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Fluid Flow through a Channel with Porous Wall under a Transverse Magnetic Field

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Abstract

The problem of steady flow of an electrically conducting incompressible newtonian fluid through a channel with a porous wall has been solved by using perturbation method. The behavior of the velocity profile for different values of Hartmann number and slip coefficient is shown graphically.

Introduction

Parallel plate membrane modules consisting of a channel bounded by two porous walls are used for filtration. Berman [2] first derived approximate expressions for fluid velocity components for flow in such a module assuming a sufficiently small Reynolds number and no slip at the porous walls. Beavers and Joseph [1] proved the existence of a non-zero tangential velocity on the surface of a permeable boundary. Singh and Lawrence [7] investigated the fluid mechanics in parallel plate filtration system assuming equally porous boundaries. The problem of steady flow of an electrically conducting incompressible non-newtonian second order fluid through a channel with porous walls has been studied by Sharma and Singh [6]. Recent works include Kuiry [4] where the steady behavior of a generalised porous wall "Couette Type " flow in the presence of a magnetic field has been discussed. Most laboratory scale parallel plate filters are asymmetric having only one porous wall, the other wall being solid. Such a geometry is convenient for conducting experiments, for permeate collection etc. A geometry of this kind is applicable in the case where membrane filtration helps to separate plasma and cellular components from the whole blood. Recently Chellam et al. [5] has derived an approximate solution for the two dimensional Navier-Stokes equations for steady, laminar flow in a channel bounded by one porous wall subject to uniform suction. The flow of an electrically conducting fluid between porous boundaries is of practical interest in problems of gaseous diffusion etc. Hence an attempt is made to study the effect of magnetic field on the flow problem discussed by [5]. As this requires a knowledge of the details of the flow, as a first step the velocity both in the axial and transverse directions are plotted for different physical parameters.

Mathematical Formulation

Consider the steady flow of an incompressible fluid in a channel with one porous wall as shown in figure 1.

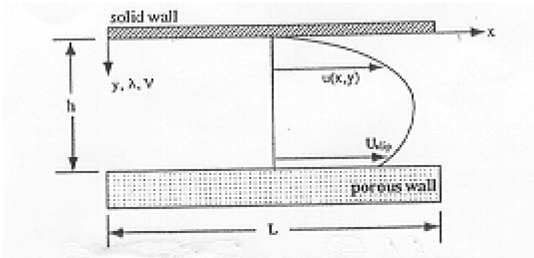

Figure 1. Schematic diagram of the channel

Let L and h denote the length and height of the channel respectively. A constant magnetic field B_O is applied normal to the axis of the channel. The flow is modeled as a two-dimensional one assuming that the width of the channel is very large compared to its height. Let p and ν denote the fluid pressure and kinematic viscosity respectively. If the induced magnetic field due to the flow is negligible and the velocity in the axial and transverse directions are u and v respectively then the simplified differential equations of motion and continuity governing the flow along with the boundary conditions are given by

$$u\frac{\partial u}{\partial x}+\frac{v}{h}\frac{\partial u}{\partial \lambda}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 u}{\partial \lambda^2}\right)-\frac{B_0{}^2\sigma}{\rho}u \quad (1)$$

$$u\frac{\partial v}{\partial x}+\frac{v}{h}\frac{\partial v}{\partial \lambda}=-\frac{1}{\rho h}\frac{\partial p}{\partial \lambda}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 v}{\partial \lambda^2}\right) \quad (2)$$

$$\frac{\partial u}{\partial x}+\frac{1}{h}\frac{\partial v}{\partial \lambda}=0 \quad (3)$$

$$u=0\,;v=0 \text{ at } \lambda=0$$

$$u=u_{slip}=-\frac{\sqrt{k}}{\alpha h}\frac{\partial u}{\partial \lambda}\,;v=v_w \text{ at } \lambda=1 \quad (4)$$

