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Stability of Taylor-Couette Flow with Axial Flow.

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Abstract

This paper describes an investigation, which uses a torque-measurement technique, of effects of axial flow on the stability of Taylor-Couette flow. Two radius ratios, 0.846 and 0.956, are used. The preliminary results show that the critical Taylor number for the onset of Taylor vortices increases monotonically with the Reynolds number of the axial flow and asymptotes to a constant value. Although our findings are in good agreement with previously published results, which were obtained with either flow visualization or pressure measurements, there are some noticeable differences with earlier studies.

Introduction

Flow between two rotating concentric cylinders, also known as Taylor-Couette flow, has long been the subject of theoretical and experimental investigation. These studies originated with the work of Couette [6] and Mallock [12], but Taylor [17] made the most significant advances in describing this flow in his 1923 classical paper "Stability of a viscous liquid between two rotating concentric cylinders". Since then, hundreds of papers on the subject have been published with most studies utilising a rotating inner cylinder and a stationary outer cylinder. Here, as the rotational speed of the inner cylinder is slowly increased from rest, the laminar axisymmetric flow (also known as Couette flow) becomes unstable. This leads to a series of flow transitions with the following flow modes: Circular Couette Flow (CCF), Taylor-Vortex Flow (TVF), Wavy Vortex Flow (WVF), Modulated Wavy Vortex Flow (MWVF) and Turbulent Taylor Vortex Flow (TTVF).

Imposing an axial flow on a Taylor-Couette flow has been shown to delay the transition to TVF, thus increasing the stability of the flow. Cornish [5] was the first to study this phenomenon. He determined the critical angular velocity of the inner cylinder for the onset of TVF for different axial flow rates. Also, he found that the beginning of TVF is accompanied by a sudden increase in the axial pressure drop across the annular region between the inner and outer cylinders.

The same phenomenon was analytically investigated by Goldstein [9] for axial Reynolds numbers of $0 \le Re_x \le 25$. In contrast to the results of [5], he found that for $0 \le Re_x \le 15$, the axial flow has a stabilizing effect, but for $15 < Re_x \le 25$ it has a destabilizing effect.

Many of the discrepancies between the early works of the 1930s were clarified by studies conducted in the 1960s (see Chandrasekhar [3], Di Prima [7], Krueger and Di Prima [10] and Sparrow, Munro and Jonsson [15]).

Chandrasekhar [3] published a monumental paper covering a more complete stability theory for the Couette-Poiseuille flow. He reported that an axial flow tends to stabilize rotational flow and, in the limit of zero viscosity, that the Rayleigh stability criterion is valid. In the same year, Di Prima [7] published his work on theoretical stability predictions, which give results similar to those of Chandrasekhar [3]. Di Prima's predictions were experimentally verified by Donnelly and Fultz [8] who showed that the critical Taylor number, Ta_{cr}, increases monotonically with axial Reynolds number (see also Snyder [14] and Schwarz, Springett and Donnelly [13]).

More recent work on this subject includes that of Chung & Astill [4], Takeuchi and Jankowski [16], Lueptow et al. [11], Buhler and Polifke [2] and Andereck, Lui and Swinney [1].

Most of the experimental investigations to date have used either flow visualization techniques and/or pressure measurements to determine the onset of transition. However, determining quantitative results from flow visualization can be quite subjective. This has prompted us to carry out the present investigation using a torque measurement technique.

Experimental Apparatus and Procedure

The experiments were conducted using a Haake RS75 rheometer, which is located in the Fluid Mechanics Laboratory at the National University of Singapore. The apparatus consists of a rotating inner cylinder with an outer radius of R_1 and a stationary outer precision, perspex cylinder with an inner radius of R_2 (see Figure 1). The inner cylinder is attached to an air bearing, which forms part of the rheometer. The rheometer, and hence the inner cylinder, can rotate up to a maximum speed of 500 rpm. In order to minimise the flow disturbances, the end of the inner cylinder and the exit of the perspex container are conically tapered at an angle of 140^o. A perspex lid is used to prevent spillage during the experiments (see Figure 2). To examine the dependence of the flow on the radius ratio, we used two different inner cylinders and the same outer cylinder. The detailed dimensions of the cylinders are given in Table 1. The kinematic viscosity (ν) of the working fluid, which is a glycerine/water mixture, was determined at the start of each experiment using the rheometer.

The axial flow rates are controlled with a specially calibrated DC pump. To minimise disturbances, the fluid flowing from the pump is diverted through four small tubes and an entry reservoir before it enters the test section.

