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Transient Mixing Events in Stably Stratified Turbulence

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Abstract

Mixing efficiency in stratified flows is the proportion of turbulent kinetic energy that goes into increasing the potential energy of the fluid by irreversible mixing. In this research direct numerical simulations (DNS) of transient, homogeneous, stably-stratified turbulence are used to determine the mixing efficiency as a function of the stratification. The DNS results are compared with data from grid experiments. There is qualitative agreement between the DNS results and the experimental data, but significant quantitative discrepancies. The grid turbulence experiments suggest a maximum mixing efficiency of about 6%, while DNS gives about 30%. We conclude that the main source of the disagreement is the inaccuracy in determining the initial turbulence energy input in the grid turbulence experiments.

Introduction

Mixing efficiency in stratified flows is the proportion of turbulent kinetic energy that goes into increasing the potential energy of the fluid by irreversible mixing. The experiments of Britter [1], Rottman & Britter [7], and Rehmann & Koseff [8] measured the mixing efficiency of grid turbulence in an initially uniformly stratified fluid. The experiments were performed by towing a bi-planar grid horizontally through water that had been stratified using salt or heat. The mixing efficiency in these experiments is measured as a function of the Richardson number $\mathrm{Ri} = (NM/U)^2$, in which N is the initial buoyancy frequency of the fluid, M is the grid mesh length, and U is the towing speed. The mixing efficiency (or the flux Richardson number R_f) is obtained by measuring the change in the potential energy of the fluid as a proportion of the work done by towing the grid through the tank. It was assumed that all of this work was transferred into turbulent kinetic energy.

The results of these experiments are all consistent. For $\mathrm{Ri} < 1$ the mixing efficiency increases approximately as $\mathrm{Ri}^{1/2}$. For larger Ri the mixing efficiency appears to approach a constant value of about 6%. These results do not appear to depend strongly on the molecular diffusivity.

There are two issues arising from these results. First, since none of the laboratory experiments could achieve values of Ri much greater than about 10, it is not conclusive whether the mixing efficiency remains constant for larger Ri. Some different types of experiments (see e.g. Linden, [4]) have suggested that R_f should decrease for sufficiently large Ri. Secondly, the maximum mixing efficiency of 6% is lower than has been measured in other types of mixing experiments where values of 10% – 20% are typical, see e.g. Linden [3] and Park, *et al.*[5].

In the research described here, direct numerical simulations (DNS) of decaying, homogeneous, stably-stratified turbulence are used to address these two issues.

Numerical Experiments

The numerical experiments described here were carried out with the pseudo-spectral DNS code used by Riley, Metcalfe & Weissman [6] and is described in detail by those authors. This code simulates a flow field with periodic boundary conditions in all three spatial directions, and with a uniform background density gradient.

The resolution of the simulations varied from 32^3 to 128^3. The simulations were carried out for values of the Reynolds number, $\mathrm{Re} = u_0 L_0/\nu$, where u_0 and L_0 are the initial turbulence velocity and (integral) length scales and ν is the kinematic viscosity, in the range $100 < \mathrm{Re} < 400$. Most of the simulations reported here were carried out for the Prandtl number $\mathrm{Pr} = 0.5$. A limited investigation of Prandtl number effects was carried out by varying the Prandtl number in the range $0.1 < \mathrm{Pr} < 2$. These values of Pr are substantially smaller than those for the laboratory experiments.

Initialization

For all our numerical experiments, the turbulent flow field was initialized as a Gaussian, isotropic and solenoidal velocity field in the usual way using random Fourier modes with a specified energy spectrum. The turbulence was then allowed to decay until approximately 99% of the initial turbulence energy had dissipated. Decay times of about ten times the initial time scale of the turbulence L_0/u_0 were typically required. The stable stratification for each transient experiment was specified by the initial Richardson number, which was varied in the range $0 < \mathrm{Ri} < 1000$. The Richardson number for these simulations is defined as $\mathrm{Ri} = (NL_0/u_0)^2$.

There is some arbitrariness in relating the different definitions of Ri used in the DNS and laboratory experiments. This relationship could be established using existing grid turbulence data at, say, $10M$ downstream from the grid. However, since our interest initially is in the trends of R_f and its maximum magnitude, we will assume the two definitions of Ri are approximately equal.