Following Chellam et al. [5] a stream function ψ can be defined as

$$\psi(x,\lambda)=(hu_0-v_w x)f(\lambda) \quad (5)$$

where u_0 is the entrance velocity, $f(\lambda)$ is an unknown function of the dimensionless distance co-ordinate λ. Now the velocities are given by

$$u = \left(u_0 - \frac{v_w x}{h}\right) f'(\lambda) \quad (6)$$

$$v = v_w f(\lambda) \quad (7)$$

Substituting for u and v from (6) and (7) into (1) and (2) we obtain

$$\left(u_0 - v_w \frac{x}{h}\right)\left[-\frac{v_w}{h}\left(f'^2 - ff''\right) - \frac{\nu}{h^2} f''' + \frac{B_0{}^2\sigma}{\rho} f' \right] = -\frac{1}{\rho}\frac{\partial p}{\partial x} \quad (8)$$

$$\frac{v_w{}^2}{h} ff' - \frac{\nu v_w}{h^2} f'' = -\frac{1}{\rho h}\frac{\partial p}{\partial \lambda} \quad (9)$$

Assuming the pressure is twice differentiable we can differentiate (9) with respect to x to get

$$\frac{\partial^2 p}{\partial x \partial \lambda} = 0 \quad (10)$$

Differentiating (8) with respect to λ and on using (10) we arrive at

$$\frac{\partial}{\partial \lambda}\left[\frac{v_w}{h}\left(f'^2 - ff''\right) + \frac{\nu}{h^2} f''' - \frac{B_0{}^2\sigma}{\rho} f' \right] = 0 \quad (11)$$

Integration of (11) gives rise to the following ordinary differential equation

$$\mathrm{Re}_w\left[f'^2 - ff''\right] + f''' - M^2 f' = K \quad (12)$$

where Re_w is the wall Reynolds number $\left(= v_w h/\nu\right)$, M^2 the square of Harmann number $\left(= h^2 B_0^2 \sigma/\mu\right)$ and K the constant of integration.

Perturbation Solution

The third order non-linear ordinary differential equation (12) is solved by a regular perturbation method. The solution of (12) for small values of Re_w may be expressed in the form of a power series as

$$f(\lambda) = f_0(\lambda) + \mathrm{Re}_w f_1(\lambda) + \mathrm{Re}_w{}^2 f_2(\lambda) + \cdots$$

$$K = K_0 + \mathrm{Re}_w K_1 + \mathrm{Re}_w{}^2 K_2 + \cdots \quad (13)$$

where f_i's and K_i's are independent of Re_w. Substituting (13) into (12) and equating terms of like powers in Re_w leads to the following

$$f_0''' - M^2 f_0' = K_0$$

$$f_1''' + f_0'^2 - f_0 f_0'' - M^2 f_1' = K_1 \quad (14)$$

The boundary conditions now transform into

$$f_i' = 0;\ f_i = 0 \text{ at } \lambda = 0,\ i \ge 0$$

$$f_0' = 0;\ f_i = 0 \text{ at } \lambda = 1,\ i \ge 1$$

$$f_i' = \phi f_i'' \text{ at } \lambda = 1,\quad i \ge 0 \quad (15)$$

where $\phi \left(= \sqrt{k}/\alpha h\right)$ is the slip coefficient. The solutions of equation (14) subject to the boundary conditions (15) are given by

$$f_0(\lambda) = A_1 + B_1 e^{-M\lambda} + C_1 e^{M\lambda} - \frac{\lambda}{M^2} K_0$$

$$\begin{aligned} f_1(\lambda) &= A_2 + B_2 e^{-M\lambda} + C_2 e^{M\lambda} \\ &+ \left(-4B_1C_1 + \frac{A_1B_1}{2} + \frac{A_1C_1}{2} + \frac{K_0{}^2}{M^6} - \frac{K_1}{M^2}\right)\lambda \\ &- K_0 B_1 \left(\frac{\lambda^4}{24} - \frac{\lambda^3}{18M} + \frac{\lambda^2}{4M^2} + \frac{\lambda}{M^3}\right) e^{-M\lambda} \\ &- K_0 C_1 \left(\frac{\lambda^4}{24} + \frac{\lambda^3}{18M} + \frac{\lambda^2}{4M^2} - \frac{\lambda}{M^3}\right) e^{M\lambda} \end{aligned}$$

where $A_i, B_i, C_i, K_i,\ i = 1,2$ are constants that depend only on the slip coefficient. Having determined f_i's the velocity profiles can now be found using the expressions below.

$$u(x,\lambda) = \left(u_0 - \frac{v_w x}{h}\right)\left(f_0' + \mathrm{Re}_w f_1'\right) \quad (16)$$

$$v(\lambda) = v_w\left(f_0 + \mathrm{Re}_w f_1\right) \quad (17)$$

Mid-channel axial and transverse velocities normalised by the local maximum values are calculated and shown graphically. It is observed that in the absence of the magnetic field and slip the results reduce to the one given by Green [3].

