r < 0.5\lambda_D$.

Pitts et al. [19] have pointed out that if the value for $C_1$ were actually as large as used by the above studies for achieving the fully resolved measurements, it would have significant practical importance since the spatial and temporal resolution requirements for a given experiment could be greatly relaxed from the requirement of resolving the Batchelor scales. Based on the experimental data from Gibson *et al.* [12], Lozano *et al.* [13], and Antonia and Mi [2], as well as those from Anselmet *et al.* [1] and Tong and Wahaft [23], these authors have proposed that the relation $\lambda_r \approx C_1 \mathrm{Re}_L^{-3/4}\mathrm{Sc}^{-1/2}L$ can provide valid estimates for the spatial resolution scale required to make accurate measurements of the scalar dissipation rate only when $C_1 \leq 2$. They take the proportionality constant to be precisely $C_1 = 1.0$. However, Friehe *et al.* [11] and Antonia *et al.* [3] showed earlier by experiments that the value for $C_1$ is approximately 2.4 for the circular jet. It was also found that $C_1 \approx 4.1$ for the planar jet [3]. Clearly, there is considerable discrepancy on the required resolution for scalar measurements.

To provide further insight into the above issues, we investigate, by using a spectral scheme, the effect of the spatial resolution of measurement on the mean square of the scalar fluctuation and the mean scalar dissipation rate in a circular jet. The objective is to assess whether a fully resolved fine-scale scalar measurement requires resolution of the Batchelor scale $\lambda_B$ or the strain-limited diffusion scale $\lambda_D$. We have chosen to use air as the working fluid to minimise the required resolution for measurements.

## Experimental details

To perform the present investigation, a small temperature differential from ambient was used to mark the scalar field. Present measurements of the temperature fluctuation $\theta$ and its streamwise derivative $\partial\theta/\partial x$ were conducted in the slightly heated air jet issuing from a long circular pipe with an inner diameter $d$ = 10 mm and a length of 70$d$. Full details of the experimental set-up are provided in [15] and so only a brief description is given here. The whole pipe was insulated so that a uniform temperature profile was achieved with less than 1.2% variation at the pipe exit plane. The exit temperature was selected to be 50°C above ambient. A single cold-wire probe was used for collecting the passive temperature signals in the far field of the jet, where the flow is fully developed and self-similar [15]. The probe consists of a short length of Wollaston wire (Pt-10%Ph) operated with an in-house constant current (0.1 mA) circuit. At this low current, the sensitivity of the wire to velocity fluctuations was negligible. The wire size is 0.63 µm in diameter and approximately 0.6 mm in length with a length-to-diameter ratio ≈ 1000, which is sufficiently large to ignore any possible low-frequency attenuation [18]. The voltage signals for temperature were offset and amplified through the circuits and then digitized by a personal computer with a 12-bit A/D converter. The signals were filtered at a maximum cutoff frequency $f_c$ = 2.8 kHz, which was chosen to eliminate high-frequency noise, and a sampling frequency of 2$f_c$ was employed. The nominal exit Reynolds number $Re_d \equiv U_o d/\nu$ (where $U_o$ denotes the exit bulk velocity) is about 16,000.

The measurement for this investigation was made at $x/d$ = 57. At this station, the centreline mean velocity $U_c$ is about 3.6 m/s and the mean velocity half radius is 56 mm, which is defined as the $y$-location where the mean velocity is 0.5$U_c$. On the axis, the Kolmogorov scale $\lambda_K$ estimated from (5) with $C_1$ = 2.4 is 0.152 mm and the Batchelor scale $\lambda_B$ from (6) is 0.182 mm. Correspondingly, the Bachelor frequency defined by $f_B \equiv U_c/2\pi\lambda_D$ is 3.1 kHz. To calculate the spectrum $\Phi_{\partial\theta/\partial x}$, Taylor's hypothesis was used so that the streamwise derivative $\partial\theta/\partial x$ was obtained from the time derivative $\partial\theta/\partial t$ via $\partial\theta/\partial x = -(1/U_c)\partial\theta/\partial t$. (In this paper, $x$, $y$ and $z$ are the coordinates, respectively, in the streamwise, spanwise and lateral directions.) This hypothesis was found to be reasonable on the axis of a circular jet in the far field by Mi and Antonia [14].