The two relevant parameters in this study are the Taylor number (Ta)[7] and the axial Reynolds number (Re_x),

$$Ta = \frac{2\eta^2 d^4 \omega^2}{\nu^2 (1-\eta^2)} \qquad (1a)$$

$$Re_x = \frac{V_m d}{\nu} \qquad (1b)$$

where η is the radius ratio, d is the gap size (see Table 1), ω is the angular velocity of the inner cylinder and V_m is the mean axial velocity.

Inner cylinder radius R_1	30.15mm	34.07mm
Outer cylinder radius R_2	35.64mm	35.64mm
Gap width $d = R_2 - R_1$	5.49mm	1.57mm
Radius ratio $\eta = R_1/R_2$	0.846	0.956

Table 1: Summary of two sets of Cylinders

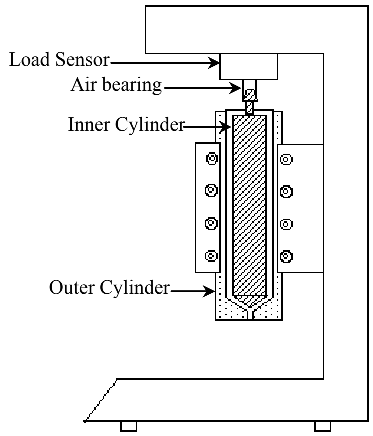

Figure 1: Modified Haake RS75 rheometer.

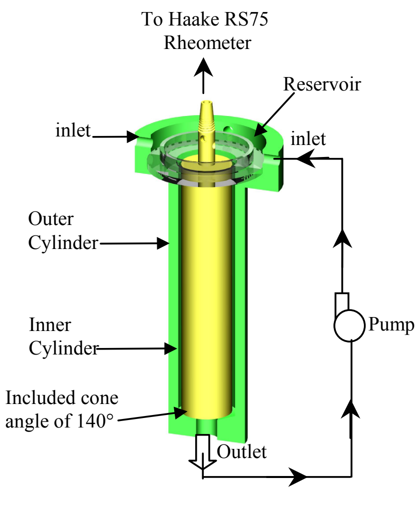

Figure 2: Drawing of the set-up.

Results and Discussion

In this investigation, the transition from circular Couette flow to Taylor vortex flow was determined by a sudden increase in the torque. Figure 3 shows a typical set of measurements, which are recorded with the rheometer as the speed of the inner cylinder is increased. The data show that the torque increases linearly in each flow regime. Thus, a curve-fit of the data yields two lines, and the intersection of the two lines gives the transition Re_x. It is worth noting that the inner cylinder is accelerated in a quasi-steady manner to reach the final flow condition. For example, the non-dimensional time ($t^*=t\nu/d^2$) taken to reach the onset of the Taylor vortices is approximately $t^*=100$.

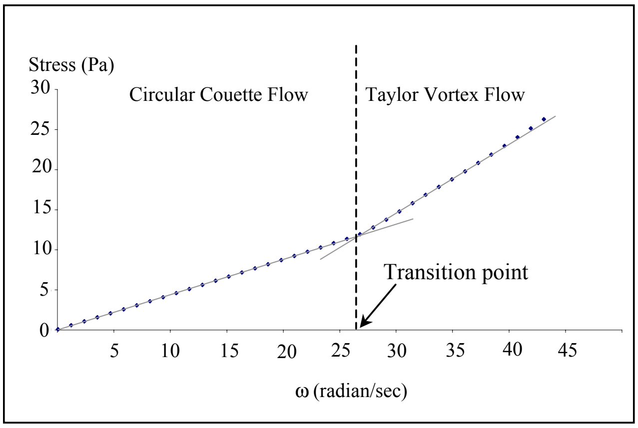

Figure 3: A typical torque-rotational speed curve recorded by rheometer.

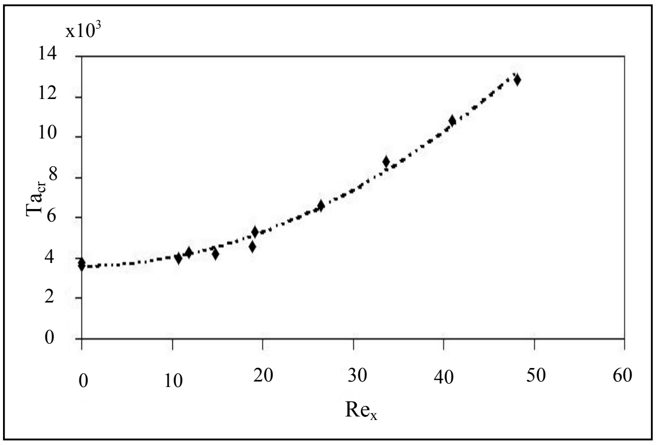

Figure 4: Plot of critical Taylor number versus axial Reynolds number for η = 0.956.