Discussions

A perturbation solution of the velocity field is given by equations (16)-(17). Typical velocity profiles are plotted for parametric values of the slip coefficient ϕ = 0, 0.1 and 0.3. Figure 2 is a plot of u versus λ for an entrance and Reynolds number of 1000. The curves are plotted for ϕ = 0.0, wall Reynolds number Re_w = 0.01 and

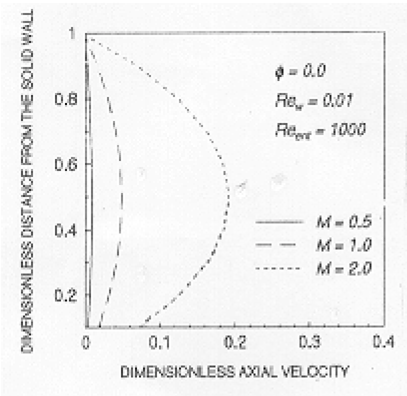

Figure 2. Axial velocity profiles in the absence of slip

Hartmann number M = 0.5, 1.0, 2.0. In the absence of slip as expected the velocity at the membrane surface ($\lambda = 1$) is 0 when ϕ = 0. This is seen in figure 1 which is in excellent agreement with Chellam et al. [5]. Figures 3 – 4 reveal that as slip increases with increasing ϕ , the wall shear decreases resulting in flatter axial

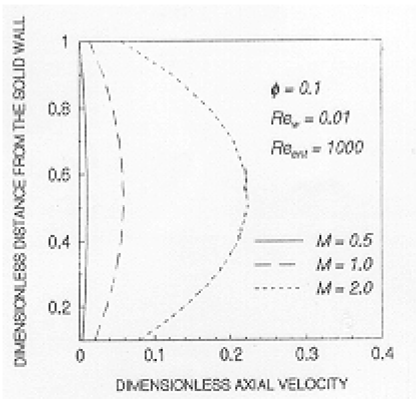

Figure 3. Axial velocity profiles in the presence of slip

velocity profiles. The effect of magnetic field for each specific value

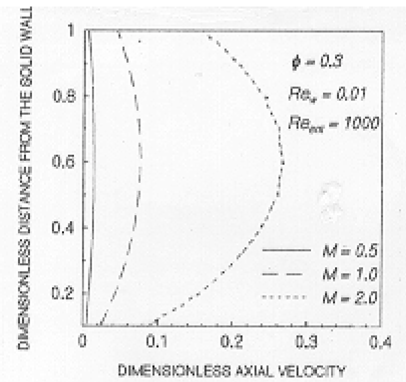

Figure 4. Axial velocity profiles in the presence of slip

of the slip is considered and we observe that this results in further flattening the velocity profiles.

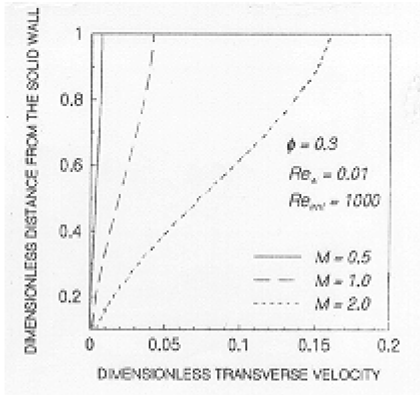

Figure 5. Transverse velocity profiles in the presence of slip

Mid-channel transverse velocity normalised by the local maximum values are shown in figure 5 for the same values of wall Reynolds number and entrance Reynolds number as considered above with the slip coefficient taken to be 0.3. It is observed that for increasing Hartmann number the magnitude of the transverse velocity increases.

Conclusions

The fluid mechanics of a membrane filtration module in the presence of a transverse magnetic field having one porous wall has been investigated using a regular perturbation method. The axial and transverse velocity profiles have been calculated. It has been observed that the effect of magnetic field along with the effect of slip results in flattening the velocity profiles.

Acknowledgements

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