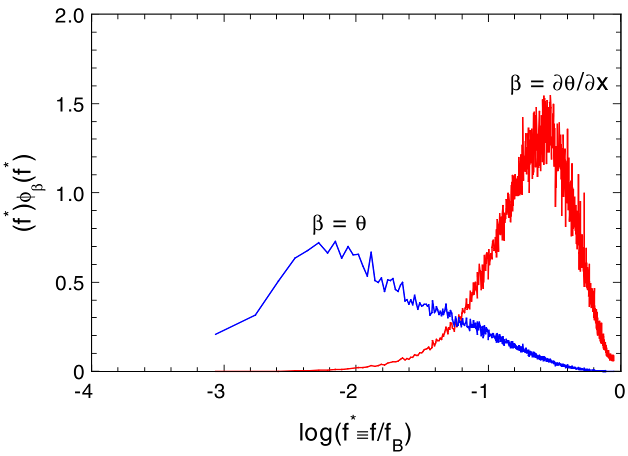


**Figure 1**. Normalized spectra of the fluctuating temperature $\theta$ and its streamwise derivative $\partial\theta/\partial x$ obtained on the axis of a circular jet at $x/d$ = 57. Here $f_B$ is the Batchelor frequency.

## Spatial resolution effect

It is well known that different scalar-mixing scales contribute differently to the mean squares of the scalar fluctuation $\theta$ and its spatial derivatives $\partial\theta/\partial p$ (here $p \equiv x$, $y$ or $z$). This can be illustrated by the power spectral density (usually called 'spectrum' for simplicity), $\Phi_\beta$, of $\beta$ ($\equiv \theta$ or $\partial\theta/\partial p$). Since the frequency spectrum of $\beta$ is defined as $\int\Phi_\beta(f)df = \langle\beta^2\rangle$ (where $f$ is frequency), it is thus likely to assess the effect of the spatial resolution scale $\lambda_r$ on $\langle\beta^2\rangle$ by calculating $\langle\beta^2\rangle$ based on $\Phi_\beta(f)$ at different low-pass filtering cut-off frequencies ($f_c$). The scalar and its derivative fluctuations at frequency higher than $f_c$ are filtered out and therefore make no contribution to their mean squares $\langle\beta^2\rangle$. Similarly, the average values of $\langle\beta^2\rangle$ over a measurement volume with dimensions $\lambda_r\times\lambda_r\times\lambda_r$ should contain little contribution from the $\beta$ fluctuations occurring at scales smaller than $\lambda_r$ as those fluctuations are not detected.

Figure 1 shows the normalized spectra of the fluctuating temperature $\theta$ and its streamwise derivative $\partial\theta/\partial x$ obtained on the jet axis at $x/d$ = 57, for the case of $f_c \approx 0.9f_B$. We present the data using the format of $f^*\Phi_\beta$ vs. $\log(f^*)$, where $f^* = f/f_B$, to highlight the low-frequency (large-scale) contents with the area under the curve of $f^*\Phi_\beta$ equal to $\langle\beta^2\rangle$. As evident from the figure, the mean square $\langle\theta^2\rangle$ is determined mainly by large scales of scalar mixing while $\langle(\partial\theta/\partial x)^2\rangle$ is determined dominantly by the small-scale fluctuations. This difference is well known, and so no further comments are needed. Nevertheless, it is worth noting that the broad peak of $f^*\Phi_{\partial\theta/\partial x}$ occurs at $f \approx 0.25f_B$ while that of $f^*\Phi_\theta$ locating at $f \approx 0.004f_B$. This suggests that the dominant small scales of scalar fluctuations are well separated from the large scales (by more than 1.5 decades). These results support the deduction that the measured value of the scalar dissipation rate is much more sensitive to spatial than the measured value of the scalar itself.