The energy spectrum of the initial turbulence was chosen to have the exponential form (e.g. Townsend, [9]) :

$$E(k) = C u_0^2 L_0^5 k^4 exp[-\frac{1}{2} k^2 L_0^2] \qquad (1)$$

where C is a constant scaling factor.

The initial energy was exclusively kinetic in form *i.e.*, the initial density fluctuations (and hence initial turbulent potential energy $PE_0 = -\frac{1}{2}\frac{g}{\rho_0}\left(\frac{\partial\overline{\rho}}{\partial z}\right)^{-1}\overline{\rho^2}$) were set to zero. Since one of our main objectives is to compare the simulation results with experimental measurements, the appropriate representation of the initial conditions is necessary. Experimental measurements downstream of grid generated turbulence in stratified fluids suggest that the turbulent potential energy close to the grid (say at $x/M = 10$) is only a small fraction of the turbulent kinetic energy, typically less than 10% ([2]). On this basis it seems that the above initialization scheme is reasonable.

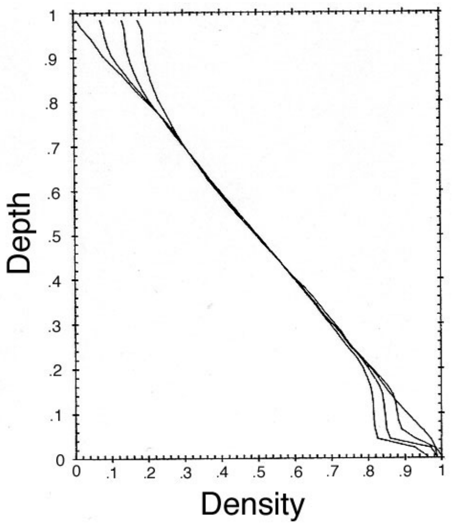

Figure 1: The evolution of the vertical distribution of density after $n = 0$, 10, 28 and 48 grid tows, for the case with $\mathrm{Ri} = 0.41$, from Rottman & Britter [7]). The vertical coordinate is scaled by the depth of the water in the tank and the density is scaled such that initially it is zero at the top of the tank and unity at the bottom.

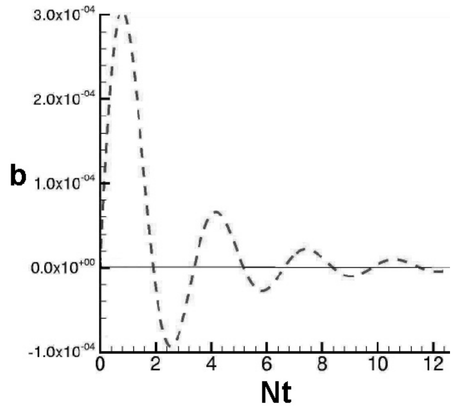

Figure 2: The buoyancy flux (with arbitrary non-dimensionalization) **b** as a function of time (non-dimensionalized by the buoyancy frequency N) computed by DNS for the case with $\mathrm{Ri} = 10$ and $\mathrm{Pr} = 0.5$.

Comparison with Experiments

For the numerical simulations, the turbulence remains homogeneous (but not isotropic) for all times. This includes the buoyancy fluxes and dissipation rates. This implies that in this idealized problem the mean fields are decoupled from the turbulence, so that there is no change in the mean density field and hence no change in the background potential energy. For the experiments, the turbulence is homogeneous in the centre of the tank, but non-homogeneous near the upper and lower boundaries. In the centre of the tank the turbulent density flux is homogeneous, but near the boundaries the gradients of turbulent density flux produce changes to the background mean density profile. These changes in the mean density profile represent an increase in the centre of mass of the fluid column and hence an increase in the background potential energy. An example of the evolution of the mean vertical density profile in the experiments is shown in figure 1. Note in this figure how uniform the mean density gradient remains in the centre of the tank.

It can be shown from the evolution equation for the potential energy that the increase in the potential energy of the fluid column in the experiments is equal to the time integrated buoyancy flux through the homogeneous region in the centre of the tank. The key to comparing the simulations with the experiments is to use the time-integrated buoyancy flux from the homogeneous simulations to represent the change in potential energy in the experiments. Therefore, in the homogeneous simulations the integral flux Richardson number R_f is computed as the time integral of the buoyancy flux divided by the change in turbulent kinetic energy over the same time period.