In Figure 4, a plot of the critical Taylor number versus the axial Reynolds number is displayed for η = 0.956. To check the accuracy of our results, the critical Taylor number at $Re_x = 0$ (no axial flow) is compared with published results for the same radius ratios - the agreement is good. In Figure 4, one can see that as the axial Reynolds number increases, the critical Taylor number also increases. At small axial Reynolds numbers $0 \le Re_x \le 20$, the delay in transition is gradual. However, as Re_x is increased beyond 20, the delay is more marked and distinct. Due to the limited capacity of the pump, the maximum axial Reynolds number that can be achieved is about $Re_x \approx 50$. For $Re_x \approx 45$, the corresponding critical Taylor number is about 3.2 times higher than without axial flow (at $Re_x = 0$).

Figure 5 shows the delay in the transition from circular Couette flow to Taylor vortex flow, which is due to the presence of the axial flow. Note that the maximum Re_x that can be achieved in this case using the same pump is substantially higher than that for the case of the high radius ratio (ie. small gap). A close examination of the figure reveals that for $0 \le Re_x \le 60$, the results follow the same trend as when η = 0.956. However, at higher Re_x, the increasing trend gradually flattens out, reaching an asymptotic value, which is near the critical Taylor number, Ta_c = 15000. Similar observation has also been made by Takeuchi & Jankowski [16] for η=0.5 (see Figure 6) using flow visualisation technique. Note that the definition of Taylor number used by Takeuchi & Jankowski is slightly different from the present study. For the purpose of comparison, our results have been re-analysed using their definition and displayed as shown in figure 6. Although both results display a similar trend, there is an obvious different between the two curves depicted in figure 6. For example, beyond the axial Reynolds number of 50, our result consistently shows a higher value of Ta/Tac, which indicates a more stable flow. The difference in the results could be attributed to the different radius ratio and aspect ratio used in the two studies. Takeuchi & Jankowski attributed the presence of the asymptotic value of Ta to the state of the axial flow. They point out that even though the critical Taylor number increases monotonically with Re_x, such a stabilising effect cannot continue indefinitely because for large Re_x, the viscous mechanism will lead to instability, even without rotation.

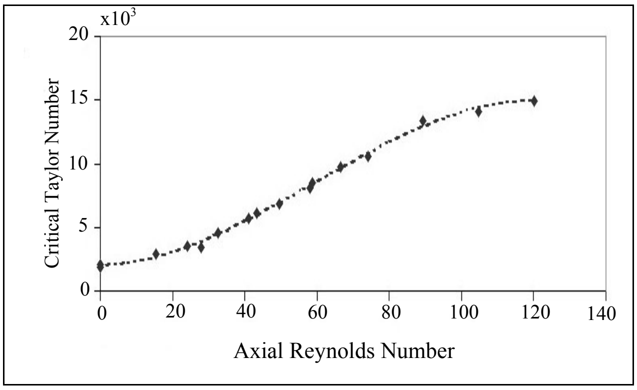

Figure 5: Plot of critical Reynolds number for η = 0.846.

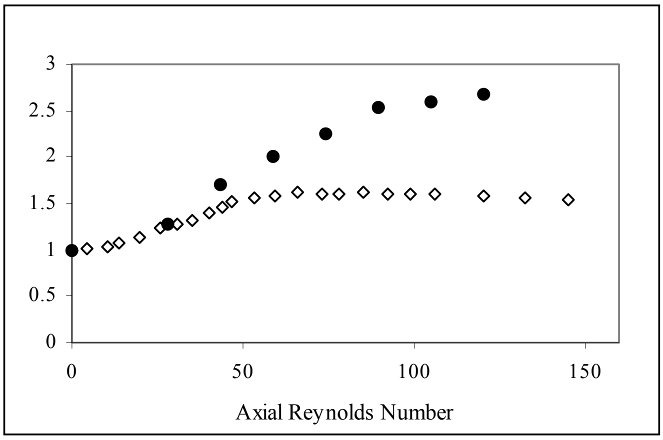

Figure 6: Experimental results of [16] for η = 0.5. Here Ta = $\omega(b-a)^2/\nu$, where b and a are the radii of the outer and inner cylinders, respectively. Note that Ta_c refers to the critical Taylor number at Rex=0. ◇: Takeuchi & Jankowski; ●: Present results, η = 0.846.