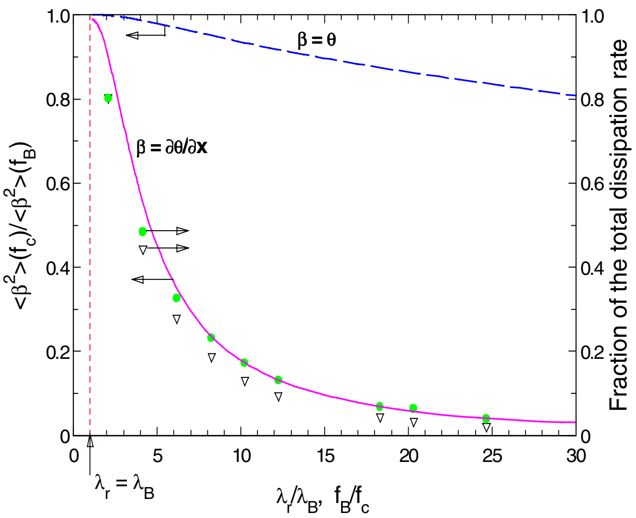


**Figure 2**. Effect of probe spatial resolution on measurements of several scalar properties in the far field of a turbulent circular jet. Mean square values of the measured scalar fluctuation θ (- - - -) and its streamwise derivative $\partial\theta/\partial x$ (—) were obtained from the spectra $\Phi_\theta$ and $\Phi_{\partial\theta/\partial x}$ at different cut-off frequencies ($f_c$). The mean scalar dissipation rates (●, ∇) were measured at different spatial resolution scales ($\lambda_r$) by Southerland & Dahm [20]. The symbol $\lambda_B$ denotes the Batchelor scale.

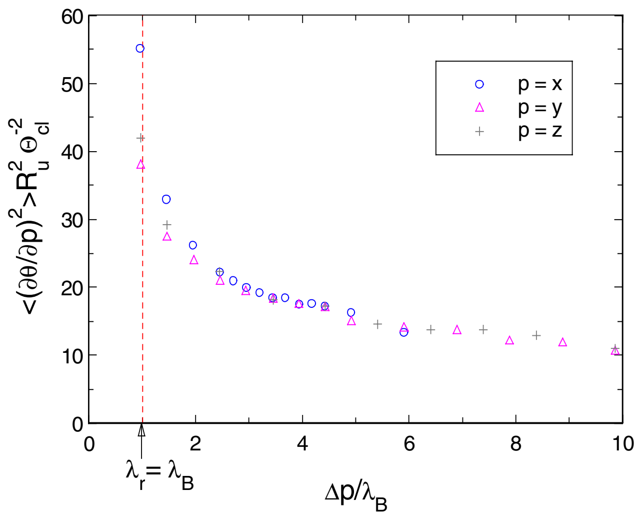


**Figure 3**. Mean square values of the measured derivatives $\partial\theta/\partial p$ ($p \equiv x, y, z$) as a function of the spatial resolution scale $\lambda_r = \Delta p$ on the axis of a circular jet at $x/d$ = 30 [16]. $R_u$ is the mean velocity half-radius and $\Theta_{cl}$ is the local mean temperature above ambient on the jet axis.

Figure 2 shows the ratios $\langle\theta^2\rangle(f_c)/\langle\theta^2\rangle(f_B)$ and $\langle(\partial\theta/\partial x)^2\rangle(f_c)/\langle(\partial\theta/\partial x)^2\rangle(f_B)$. Here the mean squares $\langle\theta^2\rangle(f_c)$ and $\langle(\partial\theta/\partial x)^2\rangle(f_c)$ were obtained from the spectra $\Phi_\theta$ and $\Phi_{\partial\theta/\partial x}$ at different values of the cut-off frequency $f_c$ ranging between $1.0f_B$ and $0.033f_B$. It is demonstrated that both $\langle(\partial\theta/\partial x)^2\rangle$ and $\langle\theta^2\rangle$ decrease as $f_c$ decreases. However, as expected, the influence of the cut-off frequency is much greater on $\langle(\partial\theta/\partial x)^2\rangle$ than on $\langle\theta^2\rangle$. For example, when $f_c$ is reduced from $1.0f_B$ to $0.05f_B$, $\langle(\partial\theta/\partial x)^2\rangle$ decreases by 93%, compared with a decrease of 13% in $\langle\theta^2\rangle$. Assuming the vadility of local isotropy, the mean squares of all scalar derivative components, and then the mean dissipation rate, obtained by a probe with $\lambda_r = 10\lambda_B$ (i.e. $f_c = 0.1f_B$), are deduced to be underestimated from the true values by about 80%.