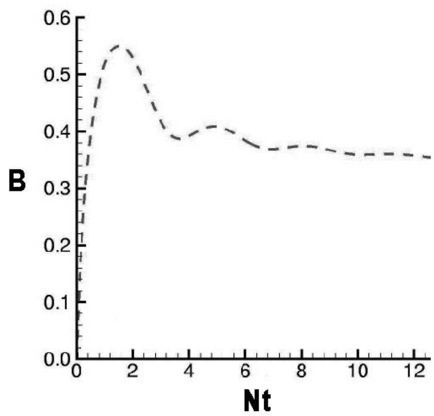

Figure 3: The evolution of the time-integrated buoyancy flux (non-dimensionalized with the initial turbulent kinetic energy) **B** as a function of time (non-dimensionalized by the buoyancy frequency N) computed by DNS for the case with $\mathrm{Ri} = 10$ and $\mathrm{Pr} = 0.5$.

Results

Figure 2 shows a typical evolution of the buoyancy flux in the simulations and figure 3 shows the evolution of the time integral of the buoyancy flux. We note that the buoyancy flux starts at zero and reaches a maximum value at $Nt \approx 1$, reduces to zero at

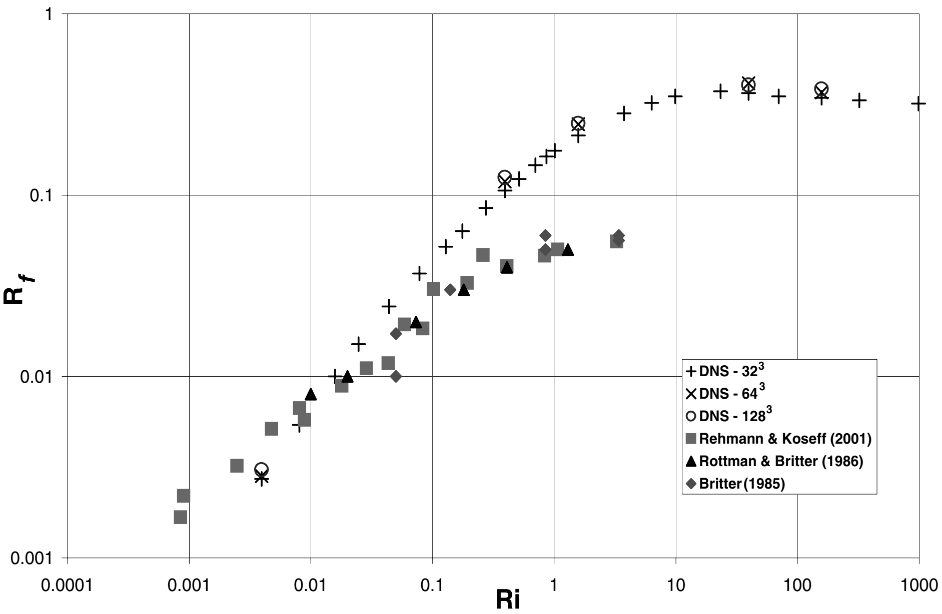

Figure 4: The mixing efficiency R_f plotted versus Ri for both experiments and DNS. ▲, experimental data from Rottman & Britter [7]; ■, experimental data from Rehmann & Koseff [8]; ⧫, experimental data from Britter [1]; $+$, DNS at 32^3 resolution; $\times$, DNS at 64^3 resolution; ◯, DNS at 128^3 resolutions. For the experiments, $\mathrm{Pr} \approx 7$ or 700, and for the DNS, $\mathrm{Pr} = 0.5$.

$Nt \approx 2$, after which it oscillates about zero with decaying amplitude and a period of approximately half the buoyancy period $2\pi/N$. The integral of the buoyancy flux attains its maximum at $Nt \approx 2$, before decreasing and asymptoting to a constant positive value after approximately one buoyancy period. This value, scaled with the initial turbulent kinetic energy, defines the mixing efficiency R_f. Since $\mathrm{Ri} = (NL_0/u_0)^2$, we note that most of the interesting dynamics, and in particular the contributions to integrated buoyancy flux, are completed by about three initial time scales L_0/u_0 of the turbulence.

Figure 4 is a plot of R_f versus Ri for a selection of the DNS and laboratory results. The results from several sets of numerical simulations at different resolutions are shown in the plot. The DNS results are for $\mathrm{Pr} = 0.5$.

The DNS results compare well qualitatively with the experimental data. The mixing efficiency initially increases with stratification but approaches a constant value of about 30% at strong stratifications. The range of Richardson numbers for the DNS is large enough to be conclusive regarding this limit. The asymptotic value is about five times larger than the maximum values measured in the experiments.