Returning to our results (Figure 5) note that at Re_x = 45, the critical Taylor number is approximately 3.39 times higher than that at Re_x = 0. Compare this with the corresponding Reynolds number for η = 0.956, which is slightly lower at 3.20. The difference is small indicating that the effect of the radius ratio may not be that strong.

In figure 7, our results are compared with those published by Snyder [14] and Schwarz, Springett and Donnelly [13] for a similar radius ratio of η = 0.956.

In general, our results are in good agreement with those of [13] and [14], although they are slightly higher. In all cases, they show a monotonic increase in the critical Taylor number as Re_x increases.

Conclusions

An experimental examination of the effects of axial flow on the stability of the Taylor-Couette flow has been performed using a torque measurement technique. To the best of our knowledge, this is probably the first time that torque measurement has been conducted on a problem of this nature. Our preliminary results show that the critical Taylor number increases monotonically with the axial Reynolds number. This finding is consistent with previous studies, which use either flow visualization or pressure measurements to determine the onset of transition. In addition, we have found that beyond a threshold axial Reynolds number the critical Taylor number asymptotes to a constant value. The work is continuing.

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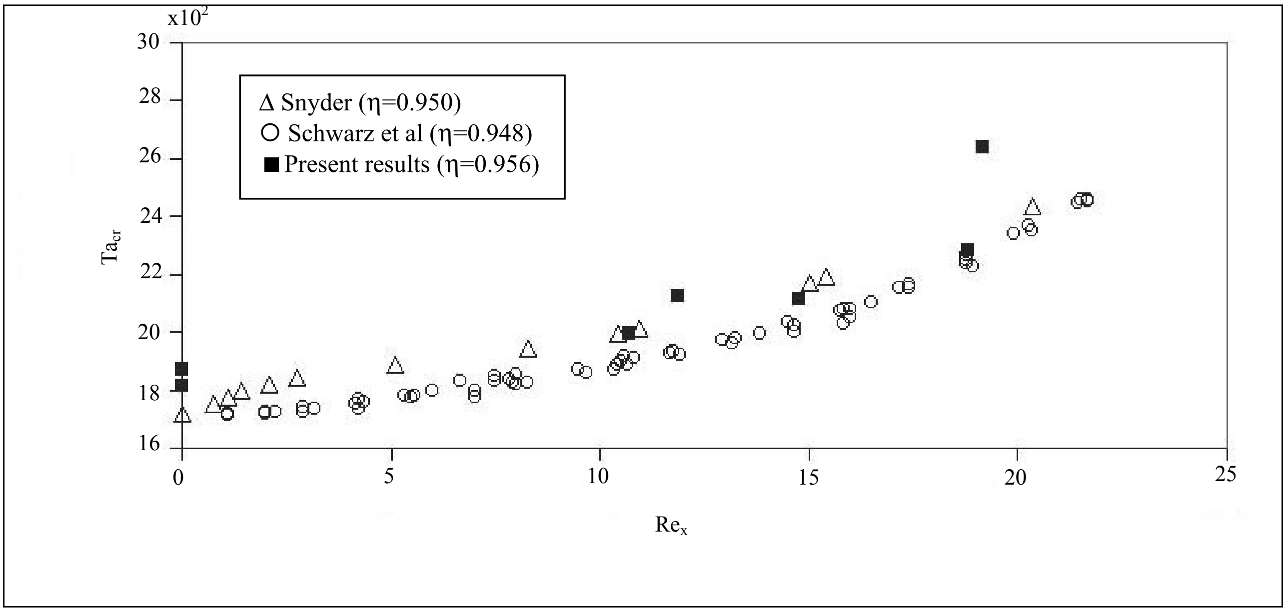

Figure 7: Comparison with other experimental work. The results of Snyder [14] for inner cylinder rotating and outer stationary, with η = 0.948 and Schwarz, Springett & Donnelly [13] with η = 0.945 are denoted by Δ's and O's, respectively. Data from the present study, for η = 0.956, are denoted by ■. Note that the results for η = 0.846 are not presented because of the different radius ratio.