This deduction agrees remarkably well with the data of Southerland and Dahm [20] who used a 3-D LIF technique to measure the scalar dissipation in the far field ($x \approx 235d$ and $y \approx 26d$) of a round water to water jet with $Sc \approx 2000$. The fraction of the total scalar dissipation rate ($\equiv D[\langle(\partial\theta/\partial x)^2\rangle+\langle(\partial\theta/\partial y)^2\rangle+\langle(\partial\theta/\partial z)^2\rangle]$) at different values of $\lambda_r$ was reported in their Figure 3.12. For comparison, we have reproduced their data in Figure 2, where their $\lambda_B$ was estimated using Eq. (6) with $C_1 = 2.4$ (note that they used $C_1 = 11.2$ for $\lambda_D$; that is, their $\lambda_D = 4.67\lambda_B$). Surprisingly, as demonstrated in Figure 2, their data points, particularly for the case of R0806 ($Re_L$ = 5000 and $\lambda_D$ = 0.209 mm), nearly perfectly follow the present curve for the ratio $\langle(\partial\theta/\partial x)^2\rangle(f_c)/\langle(\partial\theta/\partial x)^2\rangle(f_B)$ at $\lambda_r/\lambda_B > 6$. It is shown, for example, that only 10% of the true dissipation can be detected by their technique when using a spatial resolution of $\lambda_r = (12\text{~}13)\lambda_B$. This result should also apply when using other meansurement techniques.

Southerland and Dahm [20] employed a measurement volume dimension typically of $\lambda_D/3$, where $\lambda_D \approx 11.2\mathrm{Re}_L^{-3/4}\mathrm{Sc}^{-1/2}L$, for their final measurements of the dissipation. It is interesting to note that they claimed that "all cases considered here are fully resolved in all three spatial dimensions" because of $\lambda_r < \lambda_D$. However, the results shown in Figure 2 and their Figure 3.12 suggest that with their resolution $\lambda_r = \lambda_D/3$ (= $1.56\lambda_B$) the total dissipation is underestimated by at least 15%. Their data also indicate that, to resolve 98% of the total scalar dissipation, their $\lambda_r$ would be required to be reduced to one fifth of the actual value. That is, Figure 2 suggests that their measurements do not in fact truly resolve the smallest scalar scales, which are the Batchelor scales.

The present results shown in Figure 2 also gain support from previous measurements of $\langle(\partial\theta/\partial p)^2\rangle$ [$p \equiv x$, $y$ or $z$] made by the first author [16] using two parallel cold-wires (0.63 μm) in a circular jet. Figure 3 shows the effect of the wire separation $\lambda_r = \Delta p$ on the directly measured $\langle(\partial\theta/\partial p)^2\rangle$ obtained on the jet axis at $x/d$ = 30, where the flow is fully developed [2]. Obviously, the decreasing trend of $\langle(\partial\theta/\partial p)^2\rangle$ with $\lambda_r$ is similar to that of $\langle(\partial\theta/\partial x)^2\rangle$ against $f_c$. At $\lambda_r = 10\lambda_B$, all three components of $\langle(\partial\theta/\partial p)^2\rangle$ appear to be underestimated by around 75%, which is close to the decrease of 80% as for $f_c = 0.1f_B$.

Furthermore, as deduced from Figure 2, a measurement probe with a very poor resolution can also lead to the mean square of scalar fluctuations to be considerably underestimated. For instance, when $\lambda_r = 30\lambda_B$ (or $f_c = f_B/30$), the mean square $\langle\theta^2\rangle$ is underestimated by 20%; i.e., the measured root mean square $\langle\theta^2\rangle^{0.5}$ is about 10% less than its true value. This calls for caution on those researchers who investigate the high-$Sc$ flows where $\lambda_B$ is extremely small (and may be smaller than is possible to resolve in many cases).

## Conclusions

Statistical dependence of the measured scalar fluctuation and scalar dissipation rate on the measurement resolution has been investigated in the far field of a turbulent circular jet. We have estimated the influence of spatial resolution on both the measured mean square of a fluctuating scalar and the measured mean rate of scalar dissipation by using a spectral method and filtering the signals at different cut-off frequencies. The results obtained from this method agree well with direct measurements obtained by Southerland and Dahm [20] and by Mi [16]. As the probe spatial resolution becomes poorer or the Batchelor scale decreases across the flow of investigation, the measured fraction of the total scalar dissipation rate is reduced rapidly while, by comparison, the mean square of the scalar fluctuation is reduced only slightly (cf. Figure 2). That is, the measurement of scalar dissipation rate is much more sensitive to spatial resolution than is the measurement of scalar fluctuations. It has been found that the Batchelor scale $\lambda_B$ as the smallest length scale of the scalar fluctuation must be resolved for accurate measurements of the mean scalar dissipation rate. By comparison, however, the resolution requirement for accurate measurements of the scalar fluctuations is much less stringent. The resulting error in the root mean square of the measured scalar fluctuations should be small (< 6%) providing the fluctuation scales $\geq 20\lambda_B$ can be resolved by the measurement probe.


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