Discussion

The remaining question that arises from this study concerns the discrepancy between the experimentally measured mixing efficiencies and the DNS results. In this section we consider possible reasons for this discrepancy.

Prandtl Number Effects

The grid turbulence experiments of Rottman & Britter [7] and Rehmann & Koseff [8] cover a range of Prandtl (or Schmidt) numbers $7 < \mathrm{Pr} < 700$ using heat and salt stratification. Measurements of the mixing efficiency for different values of the Prandtl number agree to within the experimental uncertainty. The numerical simulations, for $0.1 < \mathrm{Pr} < 2$, also show only small Prandtl number effects in the regime $\mathrm{Ri} < 10$. However, simulations of strongly stable cases, $\mathrm{Ri} > 10$, suggest that Prandtl number effects increase with stratification. In particular, the mixing efficiency decreases with increasing Prandtl number in this regime. It is uncertain whether these molecular effects are associated with the low Reynolds numbers of the simulations. Since the experiments were all in the regime $\mathrm{Ri} < 10$, we conclude that molecular processes cannot account for the observed difference in mixing efficiency between the experiments and the numerical simulations.

Initial Conditions

An alternative explanation for the low mixing efficiencies obtained in the experiments concerns the validity of the assumption that all the work done in towing the grid goes into turbulent kinetic energy which is then available for mixing the fluid. The possibility that some of this work goes into internal or surface wave energy is ignored despite observations reported by Rottman & Britter [7] of significant surface disturbances.

The average energy per unit volume, w, input into the water by towing the grid through it is

$$w = C_D \frac{1}{2} \bar{\rho} U^2, \tag{2}$$

in which C_D is the drag coefficient of the grid, $\bar{\rho}$ is the mean density of the water in the tank and U is the tow speed. The ratio of the energy input in towing the grid to the turbulent kinetic energy may be expressed as

$$w / \frac{1}{2} \bar{\rho} q_0^2 = C_D U^2 / q_0^2, \tag{3}$$

where q_0 is the turbulent velocity magnitude with the subscript 0 indicating that the location is just downstream of the grid. In the analysis of the experiments it was assumed that this ratio is unity. Drag coefficients were reported as $C_D \approx 1$. Values of $q_0/U \approx 0.45$ would increase the experimentally determined mixing efficiencies by a factor of five, which would agree with DNS results. Direct turbulence measurements were not made in these experiments but other grid turbulence measurements typically yield $q/U \approx 0.1$ at $x/M = 10$. It therefore seems reasonable to conclude that only a fraction of the towing work is actually emerging in the form of turbulent kinetic energy downstream of the grid.

We speculate that small amplitude free-surface standing waves, or ”sloshing” modes, are initiated by the grid tows and that the energy in these waves can account for the remaining energy. Linear theory gives an estimate of the energy E_s in the fundamental “sloshing” mode (which has a wavelength equal to twice the length of the tank) as

$$E_s = (1/4) \bar{\rho} g a^2 L d \tag{4}$$

where g is the acceleration due to gravity, a is the wave amplitude, L is the length of the tank, and d is the width of the tank. As a proportion of the work done in towing the grid the energy in the sloshing mode may be expressed as

$$E_s/W = \frac{1}{2} \frac{1}{C_D} \left(\frac{gh}{U^2} \right) (a/h)^2, \tag{5}$$

where $W = w d h L$ is the total work done in towing the grid one length of the tank and h is the depth of the water in the tank. For typical values of gh/U^2 a sloshing amplitude of less than 5% of the water depth could account for all of the towing work.

Conclusions

We have used DNS to study mixing efficiency in decaying, homogeneous, stably-stratified turbulence, and compared the results with grid turbulence experiments. The DNS estimates of mixing efficiency have the same qualitative behaviour with stratification as seen in the experiments. That is, the mixing efficiency increases with stratification for small Ri and approaches a constant value at large values of Ri. The DNS results confirm the tentative conclusion based on experiments that the mixing efficiency remains constant and does not decrease for strong stratification. However, the DNS estimate for the magnitude of the maximum mixing efficiency (30%) is much larger than the experimental value (6%). We conclude that the discrepancy is due to the incorrect assumption in the experiments that all the work done in towing the grid goes into the turbulent kinetic energy. We suggest that most of the energy goes into wave motions.

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